

Radiation Pressure and $E = mc^2$

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Abstract. To show that the equation $E = mc^2$ was already implicit in Maxwell's 1861 paper "*On Physical Lines of Force*" and that it doesn't mean that mass is equivalent to energy, but rather it relates to the propagation of electromagnetic radiation through an elastic solid.

The Speed of Light in an Elastic Solid

I. In Part III of Maxwell's 1861 paper "*On Physical Lines of Force*" [1], he introduces *displacement* (and later displacement current) within the context of a perfect elastic solid. Maxwell applies elasticity theory in order to link the dielectric constant with the transverse elasticity of this solid medium. He is able to utilize the result of an experiment performed in 1855 by German physicists Wilhelm Eduard Weber and Rudolf Kohlrausch [2] in order to equate the speed of light with the ratio of the dielectric constant to the magnetic permeability, and hence also with the ratio of the transverse elasticity to density in this luminiferous medium. Since the latter ratio forms one side of Newton's equation for the speed of a wave in an elastic solid, Maxwell is able to conclude that light is a transverse wave in an elastic solid that is also the cause of electric and magnetic phenomena. Hooke's law appears at equation (105) in Maxwell's 1861 paper in the form,

$$R = - 4\pi k^2 h \tag{1}$$

where R is electromotive force, k is the dielectric constant, and h is displacement. Maxwell doesn't actually use the letter k for dielectric constant but it has been chosen here in order to avoid confusion with the normal symbol for energy which will appear later in the analysis.

The purpose now will be to show how equation (1) can be related to Newton's equation,

$$V^2 = \sigma/\rho \tag{2}$$

where V is the speed of a wave in an elastic solid, σ is the coefficient transverse elasticity, and ρ is the density. Maxwell didn't actually use the symbol σ for

transverse elasticity in this equation (equation (132) in his paper), but it will be used here to avoid confusion with the usual symbol for mass which appears later in the analysis. Maxwell equated the transverse elasticity σ to the dielectric constant k through equation (108) in his paper as per,

$$k^2 = \pi\sigma \quad (3)$$

and he equated density ρ to magnetic permeability μ (which he called the coefficient of magnetic induction) through equation (133) in his paper as per,

$$\mu = \pi\rho \quad (4)$$

Hence combining equations (2), (3), and (4), we obtain,

$$V^2 = k^2/\mu \quad (5)$$

which is equation (135) in Maxwell's paper. Maxwell didn't know the actual density of his elastic solid but he was only concerned with the ratio k^2/μ , and by comparison with the results of the 1855 experiment of Weber and Kohlrausch in which the ratio of electrostatic units of charge to electrodynamic units of charge had been established by the discharging of a Leyden jar (a capacitor) [2], he was then able to establish that the dielectric constant k equated with the speed of light (equation 131 in Maxwell's paper) as measured optically by Fizeau.

The objective now will be to show how equation (5) can be derived from the sea of tiny aethereal vortices described in Part I of Maxwell's paper. In this part, in order to explain the magnetic field and magnetic repulsion, Maxwell utilizes the concept of tiny aethereal molecular vortices that press against each other with centrifugal force while striving to dilate [3].

The Fine-Grained Vortex Nature of the Elastic Solid

II. It is now further proposed that Maxwell's molecular vortices will be dipolar, having both a sink (electron) and a source (positron) in mutual orbit around the edge of the vortex [4], [5], and since Maxwell has all immediately neighbouring vortices spinning in the same direction, the effective speed for the purposes of centrifugal potential energy will be the mutual transverse speed, which will be twice the circumferential speed. Centrifugal potential energy is the same thing as transverse kinetic energy, and summed over the two particles of each dipolar vortex this will be equal to $m(2V)^2$, or $4mV^2$, where $2m$ is the combined mass of the two particles, and where V is their circumferential speed. Mass is considered to be a measure of the amount of aether. The centrifugal potential energy will be

equal to the maximum linear kinetic energy as resolved along a diameter in relation to the projected simple harmonic motion. This in turn will be equal to the maximum potential energy that we obtain from Hooke's law. Since we are dealing with shared elasticity over the two particles within each dipole, this maximum potential energy will be $2\pi k^2 h^2$. Therefore,

$$4mV^2 = 2\pi k^2 h^2 \quad (6)$$

and hence,

$$2mV^2 = \pi k^2 h^2 \quad (7)$$

The centrifugal potential energy, $4mV^2$, is the resultant of an outward centrifugal force and an equal and opposite inward centrifugal force generated by the neighbouring dipoles. As such, if we double the outward centrifugal potential energy we will split the dipole. The input energy needed to split an electron-positron dipole is therefore $2mV^2$. We also know from the 1932 Carl D. Anderson experiment that this energy is the 1.02 MeV associated with gamma radiation, and that it corresponds exactly to $2mc^2$, where c is the speed of light [6]. Hence it follows that the circumferential speed of the electrons and positrons in the dipoles that make up of this elastic solid is equal to the speed of light [7], [8], and that,

$$c^2 = k^2/\mu \quad (8)$$

where μ is the areal density, $2m/\pi h^2$, of an electron-positron dipole. Equation (8) is equivalent to equation (135) in Maxwell's 1861 paper and it is more familiar nowadays in the form,

$$c^2 = 1/\mu\varepsilon \quad (9)$$

where ε is the electric permittivity and where μ is the magnetic permeability. By multiplying the top and bottom lines of equation (9) by area, we end up with,

$$\mathbf{E} = mc^2 \quad (10)$$

where \mathbf{E} is the centrifugal potential energy. It's the compressed orbit syndrome that gets rid of the factor of one half that appears in the standard formula for kinetic energy.

Maxwell never knew the size of his molecular vortices, but it would be reasonable to assume that they are small enough to flow through the interstitial spaces between the atoms and molecules of ponderable matter, as like water flows through a basket. We could assume that the circumference of these dipolar vortices is equal to half of the Compton wavelength for an electron,

since gamma radiation of this wavelength, or lower, can resonate with the dipoles and split them apart, as has just been explained above. This would make their diameter 0.3863 picometres, hence setting them at about one thousandth the size of the average atom. The density of the vortex sea will however be difficult to calculate because the balance between the electrostatic force in the axial direction and the centrifugal force in the equatorial plane would point to inter-particle spacings between neighbouring vortices on the femtometre scale. Since this is very much less than their actual diameters, the magnetic lines of force will in effect become tubes of force. Then on the issue of the density, as an absolute minimum, if we were to simply consider only the diameter of the vortices, the density of the vortex sea will already be into the region of fourteen hundred times denser than lead, but it will surely be many orders of magnitude yet higher than that still.

Conclusion

III. Hooke's law at equation (1) in section I became Maxwell's fifth equation in the original list of eight "*Maxwell's Equations*" in his 1864 paper "*A Dynamical Theory of the Electromagnetic Field*" [9], [10]. This is the electric displacement equation from which Maxwell derived the concept of displacement current which was later used in the 1864 paper in conjunction with Ampère's Circuital Law in the derivation of the electromagnetic wave equation. It is often forgotten though that the connection between the electromagnetic wave equation and the speed of light is not a theoretically derived fact. It arises experimentally from the 1855 Weber-Kohlrausch experiment [2]. Maxwell saw a way of linking the result to the transverse elasticity in the displacement mechanism within a background elastic solid, where the equation $\mathbf{E} = mc^2$ is tied up in this relationship through fine-grained centrifugal pressure. All of this preceded the electromagnetic wave equation of 1864.

References

[1] Clerk-Maxwell, J., "*On Physical Lines of Force*", Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)

http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

[2] Tombe, F.D., "*The 1855 Weber-Kohlrausch Experiment*" (2019)

<http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7711>

[3] Whittaker, E.T., "*A History of the Theories of Aether and Electricity*", Chapter 4, pages 100-102, (1910)

"All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess,

and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools.”

[4] Tombe, F.D., “**The Double Helix Theory of the Magnetic Field**” (2006)
Galilean Electrodynamics, Volume 24, Number 2, p.34, (March/April 2013)
<http://gsjournal.net/Science-Journals/Research%20Papers-Mathematical%20Physics/Download/6371>

[5] Tombe, F.D., “**Induction of Electrostatic Repulsion by Strong Gravity**” (2017)
<http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7167>

[6] Simhony, M., “**The Electron-Positron Lattice Space, Cause of Relativity and Quantum Effects**”,
Physics Section 5, The Hebrew University, Jerusalem (1990)
<http://web.archive.org/web/20040606235138/www.word1.co.il/physics/mass.htm>

[7] Lodge, Sir Oliver, “**Ether (in physics)**”, Encyclopaedia Britannica,
Fourteenth Edition, Volume 8, Pages 751-755, (1937)
<http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105>
In relation to the speed of light, “*The most probable surmise or guess at present is that **the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion**, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed*”

[8] O’Neill, John J., **PRODIGAL GENIUS, Biography of Nikola Tesla**, Long Island, New York, 15th July 1944
<http://www.rastko.rs/istorija/tesla/oniell-tesla.html>
“*Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. **The primary substance, thrown into infinitesimal whirls of prodigious velocity**, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance*”.

[9] Clerk-Maxwell, J., “**A Dynamical Theory of the Electromagnetic Field**”, Philos. Trans. Roy. Soc. London 155, pp 459-512 (1865). Abstract: Proceedings of the Royal Society of London 13, pp. 531--536 (1864). The original eight Maxwell’s equations are found in the link below in Part III entitled ‘*General Equations of the Electromagnetic Field*’ which begins on page 480,
http://www.zpenergy.com/downloads/Maxwell_1864_3.pdf
Maxwell’s derivation of the electromagnetic wave equation is found in the link below in Part VI entitled ‘*Electromagnetic Theory of Light*’ which begins on page 497,
http://www.zpenergy.com/downloads/Maxwell_1864_4.pdf

[10] Tombe, F.D., “**Maxwell’s Original Equations**” (2011)
<http://gsjournal.net/Science-Journals/Essays-Mechanics%20/%20Electrodynamics/Download/3889>