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The Mathematical Universe

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Abstract: We Humans, a curious beings developed from the **Darwin's principle of natural selection**, are accustomed into an inquisition. The question is not 'do we know everything from the triumph of the Higgs boson to the underlying discomfort of ultimate question of life, the universe, and everything?' or it is **'do we know enough?'** But how the creative principle resides in mathematics? There's something very mathematical about our gigantic Cosmos, and that the more carefully we look, the more equations are built into nature: **From basic arithmetic to the calculation of rocket trajectories**, math provides a good understanding of the equations that govern the world around us. Our universe isn't just described by math, but that universe is a "grand book" written in the language of mathematics. We find it very appropriate that mathematics has played a striking role in our growing understanding of the events around us, and of our own existence.

Keywords: Equations; Math; Energy; Vibration; Quantum mechanics; Physical Constants; Universe.

"But the creative principle resides in mathematics. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed." **— Albert Einstein**

⇨ **Newton's laws of motion** tie into almost everything we see in everyday life.

- **Law 1 (law of inertia):** An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.
- **Law 2:** Force equals mass times acceleration (**F = ma**).
- **Law 3:** For every action, there is an equal and opposite reaction.

⇨ As a remarkable consequence of the **uncertainty principle of quantum mechanics** (which implies that certain pairs of quantities, such as the energy and time, cannot both be predicted with complete accuracy), the empty space is filled with what is called vacuum energy.

Because $E=MC^2$:

- Mass is just energy in disguise.
- A small amount of mass can equal a large amount of energy.

If the mass of the star < 1.4 solar masses

- Electrons prevent further collapse.
- The core will thus continue to collapse and form a **white dwarf**.

If the mass of the star > 1.4 solar masses but mass < 3 solar masses

- Electrons + protons combine to form neutrons.
- Neutrons prevent further collapse.
- The core will thus continue to collapse and form a **neutron star**.

If the mass of the star > 3 solar masses

- Gravity wins! Nothing prevents collapse.
- The core will thus continue to collapse and form a black hole.

*Any object with a physical radius smaller than its **Schwarzschild radius** will be a black hole.*

All the laws of physics that we know, breaks down –

- Below this time: (**Planck Time**)
- Below this length: (**Planck Length**)
- Above this temperature: (**Planck Temperature**)

Density parameter (Ω): The ratio of the total amount of matter in the universe divided by the minimum amount of matter needed to cause the **big crunch**.

- **Ω < 1:** the Universe will continue to expand forever.
- **Ω > 1:** the Universe will eventually halt its expansion and recollapse.
- **Ω = 1:** the Universe contains enough matter to halt the expansion but not enough to recollapse it.

If a black hole has a mass less than the **Planck mass**, its quantum mechanical size could be outside its event horizon. This wouldn't make sense, **Planck mass** is the smallest possible black hole.

Absolute zero: (–273ºC) − the lowest possible temperature, at which substances contain no heat energy and all vibrations stop — almost.

⇨ If the two quarks would have occupied precisely the same point with the same properties, they would not have stayed in the same position for long. And quarks would have not formed separate, **well-defined protons and neutrons**. And nor would these, together with electrons have formed separate, well-defined atoms. And the world would have collapsed before it ever reached its present size.

- When 2 similar waves are added, the resultant wave is bigger (**constructive interference**).
- When 2 dissimilar waves are added, they cancel each other out (**destructive interference**).

The different frequencies of light appear as different colors.

Proton charge + Electron charge = 0

Just what it is if **electromagnetism** would not dominate over **gravity** and for the universe to remain electrically neutral.

- It's not their energy; it's their zero rest mass that makes **photons** to travel at the speed of light.
- Just like a dozen is 12 things, a mole is simply **Avogadro's number of particles**.

Undisturbed space + rigid mass = distorted space.

3 dimensions of space + one dimension of time = **single four-dimensional continuum** (space-time).

Since **h** − which is one of the most **fundamental numbers in physics**, ranking alongside the speed of light "**c**" and confined most of these radical departures from life-as-usual to the microscopic realm − is incredibly small (i.e., **6 × 10 to the power of – 34** − a decimal point followed by 33 zeros and a 6 − of a joule second), the frequency of the photon is always greater than its energy, so it do not take many quanta to radiate even ten thousand megawatts.

What is GRAVITY?

- **Newtonian view:** Force tells mass how to accelerate. Accelerated mass tells what gravity is.
- **Einsteinian view:** Mass tells space how to curve. Curved space tells what gravity is.

All objects emit **electromagnetic radiation** according to their temperature. Colder objects emit waves with very low frequency (such as **radio or microwaves**), while hot objects emit visible light or even **ultraviolet** and higher frequencies.

- Longer half-life of nucleus → **Slow Radioactive Decay**.
- **Shorter half-life of nucleus** → Fast Radioactive Decay.

".. Physics at the atomic and subatomic level ..."

… Weird things are possible:

- Energy is quantized (**E = nhυ**).
- Momentum is quantized (**L = nħ**).
- Charge is quantized (**Q = ne**).

Because

$$E = h\upsilon, \qquad c = \lambda\upsilon, \qquad E = \frac{hc}{\lambda} = pc$$

$$\lambda = \frac{h}{p}$$

Every particle or quantum entity may be partly described in terms not only of particles, but also of waves.

The Thermodynamic Laws think big: they dictate energy behavior…

- **1 Law:** Energy is conserved; its form can be converted.
- **2 Law:** Energies can flow, equilibrate.
- **3 Law:** "Driving force" for equilibration uniquely defined.
- **0 Law:** Thermal equilibrium is transitive.

The Life of a Star:

"More mass

More pressure and temperature

Faster Fusion

Shorter life"

"Less mass

Less pressure and temperature

Slower Fusion

Longer life"

String Theory (A theory that tries to adjust / harmonize / reconcile **General Relativity** and **Quantum mechanics**):

- Different vibrations → Different particles.
- String combinations → Particle interactions.

The universe is made of

- 21% Dark Matter.
- 74% Dark Energy.
- 4% Normal Matter.

MATTER UNDER EXTREME CONDITIONS

$$\text{Nuclei} + \text{heat} + \text{pressure} \rightarrow \text{quark-gluon plasma}$$

- **Hydrogen atom:** Diameter about a Billionth of an inch.
- **Electron:** Diameter at least 1000 times smaller than that of proton.
- **Proton:** Diameter about 60,000 times smaller than H atom.
- **Probability distribution** is the only way to locate an electron in an atom.

The Gas laws deal with how gases behave with respect to pressure, volume, temperature …

- **Boyle's law:**

Volume and pressure are inversely proportional.

- **Charles' law:**

Volume is proportional to temperature.

- **Pressure law:**

Pressure is proportional to temperature.

All three combined:

$$\frac{PV}{T} = \text{constant}$$

n → number of neutrons

p → number of protons

- **If (n / p) ≈1 (atomic number between 1 and 20):**

Nucleus has equal number of protons and neutrons to become stable.

- **If (n / p) >1 (atomic number between 20 and 83):**

Number of protons increase and repulsion between them also increases. To balance this force number of neutrons also increases.

- **If (n / p) >1 (atomic number > 83):**

Nucleus having atomic number higher than 83 has great number of protons and repulsion force between protons. Since the amount of force is too high, number of neutrons cannot balance them and nucleus stays unstable. Thus, we can say that nuclei having atomic number greater than 83 generally undergo **transmutation**, **alpha decay** or **beta decay**.

- Weak nuclear forces + **Maxwell equations** → Electro weak theory.
- Electro weak theory + Quantum Chromodynamics (**QCD**) → Standard Model of particle physics.
- **Standard Model** of particle physics → explains everything except **gravity**.

4 NUMBERS describe the characteristics of electrons and their orbitals:

- **Principal quantum number:** a number that describes the average distance of the orbital from the nucleus and the energy of the electron in an atom.
- **Angular momentum quantum number:** a number that describes the shape of the orbital.

- **Magnetic quantum number:** a number that describes how the various orbitals are oriented in space.
- **Spin quantum number:** a number that describes the direction the electron is spinning in a magnetic field — either clockwise or counterclockwise.

The square of the periods of the planets (**the times for them to complete one orbit**) is proportional to the cubes of their average distance from the Sun. A consequence of this is that the **inner planets** move rapidly in their orbits. Venus, Earth and Mars move progressively less rapidly about the Sun. And the **outer planets**, such as Jupiter and Saturn, move stately and slow.

Wavelength of UV radiation < Wavelength of IR radiation < Wavelength of microwave radiation

- Molecule dissociates (when it absorbs UV radiation).
- Molecule vibrates (when it absorbs IR radiation).
- Molecule rotates (when it absorbs microwave radiation).

⇨ If the expansion of space had overwhelmed the pull of gravity in the beginning − stars, galaxies and humans would never have been able to form. If, on the other hand, gravity had been 5% stronger− stars and galaxies might have formed, but they would have quickly collapsed in on themselves and each other to form a sphere of roughly infinite density.

Neutrons have a mass of 939.56 MeV.

If the mass of a neutron was a seventh of a percent more than it is, stars like most of those we can see would not have existed. If the neutron mass was 0.085% less than it is, the Universe would have been full of neutrons and nothing else.

If we cut the surface of a sphere up into faces, edges and vertices, and let F be the number of faces, E the number of edges and V the number of vertices, we will always get:

$$\mathbf{V - E + F = 2}.$$

Fibonacci numbers − 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

- Each number is the sum of the previous two.
- The ratio between the numbers = 1.618034 (**golden ratio**).
- From **pinecones to the Hurricane Sandy**, Fibonacci reflects various patterns found in nature.

The paths of anything you throw have the same shape, called an **upside-down parabola**.

When we observe how objects move around in **gravitationally curved trajectories** in space, we discover another recurring shape: **the ellipse**.

All material particles have properties such as charge and spin.

Space itself has properties such as dimensions.

- These properties are purely mathematical.

Equations aren't the only hints of mathematics that are built into nature: **there are also numbers involving not only motion and gravity**, but also areas as disparate as classical physics, quantum mechanics, and astronomy.

Strong force → force that is responsible for binding together the fundamental particles of matter to form larger particles.

- **If stronger:** No hydrogen would have formed; atomic nuclei for most life-essential elements would have been unstable; thus, there would have been no life chemistry.
- **If weaker:** No elements heavier than hydrogen would have formed: again, no life chemistry.

Weak force → force that is responsible for the radioactive decay of atoms.

- **If stronger:** Too much hydrogen would have been converted to helium in the big bang; hence, stars would have converted too much matter into heavy elements making life chemistry impossible.
- **If weaker:** Too little helium would have been produced from big bang; hence, stars would have converted too little matter into heavy elements making life chemistry impossible.

Electromagnetic force → force that is responsible for most of the interactions we see in our environment today.

- **If stronger:** Chemical bonding would have been disrupted; elements more massive than boron would have been unstable to fission.
- **If weaker:** Chemical bonding would have been insufficient for life chemistry.

c = 299,792,458 meters per second − serves as the single limiting velocity in the universe, being an upper bound to the propagation speed of signals and to the speeds of all material particles.

Ratio of electromagnetic force to gravitational force

- **If larger:** All stars would have been at least 40% more massive than the sun; hence, stellar burning would have been too brief and too uneven for life support.
- **If smaller:** All stars would have been at least 20% less massive than the sun, thus incapable of producing heavier elements.

Ratio of electron to proton mass

- **If larger or smaller:** Chemical bonding would have been insufficient for life chemistry.

Mass of the neutrino

- **If smaller:** Galaxy clusters, galaxies, and stars would have not formed.
- **If greater:** Galaxy clusters and galaxies would have been too dense.

Ratio of exotic matter to ordinary matter

- **If larger:** the universe would have collapsed before the formation of solar-type stars.
- **If smaller:** no galaxies would have formed.

Number of effective dimensions in the early universe

- **If larger or smaller:** Quantum mechanics, gravity, and relativity could not have coexisted; thus, life would have been impossible.

Entropy level of the universe

- **If larger:** Stars would have not formed within proto-galaxies.

- **If smaller:** No proto-galaxies would have formed.

Polarity of the water molecule

- **If greater:** Heat of fusion and vaporization would have been too high for life.
- **If smaller:** Heat of fusion and vaporization would have been too low for life; liquid water would not have worked as a solvent for life chemistry; ice would not have floated, and a runaway freeze-up would have resulted.

From the properties of subatomic particles and the **realm of quantum physics to the formation of a giant mathematical object** (**universe**), math proves unquestionably effective in describing and predicting their physical reality. However, a question that lies at the intersection of philosophy and science arises: **Is Math the Language of the Universe?**

$$\frac{m_{proton}}{m_{electron}} = 1836.152\ 673\ 89$$

Changing their values changes the physical phenomena.

$$\text{Avogadro number} = 6.022140857 \times 10^{23}\ \text{mol}^{-1}$$

Connects macroscopic properties (**e.g. molar mass, Gas constant**) with microscopic properties (e.g. atomic mass, Boltzmann's constant).

Because:

$$T = \frac{\hbar c^3}{8\pi k G M}$$

Large black holes emit less radiation than small black holes.

Because:

$$\lambda_{max}\, T = 2.897771955...\times 10^{-3}\ mK$$

Hotter objects emit most of their radiation at shorter wavelengths.

Because:

$$r = \frac{3GM}{c^2}$$

Photon spheres can only exist in the space surrounding an extremely compact object (a **black hole** or possibly an "**ultracompact**" neutron star).

$$F = \frac{GMm}{r^2}$$

Causes the moon to move in an orbit around the earth and causes the earth and the planets to follow orbital paths around the sun.

Because:

$$F = ma$$

The acceleration is twice as great if the force is twice as great.

Because:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

The energy which a particle has due to its motion will add to its mass. In other words, it will make it harder to increase its speed. This effect is only really significant for particles moving at speeds close to the speed of light. For example, at 10 percent of the speed of light a particle's mass is only 0.5 percent more than its rest mass, while at 90 percent of the speed of light it would be more than twice its rest mass. As a particle approaches the speed of light, its mass rises ever more quickly, so it takes more and more energy to speed it up further. It can in fact never reach

the speed of light, because by then its mass would have become infinite, and by the equivalence of mass and energy, it would have taken an infinite amount of energy to get it there. For this reason, any particle is forever confined by relativity to move at speeds slower than the speed of light. Only photons that have no rest mass can move at the speed of light.

Equivalence principle → inertial mass and gravitational mass are the same thing.

Because of **violation of CP-symmetry:**

There is dominance of matter over antimatter in the present Universe.

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