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Pythagoras's Theorem and Special Relativity

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***Abstract*. The *Minkowski Metric Tensor* in special relativity has pretensions of occupying a so-called four-dimensional space-time continuum where *Pythagoras's theorem* continues to operate as normal. It will now be argued that if Pythagoras's Theorem can hold outside of three dimensions, then the only possibility might be in the special case of seven dimensions, but that even this would be highly doubtful.**

Introduction

I. Articles that are critical of Einstein's Special Theory of Relativity usually focus on the absurdity of the inherent implication that two clocks could both be ticking slower than each other [1]. This article will instead challenge that surreal final coating that encases the relativity package. A double application of Pythagoras's theorem led Hermann Minkowski to invoke the four-dimensional space-time continuum which relativists believe to be a physical reality. The purpose of this article is to show that Pythagoras's theorem only exists in three-dimensional space, and the special case of seven dimensions will be fully investigated as an illustration.

Pythagoras's Theorem

II. The square of the hypotenuse of a right-angle triangle is equal to the sum of the squares of the other two sides. This ancient rule in Euclidean geometry, better known as *Pythagoras's Theorem*, is actually only a special case of the more general cosine rule,

$$c^2 = a^2 + b^2 - 2ab\cos\theta \qquad \textbf{(1)}$$

as applied to cases where the angle θ is equal to ninety degrees. When θ is ninety degrees, we will have a right-angle triangle with adjacent side length *a*, opposite side *b*, and hypotenuse *c*. The concept of *'angle'* relates to rotation in a two-dimensional plane with the rotation axis being in the third dimension. If, however a body were to be rotating in a two-dimensional plane in a four dimensional space, we would have to decide which of the other two dimensions the axis of rotation would occupy. And if we have difficulty trying to imagine rotation in a four-dimensional space, so also will we have difficulty trying to imagine the cosine rule, and hence Pythagoras's theorem in 4D.

It's assumed however in pure mathematics and relativistic circles that Pythagoras's theorem can actually apply in spaces of any dimensions. In fact, on first examination, Pythagoras's theorem does appear to hold in four dimensions. The apparent legitimacy hinges on the operation of the *"scalar product"* or *"dot product"* of two vectors, $\mathbf{a}.\mathbf{b} = ||\mathbf{a}||\ ||\mathbf{b}||\cos\theta$. If we define the sides of a triangle using vectors, each expressed in terms of four mutually orthogonal unit vectors, we do in fact find consistency with Pythagoras's theorem. A simple example is enough to demonstrate this. Consider two vectors in a 4D space. Let vector **a** be defined as $\mathbf{a} = a_1\mathbf{i} + 0\mathbf{j} + a_3\mathbf{k} + 0\mathbf{l}$ and let vector **b** be defined as $\mathbf{b} = 0\mathbf{i} + b_2\mathbf{j} + 0\mathbf{k} + b_4\mathbf{l}$, where the four unit vectors **i**, **j**, **k**, and **l**, are mutually orthogonal. Those components were specially chosen to make vectors **a** and **b** orthogonal so that they can represent two sides of a right-angle triangle in a hypothetical four-dimensional space. If we assume Pythagoras's theorem to be applicable in this four-dimensional space, then it will follow that $||\mathbf{a} + \mathbf{b}||^2$ should be equal to $||\mathbf{a}||^2 + ||\mathbf{b}||^2$, and indeed this turns out to be the case,

$$||\mathbf{a} + \mathbf{b}||^2 = a_1^2 + b_2^2 + a_3^2 + b_4^2 \qquad \mathbf{(2)}$$

Meanwhile,

$$||\mathbf{a}||^2 + ||\mathbf{b}||^2 = (a_1^2 + a_3^2) + (b_2^2 + b_4^2) = a_1^2 + b_2^2 + a_3^2 + b_4^2 \qquad \mathbf{(3)}$$

This is enough to convince relativists and pure mathematicians that Pythagoras's theorem can hold in spaces of any dimensions. But this application of the scalar product only involved parallel and orthogonal vectors. There was no evidence that it would hold more generally for the in-between angles if such a concept as *'angle'* could even be imagined in 4D.

The Minkowski Metric Tensor of Relativity

III. The metric tensor of special relativity has its roots in two consecutive applications of Pythagoras's Theorem. The first application is in respect of the right-angle triangle that is used to derive the time dilation formula in special relativity, while the second application relates to splitting one of the sides of that triangle into three mutually orthogonal Cartesian components. The end result has pretensions of 4D space-time, giving the superficial impression that Pythagoras's theorem continues to apply as normal in four dimensions, and that the consecutive application of Pythagoras's theorem in 3D has mysteriously morphed into a 4D case scenario. If however, Pythagoras's theorem should indeed hold in 4D, then we would expect the Pythagorean Trigonometric Identity to hold likewise. This matter will be examined in the next section.

The Pythagorean Trigonometric Identity

IV. Consider the "*Pythagorean Trigonometric Identity"*,

$$\sin^2\theta + \cos^2\theta = 1 \qquad \textbf{(4)}$$

where the sine and cosine terms are derived from the squares of the two perpendicular sides of a right-angle triangle. Re-arranging equation **(4)** and multiplying across by $||\mathbf{a}||^2||\mathbf{b}||^2$ we get,

$$||\mathbf{a}||^2||\mathbf{b}||^2(1 - \cos^2\theta) = ||\mathbf{a}||^2||\mathbf{b}||^2\sin^2\theta \qquad \textbf{(5)}$$

This introduces another binary operation known as the *"vector product"* or the *"cross product"*. The right-hand side of equation **(5)** is the square of this vector cross product. The cross product is defined as,

$$\mathbf{a} \times \mathbf{b} = ||\mathbf{a}||\ ||\mathbf{b}||\sin\theta\hat{\mathbf{n}} \qquad \textbf{(6)}$$

where $\hat{\mathbf{n}}$ is a unit vector mutually perpendicular to **a** and **b**. The underlying principle behind the cross product is the idea that a bilinear map involving any two elements in a set of three can yield the third. The original inspiration came in 1843 when Sir William Rowan Hamilton was walking along the towpath by the Royal Canal in Dublin. In a flash of genius, Hamilton discovered the three-dimensional formula, $i^2 = j^2 = k^2 = ijk = -1$, and in his excitement he cut it on a stone of Brougham Bridge at Cabra. A plaque commemorating the occasion can be seen at the location today. Hence equation **(5)** can be written in the form,

$$||\mathbf{a}||^2||\mathbf{b}||^2 - ||\mathbf{a}.\mathbf{b}||^2 = ||\mathbf{a} \times \mathbf{b}||^2 \qquad \textbf{(7)}$$

Equation **(7)** is known as the Lagrange Identity. The Lagrange identity itself holds in any dimensions and it takes the more general form,

$$||\mathbf{a}||^2||\mathbf{b}||^2 - ||\mathbf{a}.\mathbf{b}||^2 = \Sigma\ (a_xb_y - a_yb_x)^2 \qquad \textbf{(8)}$$

where $1 \le x < y \le n$, but it's only in the case of three dimensions that it satisfies the Pythagorean trigonometric identity. In the specific form expressed in equation **(7)**, it is impossible to write the Lagrange identify in any higher dimensions, apart from one interesting exception, that being the case of seven dimensions, which will be discussed in the next section.

The Special Case of Seven Dimensions

V. While it's impossible to set up a bilinear vector cross product in an even number of dimensions, it is nevertheless possible to set one up for any odd number of dimensions. We can set up a table for five mutually orthogonal unit vectors and map each unto two pairs from amongst the rest. We will not however be able to fit the result into the Lagrange identity. And if we can't fit it with the Lagrange identity, then neither can we fit it with the Pythagorean trigonometric identity. Only in the case of three and seven dimensions, ignoring the trivial cases of zero and one, can we fit the result with the Lagrange identity. This restriction follows from a nineteenth century theorem on composition algebras by Adolf Hurwitz's (1859 - 1919). More formal proofs only came about relatively recently **[2]**. It will now be shown that the equality,

$$\Sigma\ (a_x b_y - a_y b_x)^2 = ||\mathbf{a} \times \mathbf{b}||^2 \qquad \textbf{(9)}$$

holds in 7 dimensions, where $1 \leq x < y \leq 7$. We can set up a table of mutually orthogonal unit vectors as follows,

$$\begin{array}{lllllll}
\mathbf{i} & = & \mathbf{j}\times\mathbf{l} & = & \mathbf{m}\times\mathbf{n} & = & \mathbf{k}\times\mathbf{o} \\
\mathbf{j} & = & \mathbf{k}\times\mathbf{m} & = & \mathbf{n}\times\mathbf{o} & = & \mathbf{l}\times\mathbf{i} \\
\mathbf{k} & = & \mathbf{l}\times\mathbf{n} & = & \mathbf{o}\times\mathbf{i} & = & \mathbf{m}\times\mathbf{j} \\
\mathbf{l} & = & \mathbf{m}\times\mathbf{o} & = & \mathbf{i}\times\mathbf{j} & = & \mathbf{n}\times\mathbf{k} \\
\mathbf{m} & = & \mathbf{n}\times\mathbf{i} & = & \mathbf{j}\times\mathbf{k} & = & \mathbf{o}\times\mathbf{l} \\
\mathbf{n} & = & \mathbf{o}\times\mathbf{j} & = & \mathbf{k}\times\mathbf{l} & = & \mathbf{i}\times\mathbf{m} \\
\mathbf{o} & = & \mathbf{i}\times\mathbf{k} & = & \mathbf{l}\times\mathbf{m} & = & \mathbf{j}\times\mathbf{n}
\end{array}$$

Table 1.

Then if vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} + a_4\mathbf{l} + a_5\mathbf{m} + a_6\mathbf{n} + a_7\mathbf{o}$, and vector $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} + b_4\mathbf{l} + b_5\mathbf{m} + b_6\mathbf{n} + b_7\mathbf{0}$, the cross product is,

$$\mathbf{a}\times\mathbf{b} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k} + c_4\mathbf{l} + c_5\mathbf{m} + c_6\mathbf{n} + c_7\mathbf{o} \qquad \textbf{(10)}$$

where,

$$
\begin{aligned}
c_1 &= a_2b_4 - a_4b_2 + a_5b_6 - a_6b_5 + a_3b_7 - a_7b_3 \\
c_2 &= a_3b_5 - a_5b_3 + a_6b_7 - a_7b_6 + a_4b_1 - a_1b_4 \\
c_3 &= a_4b_6 - a_6b_4 + a_7b_1 - a_1b_7 + a_5b_2 - a_2b_5 \\
c_4 &= a_5b_7 - a_7b_5 + a_1b_2 - a_2b_1 + a_6b_3 - a_3b_6 \\
c_5 &= a_6b_1 - a_1b_6 + a_2b_3 - a_3b_2 + a_7b_4 - a_4b_7 \\
c_6 &= a_7b_2 - a_2b_7 + a_3b_4 - a_4b_3 + a_1b_5 - a_5b_1 \\
c_7 &= a_1b_3 - a_3b_1 + a_4b_5 - a_5b_4 + a_2b_6 - a_6b_2
\end{aligned}
$$

Table 2.

In order to expand $||\mathbf{a} \times \mathbf{b}||^2$ we therefore need to take the sum of the squares of c_1 through to c_7 and this multiplies out to 252 terms. These 252 terms can be split into two groups. There is a group of 84 terms, which can in turn be reduced to 21 squared terms in brackets (shown in black). It is this group of 21 alone that makes the equality between $||\mathbf{a} \times \mathbf{b}||^2$ and $\Sigma\ (a_xb_y - a_yb_x)^2$. Then there is a second group of 168 terms (shown in red) which cancels out completely. It is this cancellation, which ignoring the special case of 3 dimensions, is unique to 7 dimensions. It will not happen in 5 dimensions, and neither will it happen in 9, 11, or any higher dimensions. The expansion is as follows,

$$
\begin{aligned}
||\mathbf{a} \times \mathbf{b}||^2 = {}& (a_2b_4 - a_4b_2)^2 + (a_5b_6 - a_6b_5)^2 + (a_3b_7 - a_7b_3)^2 + 2(a_2b_4 - a_4b_2)(a_5b_6 - a_6b_5) + 2(a_2b_4 - a_4b_2)(a_3b_7 - a_7b_3) + 2(a_5b_6 - a_6b_5)(a_3b_7 - a_7b_3) \\
& + (a_3b_5 - a_5b_3)^2 + (a_6b_7 - a_7b_6)^2 + (a_4b_1 - a_1b_4)^2 + 2(a_3b_5 - a_5b_3)(a_6b_7 - a_7b_6) + 2(a_3b_5 - a_5b_3)(a_4b_1 - a_1b_4) + 2(a_6b_7 - a_7b_6)(a_4b_1 - a_1b_4) \\
& + (a_4b_6 - a_6b_4)^2 + (a_7b_1 - a_1b_7)^2 + (a_5b_2 - a_2b_5)^2 + 2(a_4b_6 - a_6b_4)(a_7b_1 - a_1b_7) + 2(a_4b_6 - a_6b_4)(a_5b_2 - a_2b_5) + 2(a_7b_1 - a_1b_7)(a_5b_2 - a_2b_5) \\
& + (a_5b_7 - a_7b_5)^2 + (a_1b_2 - a_2b_1)^2 + (a_6b_3 - a_3b_6)^2 + 2(a_5b_7 - a_7b_5)(a_1b_2 - a_2b_1) + 2(a_5b_7 - a_7b_5)(a_6b_3 - a_3b_6) + 2(a_1b_2 - a_2b_1)(a_6b_3 - a_3b_6) \\
& + (a_6b_1 - a_1b_6)^2 + (a_2b_3 - a_3b_2)^2 + (a_7b_4 - a_4b_7)^2 + 2(a_6b_1 - a_1b_6)(a_2b_3 - a_3b_2) + 2(a_6b_1 - a_1b_6)(a_7b_4 - a_4b_7) + 2(a_2b_3 - a_3b_2)(a_7b_4 - a_4b_7) \\
& + (a_7b_2 - a_2b_7)^2 + (a_3b_4 - a_4b_3)^2 + (a_1b_5 - a_5b_1)^2 + 2(a_7b_2 - a_2b_7)(a_3b_4 - a_4b_3) + 2(a_7b_2 - a_2b_7)(a_1b_5 - a_5b_1) + 2(a_3b_4 - a_4b_3)(a_1b_5 - a_5b_1) \\
& + (a_1b_3 - a_3b_1)^2 + (a_4b_5 - a_5b_4)^2 + (a_2b_6 - a_6b_2)^2 + 2(a_1b_3 - a_3b_1)(a_4b_5 - a_5b_4) + 2(a_1b_3 - a_3b_1)(a_2b_6 - a_6b_2) + 2(a_4b_5 - a_5b_4)(a_2b_6 - a_6b_2)
\end{aligned} \quad \textbf{(11)}
$$

Re-arranging the black subscripts numerically and segregating the black terms from the red terms which have now been expanded, we see more clearly how the black terms are equal to $\Sigma\ (a_xb_y - a_yb_x)^2$, where $1 \le x < y \le 7$,

$$
\begin{aligned}
\|\mathbf{a} \times \mathbf{b}\|^2 &= (a_1b_2 - a_2b_1)^2 + (a_1b_3 - a_3b_1)^2 + (a_1b_4 - a_4b_1)^2 + (a_1b_5 - a_5b_1)^2 + (a_1b_6 - a_6b_1)^2 + (a_1b_7 - a_7b_1)^2 + (a_2b_3 - a_3b_2)^2 + (a_2b_4 - a_4b_2)^2 \\
&+ (a_2b_5 - a_5b_2)^2 + (a_2b_6 - a_6b_2)^2 + (a_2b_7 - a_7b_2)^2 + (a_3b_4 - a_4b_3)^2 + (a_3b_5 - a_5b_3)^2 + (a_3b_6 - a_6b_3)^2 + (a_3b_7 - a_7b_3)^2 + (a_4b_5 - a_5b_4)^2 + (a_4b_6 - a_6b_4)^2 \\
&+ (a_4b_7 - a_7b_4)^2 + (a_5b_6 - a_6b_5)^2 + (a_5b_7 - a_7b_5)^2 + (a_6b_7 - a_7b_6)^2 \\
&+2[a_2b_4a_5b_6 - a_2b_4a_6b_5 - a_4b_2a_5b_6 + a_4b_2a_6b_5 \\
&+ a_2b_4a_3b_7 - a_2b_4a_7b_3 - a_4b_2a_3b_7 + a_4b_2a_7b_3 \\
&+ a_5b_6a_3b_7 - a_5b_6a_7b_3 - a_6b_5a_3b_7 + a_6b_5a_7b_3 \\
&+ a_3b_5a_6b_7 - a_3b_5a_7b_6 - a_5b_3a_6b_7 + a_5b_3a_7b_6 \\
&+ a_3b_5a_4b_1 - a_3b_5a_1b_4 - a_5b_3a_4b_1 + a_5b_3a_1b_4 \\
&+ a_6b_7a_4b_1 - a_6b_7a_1b_4 - a_7b_6a_4b_1 + a_7b_6a_1b_4 \\
&+ a_4b_6a_7b_1 - a_4b_6a_1b_7 - a_6b_4a_7b_1 + a_6b_4a_1b_7 \\
&+ a_4b_6a_5b_2 - a_4b_6a_2b_5 - a_6b_4a_5b_2 + a_6b_4a_2b_5 \\
&+ a_7b_1a_5b_2 - a_7b_1a_2b_5 - a_1b_7a_5b_2 + a_1b_7a_2b_5 \\
&+ a_5b_7a_1b_2 - a_5b_7a_2b_1 - a_7b_5a_1b_2 + a_7b_5a_2b_1 \\
&+ a_5b_7a_6b_3 - a_5b_7a_3b_6 - a_7b_5a_6b_3 + a_7b_5a_3b_6 \\
&+ a_1b_2a_6b_3 - a_1b_2a_3b_6 - a_2b_1a_6b_3 + a_2b_1a_3b_6 \\
&+ a_6b_1a_2b_3 - a_6b_1a_3b_2 - a_1b_6a_2b_3 + a_1b_6a_3b_2 \\
&+ a_6b_1a_7b_4 - a_6b_1a_4b_7 - a_1b_6a_7b_4 + a_1b_6a_4b_7 \\
&+ a_2b_3a_7b_4 - a_2b_3a_4b_7 - a_3b_2a_7b_4 + a_3b_2a_4b_7 \\
&+ a_7b_2a_3b_4 - a_7b_2a_4b_3 - a_2b_7a_3b_4 + a_2b_7a_4b_3 \\
&+ a_7b_2a_1b_5 - a_7b_2a_5b_1 - a_2b_7a_1b_5 + a_2b_7a_5b_1 \\
&+ a_3b_4a_1b_5 - a_3b_4a_5b_1 - a_4b_3a_1b_5 + a_4b_3a_5b_1 \\
&+ a_1b_3a_4b_5 - a_1b_3a_5b_4 - a_3b_1a_4b_5 + a_3b_1a_5b_4 \\
&+ a_1b_3a_2b_6 - a_1b_3a_6b_2 - a_3b_1a_2b_6 + a_3b_1a_6b_2 \\
&+ a_4b_5a_2b_6 - a_4b_5a_6b_2 - a_5b_4a_2b_6 + a_5b_4a_6b_2] \qquad \textbf{(12)}
\end{aligned}
$$

(In order to check that the 168 red terms exactly cancel, a larger version has been prepared in ***Appendix I*** *for the purposes of printing and marking off)*

Despite this miraculous cancellation though, the fact that the seven-dimensional vector cross product can't easily be linked to rotation, casts serious doubt on the idea that Pythagoras's theorem could hold in seven dimensions. The Jacobi Identity, $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) + \mathbf{b}\times(\mathbf{c}\times\mathbf{a}) + \mathbf{c}\times(\mathbf{a}\times\mathbf{b}) = 0$, is closely associated with rotation, but the fact that it doesn't apply to the seven dimensional cross product further casts doubt on the validity of Pythagoras's Theorem in 7D.

The Inertial Path

VI. Consider a body in motion in an inertial frame of reference. We can write the position vector of this body relative to any arbitrarily chosen polar origin as,

$$\mathbf{r} = r\hat{\mathbf{r}} \qquad \textbf{(13)}$$

where the unit vector $\hat{\mathbf{r}}$ is in the radial direction and where r is the radial distance. Taking the time derivative and using the product rule, we obtain the velocity,

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\omega\hat{\mathbf{s}} \qquad \textbf{(14)}$$

where $\hat{\mathbf{s}}$ is the unit vector in the transverse direction and where ω is the angular speed about the polar origin. Taking the time derivative a second time, we obtain the expression for acceleration in the inertial frame,

$$\ddot{\mathbf{r}} = \ddot{r}\hat{\mathbf{r}} + \dot{r}\omega\hat{\mathbf{s}} + \dot{r}\omega\hat{\mathbf{s}} + r(\partial\omega/\partial t)\hat{\mathbf{s}} - r\omega^2\hat{\mathbf{r}} \qquad \textbf{(15)}$$

Re-arranging and multiplying across by mass m leads to,

$$m\ddot{\mathbf{r}} = m(\ddot{r} - r\omega^2)\hat{\mathbf{r}} + m(2v_r\omega + r\partial\omega/\partial t)\hat{\mathbf{s}} \qquad \textbf{(16)}$$

†see the note at reference **[3]** *regarding Maxwell's equation (77)*

where ω is the angular speed and v_r is the radial speed. The radial component of equation **(16)** contains a centrifugal force, $m\ddot{r}$, and an inertial centripetal force, $-mr\omega^2$, while the transverse component contains a Coriolis force, $mr\partial\omega/\partial t$, which equals $2mv_r\omega$ when angular momentum is conserved. In the case of uniform straight-line motion, the total acceleration is zero, but when a constraint is introduced, an imbalance occurs in the inertial symmetry. For example, if the body is tethered to a pivot, the inertial centrifugal force pulls on the constraint, hence inducing a reactive centripetal tension within the material of the constraint. This tension cancels with the inertial centrifugal force and the resultant is a net inertial centripetal force which curves the path of motion.

The inertial centripetal force $-mr\omega^2$ in equation **(16)** with respect to one polar origin, is an inertial centrifugal force with respect to the origin at the same distance along a line through the moving body on the other side of it. From the perspective of the moving body, there is therefore a centrifugal force to every point in space giving rise to a cylindrical vector field in the likeness of the magnetic field that surrounds an electric current. The centrifugal force to any point on a particular cylindrical shell, concentric to the path of motion, will be a resolution of the centrifugal force to a point on the shell, that acts perpendicularly to the path of motion. The perpendicular centrifugal force will drop off with an inverse cube law in distance from the moving body (see equation **(18)**). Since centrifugal force is the radial gradient of kinetic energy, it is now proposed that this cylindrical vector field represents the extension of the body's kinetic energy.

The idea that a moving entity could yield up energy to a surrounding medium and have it returned during deceleration is observed in the case of an electromagnetic field. When the power supply to an electric circuit is disconnected, its magnetic field collapses and its stored energy, $\frac{1}{2}LI^2$, flows back into the circuit giving the current a final surge forward. Another rather obvious connection between the inertial forces and magnetism is the fact that the Coriolis force has a similar form to the magnetic force, $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$, if we adopt Maxwell's idea that it is caused by a sea of molecular vortices pressing against each other with centrifugal force while striving to dilate **[3]**, **[4]**, **[5]**, **[6]**, and where the vorticity, $\mathbf{H} = 2\boldsymbol{\omega}$, represents the magnetic intensity, where $\boldsymbol{\omega}$ is the circumferential angular speed of the vortices and where $\mathbf{B} = \mu\mathbf{H}$.

It is therefore proposed that kinetic energy, $\frac{1}{2}mv^2$, is a pressure, and an extended pressure field which drops off with an inverse cube law in distance, and that it is induced by the fine-grained centrifugal force interaction between the immediately surrounding vortices and the molecules of the moving body as they shear past each other. These vortices will be the rotating electron-positron dipoles introduced in section **I**, and they will form double helix vortex rings around the moving body, centred on the line of motion, similar in principle to smoke rings. To the front and rear of the motion, the vortices would therefore have to be continually aligning and de-aligning, and the associated precession of the vortices would be fully compatible with a Coriolis force acting equally and oppositely at the front and the rear of the motion. This process would be identical in principle to Maxwell's explanation for Ampère's Circuital Law. The kinetic energy pressure field, or inertial field, that accompanies a moving body is therefore in principle just a variation on the magnetic field theme. It is a weak magnetic field.

The Inertial Frame of Reference

VII. The inertial frame of reference is a relatively recent concept, introduced mainly in connection with Einstein's theories of relativity and retrospectively applied to Newtonian mechanics. Newton only ever considered the background stars as the significant frame of reference **[7]**. As a proposition we'll take the inertial frame of reference to be Maxwell's sea of molecular vortices with the vortices being rotating electron-positron dipoles **[8]**, **[9]**. There is supposed to be no gravitational field in an inertial frame of reference, yet if we want to have one in practice, we have little choice but to choose a region of the electron-positron sea which is entrained within the gravitational field of a planet. This way we can have an inertial frame of reference providing that we ignore the gravitational force. This is fine therefore when solving problems where gravity is negligible. If on the other hand we are dealing with planetary orbital problems where two inertial frames of reference are shearing past each other while generating centrifugal force at the interface, this changes the physical basis upon which the inertial forces are induced, and the only physically significant directions are radial and transverse. In planetary orbits, conservation of angular momentum causes the total transverse term in equation **(16)** to vanish. This is recognized in Kepler's second law, which is the law of equal areas. Meanwhile the gravity sinks distort the inertial mechanism. Gravitational tension has a physically cancelling effect on the centrifugal pressure forces that are measured relative to the gravitating centres. Writing the centrifugal term in the form $+r\omega^2$, the problem in the radial direction reduces to the scalar equation,

$$\ddot{r} = -k/r^2 + r\omega^2 \qquad \textbf{(17)}$$

where k is the gravitational constant. Taking l to be the angular momentum constant equal to $r^2\omega$, we can write Leibniz's equation in the form,

$$\ddot{r} = -k/r^2 + l^2/r^3 \qquad \textbf{(18)}$$

Between the two planets, the inter-play between the gravitational inverse square law attractive force, which is a tension, and the inverse cube law centrifugal repulsive force, which is a pressure, involves two different power laws, and this leads to stable orbits that are elliptical, circular, parabolic, or hyperbolic. And since the gravitational tails on the far sides of the planets will undermine the inertial centripetal mechanism, then centrifugal force and gravity are the only real forces acting in the radial direction.

Conclusion

VIII. The connection between rotation, Pythagoras's theorem, the cosine rule, the inertial forces, and electromagnetism, along with the fact that the Pythagorean trigonometric identity only holds in three dimensions, suggests unequivocally that space is a three-dimensional construction stabilized on cylindrical symmetry. It is proposed that space is densely packed with tiny dipolar vortices in which the default alignment is double helix toroidal vortex rings forming magnetic lines of force. These vortices are responsible for the inertial forces, magnetic force, electromagnetic induction, and electromagnetic radiation, and they also absorb the vorticity out of the large-scale gravitational sinks. A vortex involves a rotation in a two-dimensional plane with the rotation axis in the third dimension. Minkowski's four-dimensional space-time continuum is a counterfeit which was devised using two consecutive applications of the ordinary Pythagoras theorem of three-dimensional Euclidean geometry.

References

[1] Dingle, H., ***"The Case Against Special Relativity"***, Nature, Volume 216, pages 119-122, (1967)
https://www.nature.com/articles/216119a0

[2] Silagadze, Z. K., "Multi-dimensional vector product" Budker Institute of Nuclear Physics, Novosibirsk, Russia, (2002)
file:///C:/Users/user/Downloads/seven%20dimensional%20cross%20product.pdf

[3] Clerk-Maxwell, J., ***"On Physical Lines of Force"***, Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)
http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf
† Equation (77) in Maxwell's paper is his electromotive force equation and it exhibits a strong correspondence to equation **(16)** in this article. The transverse terms $2mv_r\omega$ (where vorticity $\mathbf{H} = \mathbf{2\omega}$) and $m\partial v_t/\partial t$ (where v_t is the transverse speed equal to $r\omega$) correspond to the compound centrifugal term $\mu\mathbf{v}\times\mathbf{H}$ and the Faraday term $-\partial\mathbf{A}/\partial t$, with m corresponding to µ, and where **A** is the electromagnetic momentum. Gauss's law then appears in equation **(17)**.

[4] Whittaker, E.T., ***"A History of the Theories of Aether and Electricity"***, Chapter 4, pages 100-102, (1910)
"All space, according to the younger Bernoulli, is permeated by ***a fluid aether, containing an immense number of excessively small whirlpools.*** *The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools;* ***for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools****."*

[5] O'Neill, John J., ***PRODIGAL GENIUS***, ***Biography of Nikola Tesla***, *Long Island, New York, 15th July 1944*
http://www.rastko.rs/istorija/tesla/oniell-tesla.html
"Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. ***The primary substance, thrown into infinitesimal whirls of prodigious velocity,*** *becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance".*

[6] Lodge, Sir Oliver, ***"Ether (in physics)"***, Encyclopaedia Britannica, Fourteenth Edition, Volume 8, Pages 751-755, (1937)
http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105

In relation to the speed of light, *"The most probable surmise or guess at present is that* ***the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion,*** *circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed"*

[7] Dingle, H., ***"On Inertia and Inertial Frames of Reference"***, Quarterly Journal of the Royal Astronomical Society, Volume 8, Page 262 (1967)
http://adsabs.harvard.edu/full/1967QJRAS...8..252D

[8] Tombe, F.D., ***"The Double Helix Theory of the Magnetic Field"*** (2006)
Galilean Electrodynamics, Volume 24, Number 2, p.34, (March/April 2013)
http://gsjournal.net/Science-Journals/Research%20Papers-Mathematical%20Physics/Download/6371

[9] Tombe, F.D., ***"Induction of Electrostatic Repulsion by Strong Gravity"*** (2017)
http://gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/7167

Appendix I

Page 12 below can be printed out for the purpose of marking it off with a pen, in order to demonstrate the total cancellation. These are 168 (2 × 84) of the 252 terms which resulted in equation **(12)** above when $||\mathbf{a} \times \mathbf{b}||^2$ was expanded in seven dimensions. This cancellation would not have worked in five dimensions, nor will it work for any higher dimensions, and a product can't even be constructed in the first place for any even dimensions, which hence rules out the case of four dimensions. While fifteen dimensions might have been the next obvious one to try, based on the series 0, 1, 3, 7, 15, **- - - - -** ($2^n - 1$), this would mean dealing with 2,940 (14×14×15) terms as opposed to the 252 (6×6×7) terms in this seven dimensional case, or the mere 12 (2×2×3) terms in the three dimensional case. However, on knowing Adolf Hurwitz's theorem on composition algebras and Silagadze's proof **[3]**, there would be little point in trying. The key below will assist with finding matching pairs on page 12.

Rows 1-21, columns A-D
(1A, 21C), (1B, 8D), (1C, 8A), (1D, 21B),
(2A, 16C), (2B, 15A), (2C, 15D), (2D, 16B),
(3A, 11B), (3B, 4D), (3C, 4A), (3D, 11C),
(4B, 11D), (4C, 11A), (5A,19C), (5B, 18A),
(5C, 18D), (5D, 19B), (6A, 14B), (6B, 7D),
(6C, 7A), (6D,14C), (7B,14D), (7C, 14A),
(8B, 21A), (8C, 21D), (9A, 17B), (9B, 10D),
(9C, 10A), (9D, 17C), (10B,17D), (10C, 17A),
(12A, 20B), (12B, 13D), (12C, 13A), (12D, 20C),

(13B, 20D), (13C, 20A), (15B, 16D), (15C, 16A),
(18B, 19D), (18C, 19A).

A B C D

$+2[a_2b_4a_5b_6 - a_2b_4a_6b_5 - a_4b_2a_5b_6 + a_4b_2a_6b_5$ 1

$+ a_2b_4a_3b_7 - a_2b_4a_7b_3 - a_4b_2a_3b_7 + a_4b_2a_7b_3$ 2

$+ a_5b_6a_3b_7 - a_5b_6a_7b_3 - a_6b_5a_3b_7 + a_6b_5a_7b_3$ 3

$+ a_3b_5a_6b_7 - a_3b_5a_7b_6 - a_5b_3a_6b_7 + a_5b_3a_7b_6$ 4

$+ a_3b_5a_4b_1 - a_3b_5a_1b_4 - a_5b_3a_4b_1 + a_5b_3a_1b_4$ 5

$+ a_6b_7a_4b_1 - a_6b_7a_1b_4 - a_7b_6a_4b_1 + a_7b_6a_1b_4$ 6

$+ a_4b_6a_7b_1 - a_4b_6a_1b_7 - a_6b_4a_7b_1 + a_6b_4a_1b_7$ 7

$+ a_4b_6a_5b_2 - a_4b_6a_2b_5 - a_6b_4a_5b_2 + a_6b_4a_2b_5$ 8

$+ a_7b_1a_5b_2 - a_7b_1a_2b_5 - a_1b_7a_5b_2 + a_1b_7a_2b_5$ 9

$+ a_5b_7a_1b_2 - a_5b_7a_2b_1 - a_7b_5a_1b_2 + a_7b_5a_2b_1$ 10

$+ a_5b_7a_6b_3 - a_5b_7a_3b_6 - a_7b_5a_6b_3 + a_7b_5a_3b_6$ 11

$+ a_1b_2a_6b_3 - a_1b_2a_3b_6 - a_2b_1a_6b_3 + a_2b_1a_3b_6$ 12

$+ a_6b_1a_2b_3 - a_6b_1a_3b_2 - a_1b_6a_2b_3 + a_1b_6a_3b_2$ 13

$+ a_6b_1a_7b_4 - a_6b_1a_4b_7 - a_1b_6a_7b_4 + a_1b_6a_4b_7$ 14

$+ a_2b_3a_7b_4 - a_2b_3a_4b_7 - a_3b_2a_7b_4 + a_3b_2a_4b_7$ 15

$+ a_7b_2a_3b_4 - a_7b_2a_4b_3 - a_2b_7a_3b_4 + a_2b_7a_4b_3$ 16

$+ a_7b_2a_1b_5 - a_7b_2a_5b_1 - a_2b_7a_1b_5 + a_2b_7a_5b_1$ 17

$+ a_3b_4a_1b_5 - a_3b_4a_5b_1 - a_4b_3a_1b_5 + a_4b_3a_5b_1$ 18

$+ a_1b_3a_4b_5 - a_1b_3a_5b_4 - a_3b_1a_4b_5 + a_3b_1a_5b_4$ 19

$+ a_1b_3a_2b_6 - a_1b_3a_6b_2 - a_3b_1a_2b_6 + a_3b_1a_6b_2$ 20

$+ a_4b_5a_2b_6 - a_4b_5a_6b_2 - a_5b_4a_2b_6 + a_5b_4a_6b_2]$ 21