

## Towards the Exact Value of Universal Gravitational Constant (Extended with CODATA 2014 Data)

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**Abstract:** This article is the extended version of the previous article [5], featuring new CODATA 2014 values published in June 2015 [1]. Using original formulas, the newly-obtained value of the universal gravitational constant is by a few orders of magnitude more accurate than the recommended CODATA value. The additions featured here are written in Arial font, as well as corrections of typos in the formula (5), Table 1 and text.

**Keywords:** Gravitational constant, Universe, Proton, Planck

### Introduction

The dimension of the universal gravitational constant  $G$  is:  $M^{-1}L^3T^{-2}$ . If it is expressed in natural units [2], it has value by definition (in Planck units, equals 1). The exact value of the constant is also possible in any other system in which  $G$ , or the values from which it could be directly derived, would by definition have exact values. That is not possible in the International System of Units [3] because in that system only the speed of light with dimensions  $L^2T^{-2}$  has exact value and can be used for determining  $G$ . For example, if in that system Planck mass and length would have the value by definition, then by using formula:  $G=c^2l_{pl}/m_{pl}$  ( $c$  – speed of light,  $l_{pl}$  – Planck length,  $m_{pl}$  – Planck mass),  $G$  would also have exact value. The same result could be obtained by applying some other combinations of the exactly defined values.

There are a large number of formulas which feature  $G$ , and still its value is known for its low accuracy in the SI. The reason for that is that the values which are included in the calculation of  $G$  are difficult to determine experimentally or cannot be determined at all. It is more common for those values to even be determined via the known  $G$ . Hence, in the following formulas at least one of the Planck values is always present:

$$G=c^2l_{pl}/m_{pl}$$

$$G=l_{pl}^3/m_{pl}t_{pl}^2$$

$$G=hc/\pi'm_{pl}^2$$

Taken from [1]:

|                 |                   |                      |
|-----------------|-------------------|----------------------|
| Planck length   | 1.616 199 e-35    | 0.000 097 e-35 m     |
| Planck mass     | 2.176 51 e-8      | 0.000 13 e-8 kg      |
| Planck time     | 5.391 06 e-44     | 0.000 32 e-44 s      |
| Planck constant | 6.626 069 57 e-34 | 0.000 000 29 e-34 Js |

Therefore we have:

Newtonian constant of gravitation    6.673 84 e-11    0.000 80 e-11  $m^3 kg^{-1} s^{-2}$

in the similar range of accuracy. On the right is the value of uncertainty expressed by  $1\sigma$ , standard deviations. In the text below, the uncertainty will be shown in brackets, after the value of the physical quantity. Therefore, for the accurate determination of  $G$  it is necessary to express this constant via the physical constants whose values can be determined experimentally with great accuracy.

In this article we will add the CODATA 2014 values [1] to the tables from the previous article [5].

## Formula for $G$ via proton

Starting from the statement "**Parts are dependent on the whole (Universe) and are also an integral part of the whole; therefore, the whole is also dependent on the parts!**" I developed a methodology which produced results.

Let's define the mathematical constants:

$$t = \log(2\pi, 2) = 2.651496\dots, \text{ Cycle, } cy = e^{2\pi} = 535.49165\dots, \text{ Half cycle, } z = e^{2\pi}/2 = 267.74582776\dots$$

The masses of proton and the universe are as follows:

$$m_p = 1.672621777E-27 \text{ kg [1], } M_u = 1.73944912E+53 \text{ kg,}$$

$p$  – the constant related to the proton is:

$$p = \log(m_u / m_p, 2) \quad (1)$$

And also:

$$z = e^{2\pi}/2 = \log(m_u / m_z, 2) \quad (2)$$

Then we can define the **proton shift,  $zp$** :

$$zp = z - p = \log(m_p / m_z, 2) = 1.9350609435 \quad (3)$$

We will also use physical constants  $\mu$  – proton-to-electron mass ratio and  $\alpha'$  – inverse fine-structure constant [1]. They can also be used to determine the proton shift:

$$zp = 2 - \frac{1}{\mu/\alpha' + 2} = 1.9350609435 \quad (4)$$

Moreover, from (3) and (4):

$$p = e^{2\pi}/2 - zp = 265.8107668 \quad (5)$$

If  $m_p$  is the proton mass and  $\lambda_p$  stands for the proton Compton wavelength, we obtain the following formula:

$$G = c^2 m_p^{-1} * \lambda_p * \sqrt{2\pi * 2^{(cy-3p)}} \quad (6)$$

All the physical quantities in (6) are related to the proton and are accurately determined experimentally.

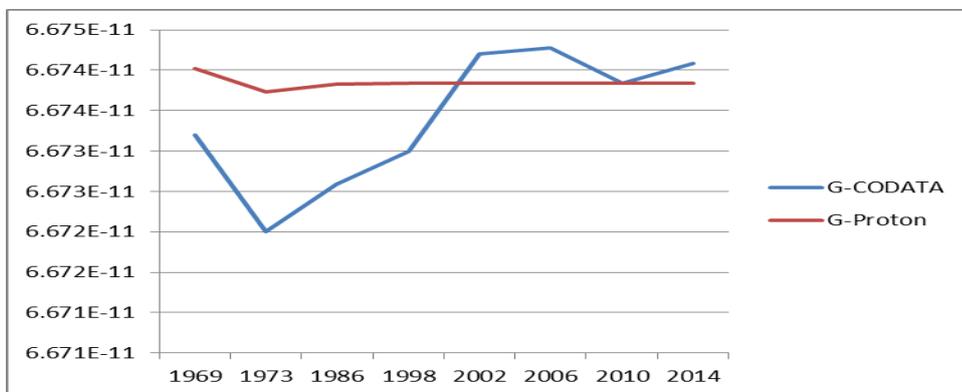
Here we will test the formula (6) by using the historical CODATA values. The CODATA values are shown in **Table 1**, columns 1-6. There, for example, we can see that each physical constants in 2010 [1] has at least two significant digits more than **G**, while the value of the speed of light **c** is exact by definition since 1973. Please note that statistics have been simplified for the purpose of this article, which does not significantly affect the result. NIST has an online document that describes the best procedure [7].

The seventh column of Table 1 shows the value of **G** determined by the formula (6), so that once the upper value **G'** is determined based on the CODATA values (**α**, **μ**, **λ<sub>p</sub>**, **m<sub>p</sub>** – for the corresponding year), and once the lower value **G**. The upper and lower values determine the uncertainty  $\pm 1\sigma$ , shown in brackets. Value  $(G' - G)/2$  is adopted to represent  $1\sigma$ .

**Table 1 determining the universal gravitational constant - G via proton**

| $p = cy/2 - 2 + 1/(\mu'/\alpha + 2)$ |                     | $G' = c^2 * m_p^{-1} * \lambda'_p * \sqrt{[2\pi * 2^{\wedge}cy - 3p]}$ |                     | formula               |                        |  |                     |
|--------------------------------------|---------------------|--|---------------------|-----------------------|------------------------|--|---------------------|
| $p' = cy/2 - 2 + 1/(\mu/a' + 2)$     |                     | $G = c^2 * m'_p^{-1} * \lambda_p * \sqrt{[2\pi * 2^{\wedge}cy - 3p']}$ |                     | value G               |                        |  |                     |
| CODATA                               | Values [1]:         | c  | Compton $\lambda_p$ | $m_p$                 | G                      | [kg <sup>-1</sup> m <sup>3</sup> s <sup>-2</sup> ] |                     |
| Year                                 | $\alpha = 1/\alpha$ | $\mu = m_p/m_e$  | (m/sec)             | * 10 <sup>-15</sup> m | * 10 <sup>-27</sup> kg | * 10 <sup>-11</sup>                                | * 10 <sup>-11</sup> |
| 1969                                 | 137.03602(21)       | 1836.1090(110)   | 299792500           | 1.3214409(90)         | 1.672614(11)           | 6.6732 (31)  | 6.674016(92)        |
| 1973                                 | 137.036040(110)     | 1836.15152(70)   | 299792458           | 1.3214099(22)         | 1.6726485(86)          | 6.6720(41)   | 6.673729(46)        |
| 1986                                 | 137.0359895(61)     | 1836.152701(37)  | 299792458           | 1.32141002(12)        | 1.6726231(10)          | 6.67259(85)  | 6.6738316(46)       |
| 1998                                 | 137.03599976(50)    | 1836.1526675(39)   | 299792458           | 1.321409847(10)       | 1.67262158(13)         | 6.673(10)  | 6.67383675(57)      |
| 2002                                 | 137.03599911(46)    | 1836.15267261(85)  | 299792458           | 1.3214098555(88)      | 1.67262171(29)         | 6.6742(10)   | 6.6738363(16)       |
| 2006                                 | 137.035999679(94)   | 1836.15267247(80)  | 299792458           | 1.3214098446(19)      | 1.672621637(83)        | 6.67428(67)  | 6.67383651(34)      |
| 2010                                 | 137.035999074(45)   | 1836.15267245(75)  | 299792458           | 1.32140985623(94)     | 1.672621777(74)        | 6.67384(80)  | 6.67383601(30)      |
| 2014                                 | 137.035999139(31)   | 1836.15267389(17)  | 299792458           | 1.32140985396(61)     | 1.672621898(21)        | 6.67408(31)  | 6.673835517(87)     |

**Table 1** shows that the value of **G** determined by the formula in year 1973 achieved the accuracy from year 2010 in [1]. The value of **G** determined by the formula for year 2010 has two significant digits more than the CODATA value.



**Figure 1 Universal gravitational constant – G in the 1969–2014 period CODATA values [1] and values achieved by formula (6)**

**Figure 1** visually presents the advantage of determining the value of G by applying the formula (6) in relation to the CODATA method.

The 2014 CODATA value shows less accordance than the 2010 one. However, that is consistent with previous years when the CODATA value changed its relation to the value obtained through the formula. In 2006 the difference between those two values was greater than in the previous report from 2002, i.e. the accordance was decreased. However, the next report from 2010 increased the accordance again. Therefore, I expect that the future 2018 report will again provide greater accordance.

Alongside the 2010 CODATA report, there is a good explanation of the method that was used to obtain the value of the universal gravitational constant for the year 2010 [6]. It is possible that if the same explanation would be published for 2014 that we explain earlier why the accordance of the value G has been decreased in the 2014 report compared to the 2010 report.

The predictive power of the formula (6) for determining the value of the universal gravitational constant G is shown by applying physical constants whose experimental determination gives the values much more accurate than the experimentally obtained G.

In the formula (6), are:

$$R_u = \lambda_p * \sqrt{2\pi * 2^{(cy-p)}} = 1.2916530E + 26 \text{ m} \quad (7)$$

$$M_u = m_p * 2^p = 1.73944912E + 53 \text{ kg} \quad (8)$$

Those are radius and mass of the universe. Note that the radius is defined as  $R_u=cT_u$  and corresponds to the age of the universe (We will say: Universe time cycle),  $T_u \approx 13.7 * 10^9$  years.

Relation  $R_u=cT_u$  is the essence of the “unity of the whole and its parts” approach. The values from the literature largely confirm the above results for the parameters of the universe. It is not possible to experimentally estimate values of these parameters, nor is it essential for this proposed model of relationships in the universe. What is certain in this model is that if **p** is known, then the parameters of the universe are precisely determined, because they are all in the function of **p** only. Therefore, the only relevant judgment of the model’s value and hence of the values of the parameters are the predictive abilities of the model itself. This will be examined here, for the case of universal gravitational constant.

Then, from (6), (7) and (8):

$$G = c^2 M_u^{-1} * R_u = M_u^{-1} * R_u^3 * T_u^{-2} \quad (9)$$

which is the basic and simple formula presenting the essence of the universal gravitational constant.

## Formula for G via the Rydberg Constant

If  $m_p$  is proton mass and  $R_\infty$  is Rydberg constant, the approach identical to the above gives the formula:

$$G = c^2 * (2 * \mu * \alpha'^2 * m_p * R_\infty)^{-1} * \sqrt{2\pi * 2^{(cy-3p)}} \quad (10)$$

All the physical quantities in (10) were determined experimentally with high precision.

Here I will test formula (10), using the historical CODATA values. The CODATA values for  $\alpha$ ,  $\mu$ ,  $R_\infty$ ,  $m_p$  are shown in columns 1, 2, 4 and 5 of **Table 2** and we can see that all the four physical constants in year 2010 [1] have at least two significant digits more than  $G$ , while the speed of light  $c$  in the third column is exact by definition.

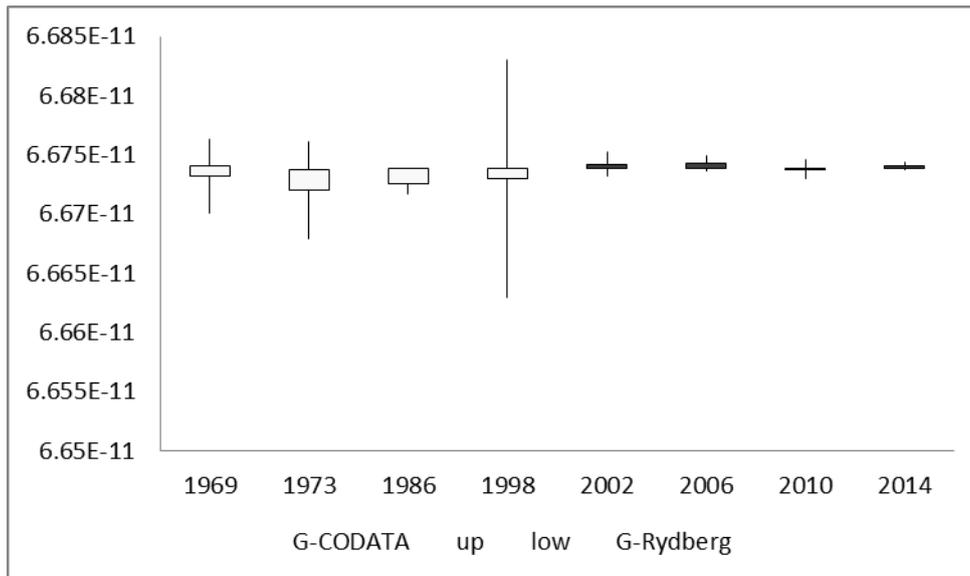
The seventh column of **Table 2** shows the value of  $G$  determined by the formula (10): the upper value  $G'$ , according to CODATA values of  $\alpha$ ,  $\mu$ ,  $R_\infty$ ,  $m_p$  for the corresponding year, and the lower value  $G$ . The upper and lower values determine the uncertainty  $\pm 1\sigma$ , shown in brackets. Value  $(G' - G)/2$  has been adopted to represent  $1\sigma$ .

**Table 2 Determining universal gravitational constant via the Rydberg constant**

|      | $p=cy/2-2+1/(\mu'/\alpha'+2)$ |                              | $G'=c^2*(2*\mu*\alpha'^2*m_p*R_\infty)^{-1}*\sqrt{[2\pi*2^{cy-3p}]}$   |   |                          | formula           |  |
|------|-------------------------------|------------------------------|--|---|--------------------------|-------------------|--|
|      | $p'=cy/2-2+1/(\mu/a'+2)$      |                              | $G=c^2*(2*\mu'*\alpha'^2*m'_p*R'_\infty)^{-1}*\sqrt{[2\pi*2^{cy-3p}]}$ |   |                          | value G           |  |
| Year | CODATA<br>$\alpha=1/\alpha$   | Values [1]:<br>$\mu=m_p/m_e$ | c<br>(m/sec)   | Rydberg const<br>$R_\infty * 10^7 m^{-1}$ | $m_p$<br>$* 10^{-27} kg$ | G<br>$* 10^{-11}$ | [kg <sup>-1</sup> m <sup>3</sup> s <sup>-2</sup> ]<br>$* 10^{-11}$ |
| 1969 | 137.03602(21)                 | 1836.1090(110)               | 299792500  | 1.09737312(11)                            | 1.672614(11)             | 6.6732 (31)       | 6.67402(11)  |
| 1973 | 137.036040(110)               | 1836.15152(70)               | 299792458  | 1.097373177(83)                           | 1.6726485(86)            | 6.6720(41)        | 6.673729(49)   |
| 1986 | 137.0359895(61)               | 1836.152701(37)              | 299792458  | 1.0973731534(13)                          | 1.6726231(10)            | 6.67259(85)       | 6.6738316(48)  |
| 1998 | 137.03599976(50)              | 1836.1526675(39)             | 299792458  | 1.0973731568549(83)                       | 1.67262158(13)           | 6.673(10)         | 6.67383674(59)   |
| 2002 | 137.03599911(46)              | 1836.15267261(85)            | 299792458  | 1.0973731568525(73)                       | 1.67262171(29)           | 6.6742(10)        | 6.6738363(12)  |
| 2006 | 137.035999679(94)             | 1836.15267247(80)            | 299792458  | 1.0973731568527(73)                       | 1.672621637(83)          | 6.67428(67)       | 6.67383653(35)   |
| 2010 | 137.035999074(45)             | 1836.15267245(75)            | 299792458  | 1.0973731568539(55)                       | 1.672621777(74)          | 6.67384(80)       | 6.67383603(31)   |
| 2014 | 137.035999139(31)             | 1836.15267389(17)            | 299792458  | 1.0973731568508(65)                       | 1.672621898(21)          | 6.67408(31)       | 6.673835517(91)  |

**Table 2** shows that the value of  $G$  determined by formula (10) already in the year 1969 achieved the accuracy from the year 2010 in [1]. The value of  $G$  determined by formula has as much as two significant digits more than the CODATA values.

It is possible to use **Table 2** to make an image that would look practically similar to Figure 1. That is why instead of showing that image we will show **Figure 2** which also includes all the uncertainties. We will show here:



**Figure 2 Universal gravitational constant – G in the 1969–2014 period CODATA values [1] and values achieved by formula (10)**

Vertical lines show G-CODATA estimates and uncertainty and the bars show G-Rydberg estimates and uncertainty. Note that Figure 2 serves only for the purpose of presentation of the formula (10) behavior during the time of CODATA reports publications and methodologically is not in full agreement with the CODATA practice.

It is evident that the CODATA value G is approaching the obtained formula (10). If we would be satisfied with six significant digits, then according to the formula the value for the year 2014 would be  $6.67384 \cdot 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$ . That value is being met by all the values obtained through the formula between 1973 and 2014 (see Table 2).

## Conclusion

The article shows the predictive power of formulas (6) and (10) for determining the value of the universal gravitational constant G by applying physical constants whose experimental determination gives the values much more accurate than the experimentally obtained G. Testing results also confirm that the other results obtained by the same approach and published at [4] are not a coincidence and mere numerology. There is also a possibility to determine G even more accurately through other constants or even exactly by redefining the International System of Units.

To put it in one sentence, the concept is: *Parts are dependent on the whole (Universe) and are also an integral part of the whole, therefore, the whole is also dependent on the parts!* This position implies that the concept is based on the fundamental importance of the relation between the parts and the whole, and if that position is wrong, then all the obtained results are coincidences.

## ACKNOWLEDGEMENT

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