The Green-Tao theorem is false

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陶哲轩和格林定理是错误的

美国 Notices of the AMS 报导陶哲轩获得 2006 菲尔茨奖，第一大成果是他和格林证明了《素数含有任意长的等差数列》。他在国际数学家大会上作了一小时报告介绍这项工作。蒋春暄 1995 年在首届《余新河数学题》研讨会散发一文，即文献 4 和定理 7，就已解决这个问题。当时数学所书记李福安等八位数学家参加这次研究会，每个人都收到这篇论文。蒋春暄从网上下载格林和陶哲轩论文，发现他们并没有证明这个问题，没有得出任何有用结果。蒋春暄花了半天只用一页纸解决这个问题（定理 1)：（1）证明有无限多素数使得等差数列都是素数，（2）找到计算素数个数渐近公式，写成论文寄到由普林斯顿高级研究院（爱因斯坦晚年生活地方）和普林斯顿大学合办《数学年刊》一个编委，指出格林和陶哲轩论文是错的（他们论文已被接受在这杂志发表），送上一文希望在你们杂志发表，10 月 10 日马上收到这杂志的来信：

Dear Professor Jiang,

Thank you for the electronic files and for submitting your paper entitled “The simplest proofs of both arbitrarily long prime arithmetic progressions” to the Annals of Mathematics（数学年刊）.

I have processed the PDF file and the paper will be given to the editors. They will contact you after they have heard from a reviewer.

Sincerely,

Maureen Schupsky

Annals staff

这封信在全国散发；只收到支持蒋春暄并想在北航建立研究小组原

北航校长沈士团来信：等待好消息。

《数学年刊》是国际顶尖杂志，普林斯顿是国际纯数学领导中心。中国民间数学家蒋春暄论文被他们重视，在中国是空前绝后的事情。他的工作被何祚庥和方舟子评为中国最大伪科学。格林和陶哲轩论文已被国际主流派数学家接受和承认，所以陶哲轩才能获得 2006 年国际数学家大会菲尔茨奖。说明全世界没有一位数学家真正理解素数理论，只是在素数大门外瞎猜和起哄，蒋否定他们的工作，也就否定 2006 年菲尔茨奖，也就否定数学诺贝尔奖，如格林和陶哲轩论文在 10 月前没有出版，那么《数学年刊》也不会出版他们的论文，蒋已把论文寄给格林和陶哲轩，并向国内外寄去 300 多份。何祚庥和方舟子，中国院士一定会写信给普林斯顿阻止他们出版蒋的论文。他们是否出版蒋的论文不是重要问题，蒋已收到他们来信也就满意了。王元对陶哲轩评价："我不敢想像天下会有这样伟大的成就"。不知王元是否理解"素数等差数列"的内容和如何去证明它。和怀尔斯证明费马大定理一样，陶哲轩和格林他们利用组合理论，遍历理论，Fourier 分析和超图证明《素数等差数列》，这些方法和素数理论没有直接关系。甚至连最简单孪生素数定理都不能证明。如果只考虑《素数等差数列》前二项就是孪生素数定理，陶哲轩说他们不能直接找到表达素数渐近公式，他们任务只能找到非平凡下界。如何找到下界也没有给出。只能说他们证明是猜想，所以陶哲轩和格林定理是错误的。

蒋春暄利用他发现 Jiang 函数，已证明了素数分布几乎所有问题。本文利用 Jiang 函数证明九个定理，已把《素数等差数列》中所有问题都已解决。

最近很多网站不发表蒋春暄文章，支持他的网：www.chinabokee.cn/bbs/已被取消，在中央关心下于 12 月 2 日恢复，蒋又可以把他论文上网，同时上网格林和陶哲轩论文：The primes contain arbitrarily long arithmetic progressions，格林在英国、捷克、法国、加拿

大和美国为广大数学听众演讲稿：Long arithmetic progressions of primes, B, Kra 为他们说明论文：The Green-Tao theorem on arithmetic progressions in the primes: an ergodic point of view, 和 A. Granville 为他们说明论文：Prime number patterns，可以相信关心中国数学发展的数学家，把蒋春暄论文定理 1 和定理 2 同格林和陶哲轩论文比较；一定可以得出结论：蒋已完全解决《素数等差数列》，而格林和陶哲轩没有接触《素数等差数列》。王元说："现在我们准备在晨兴数学中心搞一个研究班，专门读陶哲轩论文。晨兴数学中心近十年来一直将数论作为首要支持项目。丘成桐、杨乐、张寿武等始终支持。这样我们就可以跟踪世界上最前沿的东西，已经有年轻人经过一年多努力、基本上弄清了陶哲轩和格林的论文细节。"为什么他们不研究蒋春暄数论工作？因为蒋被他们评为中国最大的伪科学，没有兴趣，不值得他们研究。

王元说："陶哲轩的工作最重的地方是用了新方法。"即 Additive combinatorics，现已被普林斯顿高级研究院列为 2007 年至 2008 年研究课题。从他们网 www.math.ias.edu 下载

Arithmetic Combinatorics

(Term I 2007-2008 Academic Year)

During tem I of the year, School faculty member Jean Bourgain and Van Vu of Rutgers University will lead a program on arithmetic combinatorics. The following is preliminary information about the program.

Additive combinatorics deal with problems in number theory with combinatorial flavor. While this theory has been developing for many decades, the field has seen exciting developments and dramatic changes in direction in recent years (a well known example is Green-Tao theorem about the existence of arithmetic progressions in primes). In this focus program, we will bring together active researches in this field and many related areas such as number theory, combinatorics and theoretical computer science.

在这里对格林和陶哲轩定理评价很高，是他们研究基础，这就是当代数论最高水平。

再说一句 10 月蒋春暄把论文寄一位支持格林和陶哲轩的数学家，是他把蒋春暄论文转交《数学年刊》，最后蒋收到这杂志的来信。

The Simplest Proofs of Both Arbitrarily Long Arithmetic Progressions of primes

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Abstract

Using Jiang functions $J_2(\omega)$, $J_3(\omega)$ and $J_4(\omega)$ we prove both arbitrarily long arithmetic progressions of primes: (1) $P_{i+1} = P_1^n + di,\quad (P_1, d) = 1$, $i = 1, 2, \cdots, k-1, n \ge 1$, which have the same Jiang function; (2) $P_{i+1} = P_1^n + \omega_g i, i = 1, 2, \cdots, k-1, n \ge 1$, $\omega_g = \prod_{2 \le P \le P_g} P$ and generalized arithmetic progressions of primes $P_i = P + i\omega_g$ and $P_{k+i} = P^n + i\omega_g$, $i = 1, \cdots, k, n \ge 2$.

The Green-Tao theorem is false, because they do not prove the twin primes theorem and arithmetic progressions of primes [3].

In prime numbers theory there are both well-known conjectures that there exist both arbitrarily long arithmetic progressions of primes. In this paper using Jiang functions $J_2(\omega)$, $J_3(\omega)$ and $J_4(\omega)$ we obtain the simplest proofs of both arbitrarily long arithmetic progressions of primes.

Theorem 1. We define arithmetic progressions of primes:

$$P_1, P_2 = P_1 + d, P_3 = P_1 + 2d, \cdots, P_k = P_1 + (k-1)d, (P_1, d) = 1. \qquad (1)$$

We rewrite (1)

$$P_3 = 2P_2 - P_1, \qquad P_j = (j-1)P_2 - (j-2)P_1, \quad 3 \le j \le k. \qquad (2)$$

We have Jiang function [1]

$$J_3(\omega) = \prod_{3 \le P} \left[(P-1)^2 - X(P) \right], \qquad (3)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=3}^{k} \left[(j-1)q_2 - (j-2)q_1 \right] \equiv 0 (\bmod P), \qquad (4)$$

where $q_1 = 1, 2, \cdots, P-1; q_2 = 1, 2, \cdots, P-1.$

From (4) we have

$$J_3(\omega) = \prod_{3 \le P < k} (P-1) \prod_{k \le P} (P-1)(P-k+1) \to \infty \text{ as } \omega \to \infty. \qquad (5)$$

We prove that there exist infinitely many primes P_1 and P_2 such that $P_3, \cdots, P_k$ are all primes for all $k \ge 3$. It is a generalization of Euclid and Euler proofs for the existence of infinitely many primes [1].

We have the best asymptotic formula [1]

$$\pi_{k-1}(N,3)=\left|\{(j-1)P_2-(j-2)P_1=\text{prime}, 3\le j\le k, P_1,P_2\le N\}\right|$$
$$=\frac{J_3(\omega)\omega^{k-2}}{2\phi^k(\omega)}\frac{N^2}{\log^k N}(1+o(1)), \qquad (6)$$

where $\omega=\prod_{2\le P} P, \phi(\omega)=\prod_{2\le P}(P-1)$, （7）

ω is called primorials, $\phi(\omega)$ Euler function.

(6) is a generalization of the prime number theorem $\pi(N)=\frac{N}{\log N}(1+o(1))$ [1].

Substituting (5) and (7) into (6) we have the best asymptotic formula

$$\pi_{k-1}(N,3)=\frac{1}{2}\prod_{2\le P<k}\frac{P^{k-2}}{(P-1)^{k-1}}\prod_{k\le P}\frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}}\frac{N^2}{\log^k N}(1+o(1)). \qquad (8)$$

From (8) we are able to find the smallest solution $\pi_{k-1}(N_0,3)>1$ for large k.

Grosswald and Zagier obtain heuristically even asymptotic formulae [2]. Let $k=2$ and $d=2$. From (1) we have twin primes theorem: $P_2=P_1+2$. The Green-Tao theorem is false, because they do not prove the twin primes theorem and arithmetic progressions of primes [3].

Example 1. Let $k=3$. From (2) we have

$$P_3=2P_2-P_1. \qquad (9)$$

From (5) we have

$$J_3(\omega)=\prod_{3\le P}(P-1)(P-2)\to\infty \text{ as } \omega\to\infty. \qquad (10)$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3 are primes. From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\frac{N^2}{\log^3 N}(1+o(1))=0.66016\frac{N^2}{\log^3 N}(1+o(1)). \qquad (11)$$

Example 2. Let $k=4$. From (2) we have

$$P_3=2P_2-P_1, \qquad P_4=3P_2-2P_1. \tag{12}$$

From (5) we have

$$J_3(\omega)=2\prod_{5\le P}(P-1)(P-3)\to\infty \text{ as } \omega\to\infty. \tag{13}$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3 and P_4 are all primes. From (8) we have the best asymptotic formula

$$\pi_3(N,3)=\frac{9}{4}\prod_{5\le P}\frac{P^2(P-3)}{(P-1)^3}\frac{N^2}{\log^4 N}(1+o(1)). \tag{14}$$

Example 3. Let $k=5$. From (2) we have

$$P_3=2P_2-P_1, \quad P_4=3P_2-2P_1, \qquad P_5=4P_2-3P_1. \tag{15}$$

From (5) we have

$$J_3(\omega)=2\prod_{5\le P}(P-1)(P-4)\to\infty \text{ as } \omega\to\infty. \tag{16}$$

We prove that there exist infinitely many primes P_1 and P_2 such that P_3, P_4 and P_5 are all primes. From (8) we have the best asymptotic formula

$$\pi_4(N,3)=\frac{27}{4}\prod_{5\le P}\frac{P^3(P-4)}{(P-1)^4}\frac{N^2}{\log^5 N}(1+o(1)). \tag{17}$$

Theorem 2. From (1) we obtain

$$P_4=P_3+P_2-P_1, \qquad P_j=P_3+(j-3)P_2-(j-3)P_1, \quad 4\le j\le k. \tag{18}$$

We have Jiang function [1]

$$J_4(\omega)=\prod_{3\le P}((P-1)^3-X(P)), \tag{19}$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=4}^{k}(q_3+(j-3)q_2-(j-3)q_1)\equiv 0(\bmod P), \qquad (20)$$

where $q_i=1,2,\cdots,P-1,\quad i=1,2,3.$

From (20) we have

$$J_4(\omega)=\prod_{3\le P<(k-1)}(P-1)^2\prod_{(k-1)\le P}(P-1)\left[(P-1)^2-(P-2)(k-3)\right]\to\infty$$

as $\omega\to\infty$. (21)

We prove there exist infinitely many primes P_1, P_2 and P_3 such that $P_4,\cdots,P_k$ are all primes for all $k\ge 4$.

We have the best asymptotic formula [1]

$$\pi_{k-2}(N,4)=\left|\left\{P_3+(j-3)P_2-(j-3)P_1=\text{prime},4\le j\le k,P_1,P_2,P_3\le N\right\}\right|$$
$$=\frac{J_4(\omega)\omega^{k-3}}{6\phi^k(\omega)}\frac{N^3}{\log^k N}(1+o(1)). \qquad (22)$$

Substituting (7) and (21) into (22) we have

$$\pi_{k-2}(N,4)$$
$$=\frac{1}{6}\prod_{2\le P<(k-1)}\frac{P^{k-3}}{(P-1)^{k-2}}\prod_{(k-1)\le P}\frac{P^{k-3}[(P-1)^2-(P-2)(k-3)]}{(P-1)^{k-1}}\frac{N^3}{\log^k N}(1+o(1)). \qquad (23)$$

From (23) we are able to find the smallest solution $\pi_{k-2}(N_0,4)>1$ for large k.

Example 4. Let $k=4$. From (18) we have

$$P_4=P_3+P_2-P_1 \qquad (24)$$

From (21) we have

$$J_4(\omega)=\prod_{3\le P}(P-1)(P^2-3P+3)\to\infty \text{ as } \omega\to\infty. \qquad (25)$$

We prove there exist infinitely many primes P_1, P_2 and P_3 such that P_4 are primes.From (23) we have

$$\pi_2(N,4)=\frac{1}{3}\prod_{3\le P}\left(1+\frac{1}{(P-1)^3}\right)\frac{N^3}{\log^4 N}(1+o(1)). \qquad (26)$$

From (1) We obtain the following equations:

$$\pi_{k-3}(N,5)=\left|\left\{P_4+(j-3)P_3-(j-2)P_2+P_1=\text{prime}, 5\le j\le k, P_1,\cdots,P_4\le N\right\}\right|$$
$$=\frac{1}{24}\frac{J_5(\omega)\omega^{k-4}}{\phi^k(\omega)}\frac{N^4}{\log^k N}(1+o(1)) \qquad (27)$$

$$\pi_{k-4}(N,6)=\left|\left\{P_5+(j-4)P_4-(j-4)P_3-P_2+P_1=\text{prime}, 6\le j\le k, P_1,\cdots,P_5\le N\right\}\right|$$
$$=\frac{1}{120}\frac{J_6(\omega)\omega^{k-5}}{\phi^k(\omega)}\frac{N^5}{\log^k N}(1+o(1)) \qquad (28)$$

Theorem 3. We define arithmetic progressions of primes:

$$P_{i+1}=P_1^2+di, i=1,2,\cdots,k-1. \qquad (29)$$

From (29) we have

$$P_3=2P_2-P_1^2, \quad P_j=(j-1)P_2-(j-2)P_1^2, \quad 3\le j\le k. \qquad (30)$$

We have Jiang function [1]

$$J_3(\omega)=\prod_{3\le P}\left[(P-1)^2-X(P)\right], \qquad (31)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{j=3}^{k}\left[(j-1)q_2-(j-2)q_1^2\right]\equiv 0(\text{mod } P), \qquad (32)$$

where $q_1=1,2,\cdots,P-1$, $q_2=1,2,\cdots,P-1$.

From (32) we have

$$J_3(\omega)=\prod_{3\le P<k}(P-1)\prod_{k\le P}(P-1)(P-k+1)\to\infty \text{ as } \omega\to\infty. \qquad (33)$$

We prove that there exist infinitely many primes P_1 and P_2 such that $P_3,\cdots,P_k$ are all primes for all $k\ge 3$. We have the best asymptotic formula [1]

$$\pi_{k-1}(N,3)=\left|\left\{(j-1)P_2-(j-2)P_1^2=\text{prime},3\le j\le k,P_1,P_2\le N\right\}\right|$$

$$=\frac{1}{2^{k-1}}\frac{J_3(\omega)\omega^{k-2}}{\phi^k(\omega)}\frac{N^2}{\log^k N}(1+o(1)). \quad (34)$$

Substituting (7) and (33) into (34) we have

$$\pi_{k-1}(N,3)=\frac{1}{2^{k-1}}\prod_{2\le P<k}\frac{P^{k-2}}{(P-1)^{k-1}}\prod_{k\le P}\frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}}\frac{N^2}{\log^k N}(1+o(1)). \quad (35)$$

Theorem 4. We define arithmetic progressions of primes:

$$P_{i+1}=P_1^5+di, i=1,2,\cdots,k-1. \quad (36)$$

From (36) we have

$$P_4=P_3+P_2-P_1^5,\quad P_j=P_3+(j-3)P_2-(j-3)P_1^5,\quad 4\le j\le k. \quad (37)$$

We have Jiang function [1]

$$J_4(\omega)=\prod_{3\le P<(k-1)}(P-1)^2\prod_{(k-1)\le P}(P-1)\left[(P-1)^2-(P-2)(k-3)\right]\to\infty$$

$$\text{as } \omega\to\infty. \quad (38)$$

We prove that there exist infinitely many primes P_1, P_2 and P_3 such that $P_4,\cdots,P_k$ are all primes for all $k\ge 4$..

We have the best asymptotic formula

$$\pi_{k-2}(N,4)=\left|\left\{P_3+(j-3)P_2-(j-3)P_1^5=\text{prime},4\le j\le k,P_1,P_2,P_3\le N\right\}\right|$$

$$=\frac{1}{6\times 5^{k-3}}\frac{J_4(\omega)\omega^{k-3}}{\phi^k(\omega)}\frac{N^3}{\log^k N}(1+o(1)). \quad (39)$$

Theorem 5. We define arithmetic progressions of primes:

$$P_{j+1}=P_1^n+di, i=1,2,\cdots,k-1, n\ge 1. \quad (40)$$

From (40) we have

$$P_3 = 2P_2 - P_1^n, \quad P_j = (j-1)P_2 - (j-2)P_1^n . \qquad (41)$$

We have Jiang function [1]

$$J_3(\omega) = \prod_{3 \le P < k}(P-1)\prod_{k \le P}(P-1)(P-k+1) \to \infty \quad \text{as} \quad \omega \to \infty . \qquad (42)$$

We prove that there exist infinitely many primes P_1 and P_2 such that $P_3, \cdots, P_k$ are all primes for all $k \ge 3$.

We have the best asymptotic formula [1]

$$\pi_{k-1}(N,3) = \left|\left\{(j-1)P_2 - (j-2)P_1^n = \text{prime}, 3 \le j \le k, P_1, P_2 \le N\right\}\right|$$
$$= \frac{1}{2 \times n^{k-2}} \frac{J_3(\omega)\omega^{k-2}}{\phi^k(\omega)} \frac{N^2}{\log^k N} . \qquad (43)$$

Substituting (7) and (42) into (43) we have

$$\pi_{k-1}(N,3) = \frac{1}{2 \times n^{k-2}} \prod_{2 \le P < k} \frac{P^{k-2}}{(P-1)^{k-1}} \prod_{k \le P} \frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}} \frac{N^2}{\log^k N}(1+o(1)) . \qquad (44)$$

Theorem 6. We define arithmetic progressions of primes:

$$P_{j+1} = P_1^n + di, i = 1, 2, \cdots, k-1, n \ge 1 . \qquad (45)$$

From (45) we have

$$P_4 = P_3 + P_2 - P_1^n, \quad P_j = P_3 + (j-3)P_2 - (j-3)P_1^n, \quad 4 \le j \le k . \qquad (46)$$

We have Jiang function [1]

$$J_4(\omega) = \prod_{3 \le P < (k-1)}(P-1)^2 \prod_{(k-1) \le P}(P-1)\left[(P-1)^2 - (P-2)(k-3)\right] \to \infty$$
as $\omega \to \infty$. (47)

We prove that there exist infinitely many primes P_1, P_2 and P_3 such that $P_4, \cdots, P_k$ are all primes for all $k \ge 4$..

We have the best asymptotic formula [1]

$$\pi_{k-2}(N,4)=\left|\left\{P_3+(j-3)P_2-(j-3)P_1^n=\text{prime}, 4\le j\le k, P_1,P_2,P_3\le N\right\}\right|$$

$$=\frac{1}{6\times n^{k-3}}\frac{J_4(\omega)\omega^{k-3}}{\phi^k(\omega)}\frac{N^3}{\log^k N}(1+o(1)). \quad (48)$$

Substituting (7) and (47) into (48) we have

$$\pi_{k-2}(N,4)$$
$$=\frac{1}{6\times n^{k-3}}\prod_{2\le P<(k-1)}\frac{P^{k-3}}{(P-1)^{k-2}}\prod_{(k-1)\le P}\frac{P^{k-3}[(P-1)^2-(P-2)(k-3)]}{(P-1)^{k-1}}\frac{N^3}{\log^k N}(1+o(1)).$$

(49)

Theorem 7. We define another arithmetic progressions of primes [1, 4]:

$$P_{i+1}=P_1+\omega_g i, i=1,2,\cdots,k-1 \quad (50)$$

where $\omega_g=\prod_{2\le P\le P_g}$ is called a common difference, P_g is called $g-$ th prime.

We have Jiang function [1, 4]

$$J_2(\omega)=\prod_{3\le P}(P-1-X(P)), \quad (51)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{i=1}^{k-1}(q+\omega_g i)\equiv 0(\bmod P), \quad (52)$$

where $q=1,2,\cdots,P-1$.

If $P|\omega_g$, then $X(P)=0$; $X(P)=k-1$ otherwise. From (52) we have

$$J_2(\omega)=\prod_{3\le P\le P_g}(P-1)\prod_{P_{g+1}\le P}(P-k). \quad (53)$$

If $k=P_{g+1}$ then $J_2(P_{g+1})=0$, $J_2(\omega)=0$, there exist finite primes P_1 such that $P_2,\cdots,P_k$ are all primes. If $k<P_{g+1}$ then $J_2(\omega)\ne 0$, there exist infinitely many

primes P_1 such that $P_2, \cdots, P_k$ are all primes. We have the best asymptotic formula [1,4]

$$\pi_k(N,2) = \left|\left\{P_1 + \omega_g i = \text{prime}, 1 \le i \le k-1, P_1 \le N\right\}\right|$$
$$= \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}(1+o(1)). \qquad (54)$$

Let $k = P_{g+1} - 1$. From (50) we have

$$P_{i+1} = P_1 + \omega_g i, i = 1, 2, \cdots, P_{g+1} - 2. \qquad (55)$$

From (53) we have [1, 4]

$$J_2(\omega) = \prod_{3 \le P \le P_g} (P-1) \prod_{P_{g+1} \le P} (P - P_{g+1} + 1) \to \infty \quad \text{as} \quad \omega \to \infty \qquad (56)$$

We prove that there exist infinitely many primes P_1 such that $P_2, \cdots, P_{P_{g+1}-1}$ are all primes for all P_{g+1}.

Substituting (7) and (56) into (54) we have

$$\pi_{P_{g+1}-1}(N,2) =$$
$$\prod_{2 \le P \le P_g} \left(\frac{P}{P-1}\right)^{P_{g+1}-2} \prod_{P_{g+1} \le P} = \frac{P^{P_{g+1}-2}(P - P_{g+1} + 1)}{(P-1)^{P_{g+1}-1}} \frac{N}{(\log N)^{P_{g+1}-1}}(1+o(1)). \qquad (57)$$

From (57) we are able to find the smallest solutions $\pi_{P_{g+1}-1}(N_0, 2) > 1$ for large P_{g+1}.

Example 5. Let $P_1 = 2$, $\omega_1 = 2$, $P_2 = 3$. From (55) we have the twin primes theorem

$$P_2 = P_1 + 2. \qquad (58)$$

From (56) we have

$$J_2(\omega)=\prod_{3\le P}(P-2)\to\infty \quad \text{as} \quad \omega\to\infty, \tag{59}$$

We prove that there exist infinitely many primes P_1 such that P_2 are primes. From (57) we have the best asymptotic formula

$$\pi_2(N,2)=2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}(1+o(1)). \tag{60}$$

Example 6. Let $P_2=3$, $\omega_2=6$, $P_3=5$. From (55) we have

$$P_{i+1}=P_1+6i, i=1,2,3. \tag{61}$$

From (56) we have

$$J_2(\omega)=2\prod_{5\le P}(P-4)\to\infty \quad \text{as} \quad \omega\to\infty. \tag{62}$$

We prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are all primes. From (57) we have the best asymptotic formula

$$\pi_4(N,2)=27\prod_{5\le P}\frac{P^3(P-4)}{(P-1)^4}\frac{N}{\log^4 N}(1+o(1)). \tag{63}$$

Example 7. Let $P_9=23$, $\omega_9=223092870$, $P_{10}=29$. From (55) we have

$$P_{i+1}=P_1+223092870i, i=1,2,\cdots,27. \tag{64}$$

From (56) we have

$$J_2(\omega)=36495360\prod_{29\le P}(P-28)\to\infty \quad \text{as} \quad \omega\to\infty \tag{65}$$

We prove that there exist infinitely many primes P_1 such that $P_2,\cdots,P_{28}$ are all primes. From (57) we have the best asymptotic formula

$$\pi_{28}(N,2)=\prod_{2\le P\le 23}\left(\frac{P}{P-1}\right)^{27}\prod_{29\le P}\frac{P^{27}(P-28)}{(P-1)^{28}}\frac{N}{\log^{28} N}(1+o(1)) \quad .$$

（66）

From (66) we are able to find the smallest solutions $\pi_{28}(N_0,2)>1$.

Theorem 8. We define another arithmetic progressions of primes:

$$P_{i+1}=P_1^n+\omega_g i, i=1,2,\cdots,k-1, n\ge 1. \quad (67)$$

We have Jiang function [1]

$$J_2(\omega)=\prod_{3\le P}(P-1-X(P)), \quad (68)$$

$X(P)$ denotes the number of solutions for the following congruence

$$\prod_{i=1}^{k-1}(q_1^n+\omega_g i)\equiv 0(\bmod P), \quad (69)$$

where $q_1=1,2,\cdots P-1$.

If $X(P)=P-1$ and $J_2(P)=0$, then there exist finite primes P_1 such that $P_2,\cdots P_k$ are primes. If $X(P)<P-1$ and $J_2(\omega)\ne 0$, then there exist infinitely many primes P_1 such that $P_2,\cdots,P_k$ are all prime for all P_g.

We have the best asymptotic formula [1]

$$\pi_k(N,2)=\left|\left\{P_1^n+\omega_g i=\text{prime}, 1\le i\le k-1, P_1\le N\right\}\right|$$
$$=\frac{1}{n^{k-1}}\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}(1+o(1)). \quad (70)$$

Example 8. Let $n=2$, $k=3$ and $\omega_g=6$. From (67) we have

$$P_2=P_1^2+6, \quad P_3=P_1^2+12, \quad P_4=P_1^2+18 \quad (71)$$

We have Jiang function [1]

$$J_2(\omega)=2\prod_{5\le P}\left(P-4-\left(\frac{-6}{P}\right)-\left(\frac{-3}{P}\right)-\left(\frac{-2}{P}\right)\right)\to\infty \text{ as } \omega\to\infty \quad (72)$$

where $\left(\frac{-6}{P}\right), \left(\frac{-3}{P}\right)$ and $\left(\frac{-2}{P}\right)$ denote the Legendre symbols.

We prove that there exist infinitely many primes P_1 such that P_2, P_3 and P_4 are all primes. We have the best asymptotic formula [1]

$$\pi_4(N,2) = \left|\left\{P_1^2 + 6i = \text{prime}, i = 1,2,3, P_1 \le N\right\}\right|$$
$$= \frac{1}{8}\frac{J_2(\omega)\omega^3}{\phi^4(\omega)}\frac{N}{\log^4 N}(1+o(1)). \quad (73)$$

We shall move on to the study of the generalized arithmetic progression of consecutive primes [5]. A generalized arithmetic progression of consecutive primes is defined to be the sequence of primes,

$$P, P+\omega_g, P+2\omega_g, \cdots, P+k\omega_g \text{ and } P^n+\omega_g, P^n+2\omega_g, \cdots, P^n+k\omega_g,$$

where P is the first term, $n \ge 2$. For example, 5, 11, 17, 23, and 31, 37, 43, is a generalized arithmetic progression of primes with $P=5$, $\omega_g=6$, $k=3$ and $n=2$.

Theorem 9. We define the generalized arithmetic progressions:

$$P_i = P + i\omega_g \quad \text{and} \quad P_{k+i} = P^n + i\omega_g$$

(74)

where $i = 1, \cdots, k, n \ge 2$.

We have Jiang function [1]

$$J_2(\omega) = \prod_{3 \le P}\left(P - 1 - X(p)\right), \quad (75)$$

$X(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k}(q+i\omega_g)(q^n+i\omega_g) \equiv 0 (\bmod P), \quad (76)$$

$q = 1, 2, \cdots P-1$.

If $X(P) = P-1$ and $J_2(P) = 0$, then there exist finite primes P such that $P_1, P_2, \cdots, P_{2k}$ are primes. If $X(P) < P-1$, $J_2(\omega) \neq 0$, then there exist infinitely many primes P such that $P_1, P_2, \cdots, P_{2k}$ are all primes.

If $J_2(\omega) \neq 0$, we have the best asymptotic formula of the number of primes $P \leq N$ [1]

$$\pi_{2k+1}(N,2) = \frac{J_2(\omega)\omega^{2k}}{n^k \phi^{2k+1}(\omega)} \frac{N}{(\log N)^{2k+1}}(1+o(1)). \quad (77)$$

Example 9. Let $\omega_g = 6, k = 3$, and $n = 2$. From (74) we have

$$P_1 = P+6, P_2 = P+12, P_3 = P+18 \quad \text{and}$$

$$P_4 = P^2+6, P_5 = P^2+12, P_6 = P^2+18. \quad (78)$$

We have Jiang function [1]

$$J_2(\omega) = 12672 \prod_{23 \leq P} \left(P - 7 - \left(\frac{-2}{P}\right) - \left(\frac{-3}{P}\right) - \left(\frac{-6}{P}\right) \right) \neq 0 \quad (79)$$

Since $J_2(\omega) \to \infty$ as $\omega \to \infty$, there exist infinitely many primes P such that $P_1, \cdots, P_6$ are all primes.

From (77) we have

$$\pi_7(N,2) = \frac{J_2(\omega)\omega^6}{8\phi^7(\omega)} \frac{N}{\log^7 N}(1+o(1)). \quad (80)$$

Remark. Theorems 1, 3 and 5 have the same Jiang function $J_3(\omega)$ and theorems 2, 4 and 6 the same Jiang function $J_4(\omega)$ which have the same character. All irreducible prime equations have the Jiang functions and the best asymptotic formulas [1]. In our theory there are no almost primes, for example $P_1 = P_2P_3 + 2$ and $N = P_1 + P_2P_3$ are theorems of three genuine primes. Using the sieve method, circle method, ergodic theory, harmonic analysis, discrete geometry, and combinatories they are not able to attack twin primes conjecture, Goldbach conjecture, long arithmetic progressions of primes and other problems of primes and to find the best asymptotic formulas. The proofs of Szemerédic's theorem are false, because they do not prove the twin primes theorem and arithmetic progressions of primes [3, 6-10].

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