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The New Prime theorem（6）

$P, jP+9-j (j=1,2,4,5,7,8)$

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Abstract

Using Jiang function we prove that there exist infinitely many primes P such that each $jP+9-j$ is a prime.

Theorem

$$P, jP+9-j (j=1,2,4,5,7,8) \qquad (1)$$

There exist infinitely many primes P such that each of $jP+9-j$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega)=\prod_P [P-1-\chi(P)], \qquad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod (jq+9-j)(j=1,2,4,5,7,8)\equiv 0 \pmod P \qquad (3)$$

$q=1,\cdots,P-1$.

From (3) we have $\chi(2)=0$, $\chi(3)=1$, $\chi(5)=3$, $\chi(7)=3$, $\chi(P)=6$ otherwise.

From (3) and (2) we have

$$J_2(\omega)=3\prod_{11\le P}(P-7)\neq 0. \qquad (4)$$

We prove that there exist infinitely many primes P such that $jP+9-j$ is a prime.

We have the best asymptotic formula [1]

$$\pi_7(N,2) = \left|\{P \le N : jP + 9 - j = prime\}\right| \sim \frac{J_2(\omega)\omega^6}{\phi^7(\omega)} \frac{N}{\log^7 N}, \quad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.