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The Derivation of Maxwell's Displacement Current

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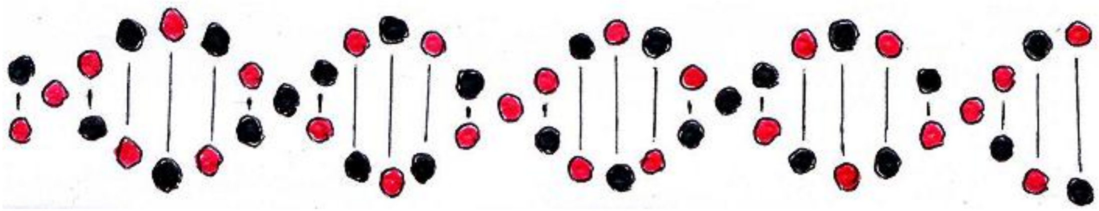

Abstract. ***An undergraduate student learning about Maxwell's equations for the first time is typically shown, that with the exception of a part of one of these equations, they can be derived from first principles with reference to nineteenth century experiments. The exception referred to relates to the displacement current term in Ampère's Circuital Law. While the existence of displacement current can be justified retrospectively due to its role in the derivation of the electromagnetic wave equations, it's very difficult to present from first principles, a concise theoretical proof of its existence. The normal textbook method is to justify its existence in connection with the conservation of electric charge in a charging or discharging capacitor, but the major problem with this method is that the displacement current term that is so derived is not the term that is used in the derivation of the electromagnetic wave equations. The textbook derivation is that of a term involving the electrostatic E field, while the E field that is involved in electromagnetic radiation arises from time-varying electromagnetic induction. The important question of how to derive the latter kind of displacement current will be the focus of this article.***

Ampère's Circuital Law

I. The textbooks present *Ampère's Circuital Law* in the dynamic state, in the form,

$$\nabla\times\mathbf{B} = \mu(\mathbf{J} + \varepsilon\partial\mathbf{E}_S/\partial t) \qquad \textbf{(1)}$$

where $\varepsilon\partial\mathbf{E}_S/\partial t$ is said to be Maxwell's displacement current, with the term $\mathbf{E}_S$, satisfying *Gauss's Law of Electrostatics*,

$$\nabla\cdot\mathbf{E}_S = \rho/\varepsilon \qquad \textbf{(2)}$$

James Clerk Maxwell himself, meanwhile, did derive this electrostatic-based form of displacement current in his 1861 paper, ***"On Physical Lines of Force"***, [1], and equation **(1)** above appears as equation (112) in Part III of that paper.

While Maxwell justified the existence of the term, $\varepsilon\partial\mathbf{E}_S/\partial t$, in connection with elastic displacement in a dielectric, the modern textbooks, on the other hand, justify it on the basis that it is required to ensure that the divergence of the right-hand-side of equation **(1)** remains zero even in the dynamic state. Either way, however, it is wrongly supposed that this displacement current is the cause of a magnetic field maintaining continuity in the space between the plates of a charging or a discharging capacitor. The Biot-Savart Law makes it clear that conduction current, **J**, is exclusively the source of all magnetic fields, and hence maintaining unbroken continuity of the magnetic field across the plates of a capacitor in the dynamic state, can be no exception. This means that there must exist a deeper undercurrent of aethereal electric fluid flowing through the dielectric between the two plates during the charging or discharging process, but this will not be represented by $\varepsilon\partial\mathbf{E}_S/\partial t$.

Also, contrary to conventional wisdom, neither is the $\varepsilon\partial\mathbf{E}_S/\partial t$ term the term that Maxwell used in his 1865 paper, ***"A Dynamical Theory of the Electromagnetic Field"***, [2], in order to derive the electromagnetic wave equation in the magnetic field, **H**. The displacement current term that was used in the 1865 paper for that derivation, involved the electric field, $\mathbf{E}_K$, that arises in time-varying electromagnetic induction, and apart from knowing that it is involved in electromagnetic radiation, there is no way of directly proving its existence experimentally. Neither is there any easy way to prove its existence theoretically from first principles. This will however be attempted in the following sections.

The Lorenz Gauge

II. Equation **(1)** in the section above is actually Ampère's Circuital Law in the *Lorenz gauge*, although the modern textbooks wouldn't state it as such, and neither would Maxwell have used that term. But it's easy to demonstrate that this is the case. We know that $\mathbf{B} = \nabla\times\mathbf{A}$, and so we can write equation **(1)** in its pure hydrodynamical form,

$$\nabla\times(\nabla\times\mathbf{A}) = -\nabla^2\mathbf{A} + \nabla(\nabla\cdot\mathbf{A}) \qquad \textbf{(3)}$$

Meanwhile, the Lorenz gauge is defined by the equation,

$$\nabla\cdot\mathbf{A} = -\mu\varepsilon\partial\psi/\partial t \qquad \textbf{(4)}$$

where ψ is the electrostatic scalar potential. Since $-\nabla\psi = \mathbf{E}_S$, then if we substitute **(4)** into **(3)**, we obtain,

$$\nabla\times(\nabla\times\mathbf{A}) = -\nabla^2\mathbf{A} + \mu\varepsilon\partial\mathbf{E}_S/\partial t \qquad \textbf{(5)}$$

While we have now just confirmed that both Maxwell's original displacement current and the textbook displacement current are a product of the Lorenz gauge, this displacement current is not, however, that which Maxwell later used to derive the electromagnetic wave equations. It will be shown that the displacement current as is used in the derivation of the electromagnetic wave equations is in fact included in that other term that appears on the right-hand-side of equation **(5)**. We're talking about $-\nabla^2\mathbf{A}$. In Maxwell's 1873 Treatise, he equated $\mu\mathbf{J}$ to $-\nabla^2\mathbf{A}$ simply by comparing the expression $\nabla\times(\nabla\times\mathbf{A})$ with Ampère's Circuital Law, [3], but a more concise way to do so has been demonstrated by Dr. Z.-C. Liang, [4]. Dr. Liang performed a mathematical derivation at a fundamental level, prior to linking any of the parameters to electromagnetism. He defined a vector field, **A**, in a manner, that when we replace his constant, α_s, with the magnetic permeability, μ, it is in effect the definition for the magnetic vector potential, **A**, in the Lorenz gauge, as in,

$$\mathbf{A}(\mathbf{r}, t) = (\mu/4\pi)\int_V (\mathbf{J}(\mathbf{r}', t')\mathrm{d}V')/r \qquad \textbf{(6)}$$

He showed in his paper referenced at, [4], that,

$$-\nabla^2\mathbf{A} = \mu\mathbf{J} \qquad \textbf{(7)}$$

In the next sections, it will be shown that the substantive displacement current that is involved in electromagnetic radiation, is a particular manifestation of the conduction current, **J**, in equation **(7)**, rather than being a separate term in its own right.

The Coulomb Gauge

III. The *Coulomb gauge*, or *transverse gauge*, or *radiation gauge*, takes the form,

$$\nabla\cdot\mathbf{A} = 0 \qquad \textbf{(8)}$$

It would be meaningless, though, to convert equation **(3)** into the Coulomb gauge without giving some kind of physical context to justify the conversion. We would ideally want a situation in which the sum of sinks and sources cancels. One consideration would be the case of a rotating electron-positron dipole in which the electron and the positron are in mutual circular orbit, [5], [6]. If the electron is a sink for the all-pervading electric fluid aether, [7], with the positron being a source, then **A** could represent a circulating momentum in a dipolar vortex in this electric fluid. In this context, **A** and **J** will merge and become the same thing, [8]. It should be noted though, that due to cyclical variations in the electric fluid density when an electromagnetic wave is passing through, that the Coulomb gauge is really only a time-average condition. The electromagnetic wave equations don't take such finer details into consideration.

When Ampère's Circuital Law is derived from a closed electric circuit, the electrostatic-based displacement current term in equation **(1)** vanishes. And when it vanishes, then from equation **(5)**, it follows that $\nabla(\nabla\cdot\mathbf{A})$ also vanishes, just as it would for $\nabla\cdot\mathbf{A} = 0$. In this closed-circuit context, which is the Coulomb gauge, and considering equations **(5)** and **(7)** along with $\mathbf{B} = \nabla\times\mathbf{A}$, it follows that,

$$\nabla\times\mathbf{B} = \mu\mathbf{J} \qquad \textbf{(9)}$$

and hence, if space is densely packed with tiny dipolar aether vortices, [9], it follows that equation **(9)** will be Ampère's Circuital Law as applied in the steady state, to the individual vortices, which are like tiny, closed electric circuits. Equation **(9)** will then take the form,

$$\nabla\times\mathbf{B} = \mu\mathbf{A} \qquad \textbf{(10)}$$

Electromagnetic Radiation

IV. It is proposed that electromagnetic radiation constitutes a relay of excess electric fluid from the positron of one vortex into the electron of its immediate neighbour, [10]. Since the driving electric field, $\mathbf{E}_K$, will be sourced between the positrons and electrons of immediately adjacent vortices, and since these particles are moving tangentially within their respective vortices, the $\mathbf{E}_K$ force will be a tangential or an axial action, causing the vortices to angularly accelerate, and this will invariably be in the form of a precession. It is proposed that this precession is what causes the excess electric fluid to emerge from its positron and swirl across into the electron of its immediate neighbour, and that this cycle repeats in a wave-like manner through the sea of vortices. Electromagnetic waves therefore constitute a relay of tiny electric currents at the most fundamental level, [11], swirling from vortex to vortex through an all-pervading sea of such vortices. This sea of vortices constitutes the electromagnetic wave-carrying medium. Wireless radiation is therefore a relay of the time-varying electromagnetic induction process on a microscopic scale, [12], and indeed more likely on a picoscopic scale, where,

$$\mathbf{E}_K = -\partial\mathbf{A}/\partial t \qquad \textbf{(11)}$$

Meanwhile, during the passage of electromagnetic waves, **A** will be in an angularly oscillating state, hence obeying the equation,

$$\mathbf{A} = -\varepsilon\partial^2\mathbf{A}/\partial t^2 \tag{12}$$

where ε is the elastic constant. Substituting equation **(12)** into equation **(10)**, we obtain,

$$\nabla\times\mathbf{B} = -\mu\varepsilon\partial^2\mathbf{A}/\partial t^2 \tag{13}$$

Then from $\mathbf{E}_K = -\partial\mathbf{A}/\partial t$, equation **(13)** then becomes,

$$\nabla\times\mathbf{B} = \mu\varepsilon\partial\mathbf{E}_K/\partial t \tag{14}$$

This is the form of Ampère's Circuital Law as would apply in starlight in space, far from any laboratory electrical apparatus. It's very important to note, though, that the displacement current term, $\mu\varepsilon\partial\mathbf{E}_K/\partial t$, in equation **(14)** is not the displacement current that was originally derived by Maxwell in his 1861 paper, and neither is it the displacement current that is derived in the modern textbooks. The displacement current in the Coulomb gauge has got nothing to do with capacitors.

Finally, combining equations, **(7)**, **(9)**, (**10**), and (**13**), leads to the primary electromagnetic wave equation in **A**, derived by Maxwell in his 1873 Treatise, **[3]**,

$$\nabla^2\mathbf{A} = \mu\varepsilon\partial^2\mathbf{A}/\partial t^2 \tag{15}$$

where $\mu\varepsilon = 1/c^2$, with c being the speed of light. Then, if we take the curl of equation **(15)**, from $\nabla\times\mathbf{A} = \mathbf{B}$ we obtain,

$$\nabla^2\mathbf{B} = \mu\varepsilon\partial^2\mathbf{B}/\partial t^2 \tag{16}$$

as was derived by Maxwell in 1864, **[2]**, for **H**, where $\mathbf{B} = \mu\mathbf{H}$, and meanwhile, if we differentiate equation **(15)** with respect to time, then from $\mathbf{E}_K = -\partial\mathbf{A}/\partial t$, we obtain,

$$\nabla^2\mathbf{E}_K = \mu\varepsilon\partial^2\mathbf{E}_K/\partial t^2 \tag{17}$$

Conclusion

V. The electrostatic-based displacement current, $\varepsilon\partial\mathbf{E}_S/\partial t$, where $\mathbf{E}_S$ is the electrostatic field of Coulomb's Law, was originally derived by Maxwell in his 1861 paper in connection with linear polarization in a dielectric, and it is derived in modern textbooks in connection with conservation of electric charge in a charging or discharging capacitor. This version of displacement current is, however, merely a reaction with no causative effect. It's a reaction to an externally applied electric field. It is not the electric current that causes continuity of the magnetic field between the plates of a charging or a discharging capacitor. The magnetic field between the plates of the charging or discharging capacitor is caused by a very real conduction current comprising a flow of the fundamental electric fluid. This electric fluid, with which everything is made, **[7]**, is otherwise known as *the aether*. It flows across the gap between the capacitor plates until it is blocked by the back-EMF caused by dielectric polarization, or by the accumulating electric charge on the plates.

Then, as regards the displacement current that is used in the derivation of the electromagnetic wave equations, it must be a special case of the closed-circuit conduction current, and in order to be compatible with the Maxwell-Faraday equation in these derivations, it cannot involve the electrostatic based $\mathbf{E}_S$ field. It must instead involve $\mathbf{E}_K$, this being the electric field, $-\partial\mathbf{A}/\partial t$, that arises from time-varying electromagnetic induction. In his 1865 paper, Maxwell's derivation of the electromagnetic wave equation in **H** did use this electromagnetic induction version of the displacement current, and so Maxwell was firmly working in what later became known as the ***Coulomb Gauge***, or the ***Radiation Gauge***, or the ***Transverse Gauge.***

Meanwhile, we also need a physical context in which to apply a Coulomb gauge-based displacement current. We need space to be filled with tiny dipolar aether vortices, **[5]**, **[6]**, **[9]**, serving as the electromagnetic wave carrying medium, with electromagnetic waves themselves constituting a relay of this electric fluid, swirling through space from vortex to vortex, from the source in one vortex, into the sink in the immediately neighbouring vortex. This fundamental electric fluid flow that occurs in conjunction with electromagnetic waves is the actual displacement current that Maxwell used in 1864 and 1873 in his derivation of the electromagnetic wave equations, and it takes the form $\varepsilon\partial\mathbf{E}_K/\partial t$, where $\mathbf{E}_K$ is the electric field that arises in time-varying electromagnetic induction.

References

[1] Clerk-Maxwell, J., ***"On Physical Lines of Force"***, Philosophical Magazine, Volume XXI, Fourth Series, London, (1861)
http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

[2] Maxwell, J.C., ***"A Dynamical Theory of the Electromagnetic Field"***, Philos. Trans. Roy. Soc. London **155**, pp 459-512 (1865). Abstract: Proceedings of the Royal Society of London 13, pp. 531-536, (1864).
The derivation of the electromagnetic wave equation in **H** begins on page 497 in the first link below. Note how the electrostatic component, Ψ, is eliminated after equation (68), hence leaving the elastic displacement mechanism in the wave as an effect that is connected exclusively with time-varying electromagnetic induction.
http://www.zpenergy.com/downloads/Maxwell_1864_4.pdf
http://www.zpenergy.com/downloads/Maxwell_1864_5.pdf

[3] Maxwell, J.C., ***"A Treatise on Electricity and Magnetism"***, Volume II, Chapter XX, *'Plane Waves'*, Section 790, pp. 389-390 (1873)
https://en.wikisource.org/wiki/A_Treatise_on_Electricity_and_Magnetism/Part_IV/Chapter_XX

[4] Liang, Z.-C., ***"Dark matter and real-particle field theory"***,
"**Appendix B**", *Derivation of field equations*, 2. ***Divergence field***, Equation (B15), (2021)
https://www.researchgate.net/publication/350620139_Dark_matter_and_real-particle_field_theory

[5] Tombe, F.D., ***"The Double Helix Theory of the Magnetic Field"***, (2006)
Galilean Electrodynamics, Volume 24, Number 2, p.34, (March/April 2013)
https://www.researchgate.net/publication/295010637_The_Double_Helix_Theory_of_the_Magnetic_Field

[6] Tombe, F.D., ***"The Double Helix and the Electron-Positron Aether***", (2017)
https://www.researchgate.net/publication/319914395_The_Double_Helix_and_the_Electron-Positron_Aether

[7] O'Neill, John J., ***"PRODIGAL GENIUS, Biography of Nikola Tesla"***, Long Island, New York, 15th July 1944, Fourth Part, paragraph 23, quoting Tesla from his 1907 paper ***"Man's Greatest Achievement"*** which was published in 1930 in the Milwaukee Sentinel,
"Long ago he (mankind) recognized that all perceptible matter comes from a primary substance, of a tenuity beyond conception and filling all space - the Akasha or luminiferous ether - which is acted upon by the life-giving Prana or creative force, calling into existence, in never ending cycles, all things and phenomena. The primary substance, thrown into infinitesimal whirls of prodigious velocity, becomes gross matter; the force subsiding, the motion ceases and matter disappears, reverting to the primary substance".
http://www.rastko.rs/istorija/tesla/oniell-tesla.html

[8] Tombe, F.D., ***"Displacement Current and the Electrotonic State"***, (2008)
https://www.researchgate.net/publication/303112543_Displacement_Current_and_the_Electrotonic_State

[9] Whittaker, E.T., ***"A History of the Theories of Aether and Electricity",*** Chapter 4, pages 100-102, (1910)
"All space, according to the younger (John) Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools."

[10] Lodge, Sir Oliver, ***"Ether (in physics)"***, Encyclopaedia Britannica, Fourteenth Edition, vol. 8, pp. 751-755, (1937)
http://gsjournal.net/Science-Journals/Historical%20PapersMechanics%20/%20Electrodynamics/Download/4105
See pp. 6-7 in the pdf file in the link above, beginning at the paragraph that starts with, ***Possible Structure. −***, and note that while the quote suggests that the ether is incompressible, this is almost certainly not the case. The quote in question, in relation to the speed of light, reads,

"The most probable surmise or guess at present is that the ether is a perfectly incompressible continuous fluid, in a state of fine-grained vortex motion, circulating with that same enormous speed. For it has been partly, though as yet incompletely, shown that such a vortex fluid would transmit waves of the same general nature as light waves— i.e., periodic disturbances across the line of propagation—and would transmit them at a rate of the same order of magnitude as the vortex or circulation speed"

[11] Lorenz, L., ***"On the Identity of the Vibrations of Light with Electrical Currents"*** Annalen der Physik, Volume 131, page 243. English translation in Philosophical Magazine Volume 34, pages 287-301, (1867)
https://www.researchgate.net/publication/323867067_Ludvig_Lorenz_1867_on_Light_and_Electricity

[12] Tombe, F.D., ***"Wireless Radiation Beyond the Near Magnetic Field"***, (2019)
https://www.researchgate.net/publication/335169091_Wireless_Radiation_Beyond_the_Near_Magnetic_Field