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Towards a Discrete Cosmology 14/17

Relativity and discreteness

Links to all series' papers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17.

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Abstract.- Leaving aside the fact that special relativity is a theory built on the infinitist spacetime continuum, and that this spacetime continuum is inconsistent (see paper 3 of this series), here it will be proved that special relativity is not compatible with discrete space and time. Therefore, the relativistic inertial length contractions, inertial time dilations and local simultaneities could be only apparent, as is apparent the continuous motion in a film, or the refractive deformations; or even continuous motion in a discrete universe. It also happens that the relativistic Lorentz factor, which is crucial in the Lorentz Transformation, coincides with the conversion factor between the continuous and the discrete versions of Pythagoras Theorem.

Keywords: special relativity, Lorentz factor, discrete space and time, Discrete Pythagoras Theorem.

1. Introduction

Although the main purpose of this article is to formally demonstrate the incompatibility of the special theory of relativity (SR) with a discrete space and time scenario, we will make a couple of preliminary considerations about SR. The first has to do with the excessive weight that, in my opinion, SR has in contemporary physics: SR seems to have become a new basic law of logic: if something does not agree with SR, that something, whatever it is, cannot be true. The other consideration has to do with the experimental support of the theory. Indeed, it is often forgotten that:

1. Experimental confirmations of SR (almost always non-inertial (!) time dilations) can also be confirming other theories. For example, of the theory of absolute space and time of H. E. Ives ([26, 24, 23, 25].

2. Some of the observed relativistic spacetime deformations may be only apparent, as are all refractive deformations.

3. Or they could be consequences of considering as continuous a spacetime that in fact is discontinuous, discrete. The last section of this paper proves the factor that converts between the continuous and the discrete version of Pythagoras' Theorem is just the relativistic Lorentz factor.

4. The confirmations have to be simultaneously symmetric. Indeed, let A and B be two inertial reference frames in relative motion:

    - In the direction of relative motion, the proper objects of the reference frame A are seen shorter from the frame B than from the frame A. And, at the same time, the proper objects of the reference frame B are seen shorter from the frame A than from the frame B.

    - The proper clocks of the reference frame A are observed slower from the frame B than from the frame A. And, at the same time, the proper clocks of the reference frame B are observed slower from the frame A than from frame B.

    - In the direction of relative motion, all events that are simultaneous in the reference frame A are not simultaneous in the reference frame B. And all events that are simultaneous in the reference frame B are not simultaneous in the reference frame A.

    - In all cases, SR requires both confirmations, but that never happens.

It is also surprising that the relativistic contractions are the same for all objects, regardless of their chemical composition and crystallographic structure. And that inertial time dilation is independent of the periodic mechanisms by which the different clocks measure time. Moreover, there is no physical cause that produces these deformations, only the fact that they are observed with different uniform relative velocities.

2. Special relativity is not compatible with discreteness

According to the Principle of Relativity, the laws of physics are the same in all inertial reference frame. And according to the Theorem of the Consistent Universe 3 (paper 5)the universe evolves under the control of a unique set of invariant and formally consistent laws. This includes all the universal constants involved in such laws. So, being Planck length l_p, and Planck time t_p two universal constants, they should be universal constants in all inertial reference frames, which poses the problem of their respective relativistic contraction and dilation. Or in other words, the problem of the relativity of the intervals of space and time below which physical laws do not apply, will now depend on the relative velocity at which the corresponding events and magnitudes are observed and measured. This problem has already been dealt with by some authors, although they have not had a great impact [4, 27, 2, 1, 22].

It could be argued that the Lorentz Transformation does not hold for lengths and times respectively less than l_p and t_p. So, let L_o be the length of a macroscopic rule in its proper inertial reference frame RF_o. In a discrete space and time, and according to the Theorem 13 (paper, 5), we will have:

$$L_o = n_o l_p;\ n_o \in \mathbb{N} \tag{1}$$

Let RF_v be another inertial reference frame that coincides with RF_o at a certain instant and from whose perspective RF_o moves with a uniform velocity $v = kc$, $0 < k < 1$, in the direction of the increasing X_v. In accordance with the Lorentz Transformation, the rule, that moves parallel to X_v, will be observed with a length L_v such that:

$$L_v = \gamma^{-1} L_o \tag{2}$$

where $\gamma = 1/(\sqrt{1-k^2}$ is Lorentz factor. If l_p is also a universal constant in RF_v, we will have:

$$L_v = n_v l_p;\ n_v \in \mathbb{N} \tag{3}$$

In consequence, it must hold:

$$n_v l_p = \gamma^{-1} L_o = \gamma^{-1} n_o l_p \tag{4}$$

$$n_v l_p = \gamma^{-1} n_o l_p \tag{5}$$

$$n_v = \sqrt{1-k^2}\, n_o \tag{6}$$

which is impossible because $\sqrt{1-k^2}$ is not a natural number. The same argument applied to any proper interval of time $t_o > t_p$ leads to:

$$t_v = t_o/\sqrt{1-k^2} \tag{7}$$

$$n_v t_p = n_o t_p/\sqrt{1-k^2} \tag{8}$$

$$n_v = n_o/\sqrt{1-k^2} \tag{9}$$

which for the same above reason is also impossible. If l_p and t_p represent respectively the minimum indivisible unit of length and time, we have to conclude that the theory of special relativity is not compatible with a discrete space and a discrete time, i.e. it is not compatible with the Corollary of the Discrete Threshold 8 (paper 5). Therefore, the special theory of relativity requires that one of the following two alternatives be satisfied:

1. The laws of physics hold for any space (time) interval arbitrarily less than the discrete space (time) unit.
2. The speed of light is undefined for any time interval and any length interval respectively different of nt_p, and nl_p, for any natural number n.

3. Pythagoras' theorem and Lorentz factor

(The text of this section is taken from [30, pp. 438-440])

If, according to Theorem 8 of the Inconsistent Continuum (paper 3), the continuum is inconsistent, then the only alternative to the spacetime continuum would be a discontinuous, i.e. discrete, space and time made of indivisible units (atoms of space and time in L. Smolin words

[39]) we are calling here respectively qusits and qutits. The interest in discrete space and time began in the first half of the twentieth century [10, 13], although only in a minority of authors. W. Heisenberg, for instance, considered the idea of space as a sort of crystal lattice composed on tiny cells of the size of an elementary particle, although this idea was not finally developed [16]. Things have begin to change, especially in the last two decades. [18, 34, 29, 20, 7, 37, 8, 5, 36, etc.].

An increasing number of physicists suspect now that, in fact, Planck length and Planck time define a sort of space and time 'granularity' that could be an efficient alternative to the infinitist spacetime continuum, an alternative that could be tested in experimental terms [35, 14, 19, 31, 12]. The discrete nature of space and time has been proposed in different areas of physics [28, 21, 41, 17, 38, 39, 40, 6, 32, 33, 3, etc.], albeit the proposed models continue to be developed within the framework of infinitist mathematics.

Although discrete geometries already exist, they exist for particular purposes, for example the combinatorial analysis of the relationships between geometric elements [9], or the development of computational algorithms for the representation of geometric objects [15, 11]. There are even general discrete geometries, whether or not applied to quantum gravity, but not independent of infinitist mathematics. The discrete geometry suggested here would be a geometry with extended and indivisible minimum units of space instead of the non-extended points of current geometries. A discrete geometry that could only be developed on a discrete and finitist basis. For this geometry, everything remains to be done, starting with the establishment of its foundational base (axioms and definitions). Even so, some non-detailed arguments, as the next one, can be made.

Though the fine structure of (a possible) discrete space is unknown, let us consider the right angled triangles depicted in Figure 1.

It can easily be tested that the number of qusits (in reality bidimensional qusits) of their corresponding hypotenuses is in each case equal to the number of qusits of the greater of their corresponding legs. This empirical version of discrete Pythagoras theorem could surely be formally proved once the foundation of discrete geometry had been formally established. If that were the case, the factor for converting between continuous and discontinuous hypotenuses would have the algebraic form of the relativistic factor γ.

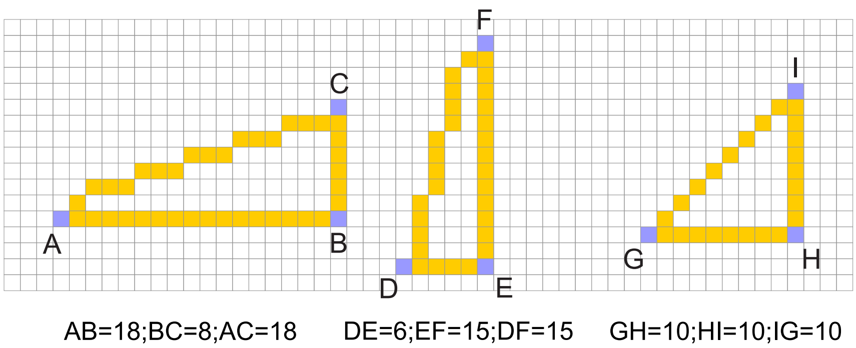

Figure 1 – A crude representation of three right angled triangles to test Pythagoras discrete theorem. The size of bidimensional space absolute quanta (qusits) has been exaggerated to makes it possible their visual counting.

Indeed, let h, x and y be the respective number of qusits of the hypotenuse and legs of a right triangle in a discrete space time, and let l_p be the length of one qusit in both the discrete and the continuous geometry. Assume $x < y$. In a discrete geometry we will have: $h = y$. In the continuous geometry the length of the hypotenuse would no longer be hl_p but $h'l_p$, being $h' > h$, because it is greater than the length hl_p, which is also the length yl_p of the greatest leg (note that while h, x and y are natural numbers, l_p and h' are real numbers). According to classical Pythagoras theorem, it can be written:

$$\text{Hypotenuse: } h'l_p = \sqrt{(yl_p)^2 + (xl_p)^2} \tag{10}$$

$$\text{leg: } yl_p = \sqrt{(h'l_p)^2 - (xl_p)^2} \tag{11}$$

$$y = \sqrt{h'^2 - x^2} \tag{12}$$

The ratio between the continuous and the discrete hypotenuse is given by:

$$\frac{h'l_p}{hl_p} = \frac{h'}{h} \tag{13}$$

$$= \frac{h'}{y} \tag{14}$$

$$= \frac{h'}{\sqrt{h'^2 - x^2}} \tag{15}$$

$$= \frac{1}{\sqrt{1 - (x/h')^2}} \tag{16}$$

where the last term on the right side of (16) as the algebraic form of the relativistic Lorentz factor γ. It can ve rewritten as:

$$\frac{h'l_p}{hl_p} = \frac{1}{\sqrt{1 - (xl_p/h'l_p)^2}} \tag{17}$$

Let a^* be a photon that moves through a vertical distance yl_p in the rest frame RF_o of its source. Assume a^* moves the same vertical distance yl_p from the perspective of another inertial frame RF_v while RF_o moves with respect to RF_v the horizontal distance xl_p at a uniform velocity v parallel to X_v for a time t_v. So, a^* moves with respect to RF_v along the hypotenuse of a right triangle whose legs are yl_p and $xl_p = vt_v$; i.e. a^* moves along the hypotenuse $h'l_p$ (10). And it will hold $h'l_p = ct_v$. Therefore, (17) can be rewritten:

$$\frac{h'l_p}{hl_p} = \frac{1}{\sqrt{1 - (vt_v/ct_v)^2}} \tag{18}$$

$$= \frac{1}{\sqrt{1 - (v/c)^2}} \tag{19}$$

$$= \gamma \tag{20}$$

which proves the ratio between the continuous hypotenuse and its corresponding discrete alternative is the relativistic Lorentz factor γ. This result suggests that a discrete interpretation of special relativity could be possible. Special relativity could be, in fact, the consequence of explaining a discrete, discontinuous, world in terms of the continuous mathematics of the spacetime continuum. Or in other more expeditious words: the result of explaining a consistent discontinuous reality in terms of an inconsistent continuous reality.

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