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Towards A Discrete Cosmology 9/17

Discrete space

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Abstract.-This paper discusses the consequences (on the physical space) of some of the formal results proved in the precedent papers, namely the Theorem 6 of Formal Dependence (paper 5)and the inconsistencies of the actual infinity and the infinite divisibility. The consequences are almost immediate and very clear, space would have to be finite, discrete and physical, i.e. real, which obviously contrast with the usual concept of space in contemporary physics. A model based on Cellular Automata Like Models is then proposed to start the discussions on the foundational basis of a new finitist and discrete paradigm of physical space.

Keywords: relative space, absolute space, finite space, infinite space, discrete space, physical space, CALM.

1. Introduction

Space is one of those concepts that have always attracted the attention of philosophers and physicists (and in this case also of theologians). Discussions about space began with the pre-Socratics and have continued to this day. And they have basically focused on three aspects of space: its real or fictitious nature; its absolute or relative nature; and its finite or infinite size. Leaving aside the *extensive* points of the early Pythagoreans and the discrete physical world proposed by the authors of the Islamic culture Kalam (or Kalām) (see paper 4), the discussion of the continuous or discrete nature of space is much more recent and reduced to a minority of authors.

The aim of this article is not to review the historical details of those discussions, for which there is an excellent literature [4, 24, 20, 5, 13, 14, 19, 15, 16, 8, 7, 23, 9, 10], but to introduce these disputed aspects of space and subsequently analyze them in the light of Theorem 2 on Finite Distances and Theorem 12 of Adjacency, both proved in paper 5.

Once the classical alternatives have been introduced, this paper discusses the formalism of one of them: the hegemonic spacetime continuum. Following this discussion, some problems are raised about the contradictory use of certain concepts related to the content of space and the relationships between the elements proper to space and the objects contained in space. These problems are rarely analyzed, though in my opinion are necessary in order to understand the role of space in the physical world.

In agreement with the formal results proved and assumed in the precedent papers (inconsistency of the actual infinity, inconsistency of the infinite division and Corollary 8 of Discrete Threshold, paper 5), this paper 9 proves some important results on the physical nature of space, which, if nature is consistent, can only be finite, discrete and physical. Where physical means real, within the real versus relative debate.

An outline is then presented of what could be a discrete and absolute physical space and its relation with physical objects, a relation that now would not be that of a simple container but that of a generator of all physical objects. In these conditions, the concept of CALM (Cellular Automata Like Model) is proposed as an inspiring instrument to begin the construction of a new finitist and discrete paradigm of space and time, and then of the universe.

One of the obvious advantages of the new discrete paradigm is its complete independence of the Axiom of Infinity. In addition, the above mentioned relationships between the elements of space and the elements of the physical objects contained in physical space are better explained in this new discrete and finitist paradigm of physical space.

Although discrete geometries (and discrete mathematics) exist, they are not the ones that would have to be developed for this new paradigm. As an informal example (informal, because it is not built on a defined foundational base), a discrete version of Pythagoras theorem will be introduced in paper 14. This example is interesting because of the role played by this theorem in the calculation of distances, and because the factor that converts between its classical and discrete versions is the relativistic Lorentz factor γ given by:

$$\gamma = 1/\sqrt{1 - v^2/c^2} = (1 - k^2)^{-1/2};\ 0 < k < 1 \qquad (1)$$

where, as is well known, $v = kc$ is the relative velocity and c the speed of light.

2. Absolute (real) and relational (fictitious) space

Is space real, independent of the existence of observable physical objects? or is it purely relational, i.e. a mental construct of our way of perceiving the physical world? This is the classical debate on the nature of space, (substantival versus relational alternatives). Or in other words, while for some authors space is a real physical entity, for others it is just a fiction, a mental way of representing objects and their relative positions. For the first ones there would be space even if it does not contain objects, and for the others there is no space if there are no objects that define it (in theoretical terms).

Practically all the authors interested in the nature of space supported one of the two alternatives, giving rise to certain discussions that are still remembered, such as that of Newton-Clarke (absolute space) versus Leibniz (relational space). The debate is still open today, although according to the hegemonic theories of relativity (special and general) space and time are only relative while spacetime is absolute, the three of them being (for most authors since Leibniz) theoretical constructs that are necessary to describe the world, but not real entities.

The debate on the nature of space is far from closed, because, in spite of being unreal, it can deform, expand and vibrate (some space deformations propagate through space itself: gravitational waves). We would have to admit the existence of an object that is not real but that can deform, vibrate and is the medium of propagation of its own vibrations. What kind of non-reality would that be? A vibration of something that does not exist, does not exist either.

On the other hand, the space that expands in the universe is not the intra-galactic space but the intergalactic space of the emptiest zones of the universe. So it is space, and only space, which expands in an accelerated way. It is also difficult to assume that an unreal object can expand, however the unreal space expands, and then self-creates continuously. Therefore, space is the cause of itself, a conclusion that goes against the Theorem 6 of Formal Dependence (paper 5).

In addition, space would be filled by fields of the four forces and by a special field that would fill the entire spacetime: the quantum vacuum field, from which continuously emerges a multitude of virtual particles that only exist for extremely short intervals of time. The debate is further complicated by the imprecise use of certain key terms such as *nothing*, *void*, *infinity*, and even *point* and *instant*.

3. Finite and infinite space

The other classic debate about the nature of space is whether its extent is finite or infinite. Almost invariably, in these discussions the concept of infinity is used in a very imprecise way. And the imprecision was justified as long as no proper formal definition of infinity was available, basically until the early 20th century, when Dedekind's Definition 3 and the Axiom of Infinity 1 were proposed and accepted (see paper 3).

Since then, it is not justified to use the concept of infinity in imprecise terms. Despite this, it is still used in these imprecise Gaussian terms (infinity as a way of speaking, see paper 3), very common in contemporary physics, especially in the secondary literature (though not uncommon in the primary literature). The same thing happens with other concepts such as *point* or *nothingness* or *void*, even with concepts alien to our discussion but very important in physics, such as *order*, *organization* and *complexity*. As noted in paper 2, it could be said that contemporary physics lacks some formal rigor in the use of ordinary language and formal language.

There is a third alternative regarding the finite/infinite extent of physical space. This third alternative proposes a finite but unbounded space, as with the curved surface of any sphere. In this case, the universe would have a finite size, but somehow it would be curved back on itself so that a straight trajectory would always end up at the initial point of that same trajectory. This alternative has, therefore, certain requirements on the geometry of the space itself (to be discussed in paper 11).

In papers 3, 4, 5 of this series of papers, some very significant results about actual infinity and infinite division were proved. In this paper 9 we will begin to see the devastating consequences of these results on the formal foundations of the physical sciences. Let us remember that there, it was demonstrated, among other, the following results:

1. ω-Ordered sets are inconsistent (Corollary 2, paper 3).
2. The Axiom of Infinity and the actual infinity are inconsistent (Theorem 6 and Corollary 3, paper 3).
3. The continuum (and then the spacetime continuum) is inconsistent (Theorem 8, paper 3).
4. A consistent universe can only have a finite number of physical objects (Theorem 12, paper 3).
5. Numbers with an infinite decimal expansion are inconsistent (Theorem 1, paper 4).
6. The actual infinite division of any finite real interval is inconsistent (Theorem 3, paper 4).
7. An interval of time can only be divided into a finite number of parts (Theorem 3, paper 4).
8. The length of any line with two endpoints is always finite (Theorem 1, paper 5).
9. In the Euclidean space $\mathbb{R}^3$, any closed line has a finite length (Corollary 1, paper 5).
10. In the Euclidean space $\mathbb{R}^3$ the distance between any two of its points is always finite (Corollary 2, paper 5).
11. In the Euclidean space $\mathbb{R}^3$ it is impossible to join any two of its points by a line of infinite length (Corollary 3, paper 5).
12. In the spacetime continuum, the distance between any two points and the time elapsed between any two instant is always finite (Theorem 2, paper 5).
13. There is an indivisible minimum of space (time) interval of which all space (time) intervals are an integer multiple (Theorem 11, paper 5)
14. The laws of physics do not apply in spaces smaller than the indivisible unit of space nor in times smaller than the indivisible unit of time, both being of non-zero extension (Corollary 8, paper 5).
15. Change is impossible in the spacetime continuum (Theorem 2, paper 6).

Physics should prepare itself to dispense with the concept of infinity in all its models and theories of the physical world. It is rare to find a physics book (especially those related to the physics of space and time) in which the word infinity does not appear dozens of times, for example more than 240 times in [22], 170 times in [4], more than 140 times in [21], more than 70 times in [11]. And in expressions like:

... escape to infinity.

... at infinite distance.

... repelled to infinity.

... extends to infinity.

... continues to infinity.

... an infinitude of positions.

... infinitely many points.

... etc.

4. The spacetime continuum

As noted above, Modern Physics assumes that space and time separately are only relative, while the spacetime continuum is absolute and modeled by the set $\mathbb{R}^4$ of all real 4-tuples, (x_i, y_i, z_i, t_i) where the first three real numbers represent the spacial coordinates of a point (a spacial position) and the fourth one t_i represent an instant of time (a time position), points and instants being primitive concepts. For most physicists, neither space, nor time, nor spacetime are real physical entities but concepts necessary for the construction of models that describe reality. We will discuss this matter in paper 11.

As noted above, the spacetime continuum is made up of points and instants, which are also primitive concepts. An additional problem about these two concepts is that our intuition about them is flawed by our experience with graphic marks and the very fast events we associate respectively with points and instants. In this regard, it is useful to recall some informal definitions of point:

1. A point is that which has no part [6, p. 153].
2. A point is that which has position but not magnitude [18, p. 8].
3. Geometric element without dimensions whose position in the plane or in space is located by means of its coordinates [3].
4. A geometrical element having no dimensions; in Cartesian space, an element that can be located by a single n-tuple of coordinates [1].
5. A geometrical construct which has position but no size [2, p. 609].

So, a point would be something that, having no size (no extension), occupies a place. The problem is how a thing that has no size can occupy a place. In paper 11 it will be formally proved that, in fact, points can have neither size nor shape; and instants can have no duration. So, points and instants can only be abstract positions defined by 4-tuple of real numbers in abstract reference frames.

As just noted, the concept of instant is as primitive as the concept of point, and even more deceptive. Almost all informal definitions (much rarer than point definitions) are circular or outright wrong:

1. Very brief portion of time [3].
2. A very short time; a moment (Google Online Dictionary).
3. A very short space of time (WordReference Online Dictionary.
4. A very short period of time (Oxford Online Dictionary).

But an instant is not an interval of time, however brief the duration of that interval may be. So, as in the case of points (which have not extension), we will have to admit that instants have not duration. The same kind of formal dissatisfaction as with points:

1. Line intervals have length but a line interval only consists of a densely ordered sequence of points (between any two points of the sequence uncountable many other different points exist), none of which has length.
2. Time intervals have duration but a time interval only consists of a densely ordered sequence of instants (between any two instants of the sequence uncountable many other different instants exist), none of which has duration.
3. Any line interval has the same number of points as any other, which is also the number of points of the whole tridimensional universe.
4. Any time interval has the same number of instants as any other, which is also the number of instants of the whole history of the universe.
5. Length and duration are defined by arbitrary metrics based on one to one correspondences between points (instants) and real numbers.

On the other hand, it is important to remember that neither in space intervals nor in time intervals does immediate successiveness (adjacency) exist: no point (instant) is the immediate successor of another point (instant). Recall that in the spacetime continuum, change is inconsistent (Theorem 2, paper 6), just because of this lack of immediate successiveness. And not forget that physics is the science of change!

And most important of all, any continuum of points (or instants) is inconsistent (Theorem 8, paper 3). Which makes the following question unavoidable:

> Can a theory that makes use of an inconsistent concept be consistent?

To this formal dissatisfaction must be added at least an additional physics dissatisfaction derived from the dynamics of the quantum void: modeling physical spacetime with the $\mathbb{R}^4$ continuum means that, for example, during one millisecond and on a line of one millimeter, as many virtual particles are created and annihilated as in the three-dimensional universe (with a radius of 45 billion light years) during its entire history (13.7 billion years).

As will be seen here and in the following articles of this series of articles, things are much simpler and more formally and physically satisfactory in the framework of a finite and discrete space and a finite and discrete time, both made up of indivisible and extensive units.

5. Problems not properly addressed

The formal basis of a science must be continuously revised, but physics does not pay attention to the formal basis of its mathematical language, nor to the lack of rigor in the use of certain key terms such as infinite, something or nothingness (See paper 2). In particular it has never questioned the consistency of the Axiom of Infinity. This is an error that could have very serious consequences. Some of them were discussed in paper 2 of this series of articles. As noted there, some words and expressions are used in physics with little formal rigor, which can lead to confusion or error, for example:

1. The word infinity is sometimes used as potential infinity and others as actual infinity, without indicating which is the case in each corresponding use. The same applies to the ellipsis "..." and to expressions as "ad infinitum," "and so on and on" etc.

2. Something and nothing as frequently used as synonymous [11, p. 7 Kindle Ed.].

3. Void and nothingness are used in a sense that is contrary to their respective original meanings.

4. How can the vacuum (which by definition has not substance) have measurable physical properties like magnetic permeability or electrical permittivity?

5. 'A fluctuation of nothingness' implies that nothingness is not nothingness but something with the capacity to fluctuate (assuming that nothingness has no capacity, not even the capacity to fluctuate).

6. 'Quantum void fluctuates continuously' means that it is not void of content but filled with something with the capacity to continuously fluctuate (assuming the void is devoid of anything that can fluctuate).

7. It is insisted ad nauseam that the velocity of a photon does not depend on the relative velocity of its emitting source, but it is almost never made clear that the velocity vector of a photon does depend on the relative velocity vector of its emitting source: the relative velocity vector of its emitting source is, or forms part of, one of its vector components (preinertia, paper 7).

8. etc.

Also relevant is the existence of problems that are systematically ignored, as if they did not exist. For example, the classical problems posed by Zeno's Dichotomy, or by change (both direct consequences of the lack of immediate successiveness, adjacency, in the spacetime continuum). Physicists act as if these problems had been solved; or as if they had no relation to physics. But they are not solved and they do have a relationship to physics. A fundamental relationship.

Recall also the questions raised in paper 7 about uniform motion that physics doesn't even ask (and don't forget that science always starts with an opportune question). Sometimes one has the impression that for some authors the strange and unusual adds value to scientific theories. I prefer simplicity. And respect for authors and theories, instead of religious veneration or contempt. We should be concerned about the almost divine position that some authors and theories occupy in the history of physics.

As noted above, another family of problems ignored by contemporary physics is that posed by the real/unreal nature of space. Indeed, for most contemporary physicists, space is not real, it is just an illusion, a fiction, like time and spacetime (see paper 11). But space is also something that expands and deform and its deformation can travel through the space itself. One could argue, as some authors do, that it is mathematical equations, and only mathematical equations, that describe deformations, expansions and vibrations of something that is not real in order to explain what happens in reality. But the question is raised again, why are these deformations necessary if there is nothing real that deforms in that way? If it is only to explain the behaviour of physical objects, that explanation would be as valid as any other explanation based on things that do not exist.

6. Physical space

In papers 3, 4 and 5 some important results about the actual infinity and the division into infinite parts were proved. They will be used in this section. But first, let us pay our attention to the following representative text of the hegemonic infinitism of our days [11, p. 5]:

> Likewise, our minds may not be able to easily understand infinity (although mathematics, which is a product of our minds, deals with it with remarkable success), but this does not mean that infinities do not exist. Our universe could be infinite in its spatial or temporal extent.

There is, however, one aspect of the mathematical analysis of infinity that is completely neglected: its critique. The whole mathematical theory of infinity rests on an axiom: the Axiom of Infinity. And an axiom is an axiom, not a religious creed. An axiom must be subject to criticism, especially when it is as unobvious as the Axiom of Infinity. Especially when it has behind it a conflicting history of twenty-seven centuries of discussions. In this sense, infinitist mathematics is more like a religion than a formal science: it even has its fanatic believers who do not admit the slightest criticism and tend to respond with insult much more than with reason. I can vouch for that.

But if only one of the more than forty proofs of the inconsistency of actual infinity developed in [12] were correct, then the hypothesis of actual infinity would be inconsistent. And all the infinitist mathematics would collapse like a house of cards. The reader of this series of articles has the opportunity to analyze one such proofs: the Theorem 5 of the Denumerable Infinity included in paper 3. In any case, and being the existence of actual infinities established by means of an axiom, science is obliged to analyze the possibility of its inconsistency.

More than three centuries after the publication of Newton's Laws of Mechanics (Newton's Principia [17]), we still do not have a detailed formal description of motion. We will try it here, at least in the sense we are interested in. Although the description has no formal consequences on our argument about the physical nature of space (which is the main objective of this section), it is proposed as an illustrative example of the type of problems that physics hardly ever deals with. And it is also proposed as a comparative reference for the model of space, time and motion that is proposed in this and in the following articles of this series of articles.

Consider any physical object Ob with a definite shape and volume and in a definite position in the physical space PS, be PS real or unreal, absolute or relative, finite or infinite, or whatever it is if it is anything at all. Assume also Ob is uniformly moving through PS in any direction d. Let S be the spacetime continuum that models PS, and let Ot be the theoretical representation of Ob with the same shape, volume, motion and position in S as Ob in PS, but made exclusively of geometrical points densely ordered as in the continuum S. As Ot moves through S it will be occupying different points of S. So. let P be any point of Ot. At a certain instant t_1 it will be in a point P_1 of S. In order to describe the motion of Ot through S we must consider the following two exhaustive and exclusive alternatives.

1. The point P remains in P_1 a null time.
2. The point P remains in P_1 for a time τ (whatsoever it be) greater than zero.

According to the first alternative, P coincides with P_1 for zero time. But that is the same thing that happens with all the points of S where P is not and never will be. Which is to say that the points where P is not, and never will be, are indistinguishable from the points where it is and will be. Under these conditions it is impossible to describe the motion of Ot and therefore to model the motion of Ob through PS.

According to the second alternative, at instant $t_2 = t_1 + \tau$ the point P will be in another point P_2 of S. Now then, since space is densely ordered in the spacetime continuum S, in the direction d and between P_1 and P_2 there is a non denumerable infinitude of points ($2^{\aleph_o}$ points) in which P has never been, and never will be. One could argue against this conclusion on the grounds that it is invalid because τ is not defined. But the conclusion is valid for all real numbers greater than zero, for all! Or to put it another way, there is no real number (for the value of τ) that makes this conclusion invalid. Notice that P is in P_1 for a time τ and then in P_2, without traversing the infinitely many points between P_1 and P_2 in the direction d of motion.

We must conclude, therefore, that in the spacetime continuum, either the points at which a moving object will be and not will be are indistinguishable, or its motion can only be described as a discontinuous process. A discontinuous (discrete) process in which the moving object jumps between two successive positions (P_1 and P_2 in our example) without passing through the intermediate points between those two successive positions. This proves the following:

Theorem 1 (of the Discrete Motion) *The continuum densely ordered spacetime cannot be used to model uniform motion.*

Let us now try to describe motion from the perspective of the Theorem 12 of Adjacency (paper 5), according to which a qutit (qusit) can only be immediately succeeded by another qutit (qusit), so that no time elapses (space exists) between them (discrete adjacency). Remember that the Theorem 12 of Adjacency (paper 5) is a formal consequence of both the Theorem 3 of the Consistent Universe (paper 5) and the Corollary 8 of Discrete Threshold (paper 5), according to which physical laws do not apply to lengths less than the discrete unit of length nor to intervals of time less than the discrete unit of time.

Under these conditions there will be an insurmountable maximum speed of one qusit per qutit, so that every physical object in uniform motion will only be able to advance one qusit after a certain integer number $t \geq 1$ of qutits. Where to advance a qusit means that a physical object (if it were of the size of a qusit) is in one qusit for at least one qutit after which it will be in another adjacent qusit, without ever being between the two qusits, simply because there is no qusit between two adjacent qusits. Motion would be, therefore, discontinuous (discrete) as in the second alternative above. It is also the same type of motion as that of the objects in a CALM (see next section). It is necessary, however, to formally complete this conclusion in the following terms and with the same words that were used in the article 6 of this series:

> The directional evolution of the universe demonstrates that this evolution is subject to a consistent set of rules, physical laws, (Theorem 3 of the Consistent Universe (paper 5). So, canonical changes have to be consistent processes, and then instantaneous[1]. The problem is that we have no idea how that is possible. As a very adventurous hypothesis, it could be proposed that qusits have two modes of existence:
>
> 1. Permanence mode: the state of each qusit remains unchanged at least for one qutit. This would be the only perceptible state of qusits.
> 2. Interacting mode: all qusits update synchronically their respective states through appropriate processes lasting at least one qutit.
>
> Although, in accordance with what was said above, the problem of change will now reappear in the terms of these changes of modes. So, we would have to admit that the interactive mode is simultaneous with the permanence mode, although it remains in an imperceptible background (such as computer applications running in the background) that changes to the permanence mode at each successive qutit (or something similar).

But this kind of discrete motion is only possible if qusits and qutits are real, otherwise every physical object would have to have the ability to measure qusits and qutits by their own means in order to determine the successive positions of its motion by itself; which in turn would imply that all of them (including elementary particles) have internal clocks and rules of measurement. It seems evident that this is not the case, at least from what we know of the physical world. Consequently, we can end this section with the following important:

subTheorem 1 (of Physical Space and Time) *The indivisible units of space and time are physical, and then real and absolute.*

Note that a subtheorem is not a theorem, but a statement supported by both formal and inductive elements. In this case, the formal elements are Theorem 3 (paper 4), 8 of the Discrete Threshold (paper 5), Theorem 12 (paper 5) and Theorem 1. The inductive support is the enormous empirical evidence that no natural object, including elementary particles, has internal clocks and rules to measure time (qutits) and distances (qusits). Corollary

7. Space in CALM

In order to start building a new foundational basis for the concept of physical space, we will now recall the space in CALMs (Cellular Automata Like Models) introduced in paper 6. In these theoretical objects:

1. Space is made of indivisible and contiguous elements we call qusits (cells in the jargon of cellular automata).

[1] As was proved in paper 6.

2. Time is a sequence of indivisible and contiguous elements we call qutits.
3. Being contiguous means that between any two qusits (qutits) no other qusit (qutit) exists (immediate successiveness or adjacency).
4. Every region of a CALM has a finite number of qusits.
5. Every interval of time in a CALM has a finite number of qutits.
6. The state if each qusit is defined by a set of variables.
7. During at least one qutit, each qusit exhibits the same state, while in the background, the interactions between the states of the different qusits are performed, which will determine their corresponding states to be exhibited in the next qutit.
8. The changes of state are driven by the laws of the CALM.
9. Some groups qusits can be temporarily or permanently linked: they are the objects of the CALM.

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