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Towards a discrete cosmology 1/17: Unsolved foundational problems

Links to papers in this series.

All the articles in this series are aimed at proposing the construction of a formal basis for a discrete cosmology, based on a finite and discrete space and time. The articles in the series can be read in any order, although there are cross-references between them.

Antonio Leon
Retired Professor. Independent researcher in the foundametals of science

Abstract.-This is the first in a series of 17 articles that aim to open a debate on the foundation of a new finitist and discrete cosmology. All the articles in the series have already been written, and will be published weekly from the last week of October 2022. The objective of this first article in the series is to expose a set of fundamental problems of logic, mathematics and physics that have not been properly resolved, and that presumably will not be, because at the base of all of them is an inconsistency directly related with the old Hypothesis of the Actual Infinity. An inconsistency that contemporary mathematics and physics insist on ignoring, despite its catastrophic consequences in the development of physical theories, particularly of cosmological theories.

Keywords: actual infinity, problem of change, infinite regress, foundation of science, laws of logic, preinertia, real and unreal space and time, irreversible time.

1.1 Introduction

I was educated first as a geologist and then as a mathematician. For that reason my clash with the actual infinity was devastating for my original scientific project (thermodynamics of organic evolution) which was interrupted forever in 1995. I was warned that the new path, the critique of infinitism, would lead me directly to ostracism. They were right[1], but I have not minded staying in

[1] Therefore, thanks to The General Science Journal for publishing my work and to the Academia and Researchgate scientific networks for contributing to its public dissemination.

academic exile for so many years. Having said that about myself, let's start talking about what we have to talk about in this series of articles.

There are at least seven foundational problems of contemporary science, some of them of pre-Socratic origin, that have been forgotten or not adequately addressed, despite their great relevance in disciplines such as physics and cosmology. They are the following:

1. The Hypothesis of the Actual Infinity.
2. The problem of change.
3. The infinite regress af arguments, definitions and causes.
4. The foundation of science.
5. Preinertia.
6. The real/unreal nature of space and time.
7. The irreversible nature of time.

The following sections introduce them, as well as the role they will play in the discussions developed in the subsequent articles of this series of articles. The main objective of these discussions is to propose a new foundational basis for a model of the universe whose main feature would be a finite and (above all) discrete space and time instead of the infinitist spacetime continuum of the current model. In the case of time, and apart from being finite and discrete, it would also be essentially directional and irreversible.

1.2 The Hypothesis of the Actual Infinity

The discussions about infinity are as old as abstract thought itself, which, as is well known, born in pre-Socratic Greece (though influenced by the empirical knowledge previously developed in the so called Mesopotamian river cultures [27, 3, 31, 24, 32]). In fact, the concept of (actual) infinity is already present, and in a very significant way, in Zeno's paradoxes [7, 4, 29, 28, 19, 14]. It is surely the most conflicting abstract concept created by man. And the greatest known source of paradoxes. Twenty-seven centuries of discussions were not enough to prove (or disprove) the existence of actual infinities. So, finally, that existence had to be established axiomatically at the beginning of the twentieth century by the Ax-

iom of Infinity, one of the foundational axioms of set theory. And one of the worst misfortunes in the history of science, as will be seen in this series of articles.

Since the beginning of the 20th century, infinitist mathematics has been absolutely hegemonic and, at the same time, intolerant of dissent. But, let me recall, an axiom is just an axiom, i.e. a statement assumed without proof, and then one that can be assumed or rejected. A statement that one has the right and the duty to put to the test, particularly if it is far from obvious. Recall that the Axiom of Infinity was (more or less explicitly) rejected by authors of the intellectual stature of L. E. J. Brouwer, C. Hermite, S. Kleene, J. König, L. Kronecker, H. Poincaré, A. Robinson, L. Wittgenstein, or H. Weyl, among others. It is, on the other hand, rather ironic that set theory, the first-born creature from the Axiom of Infinity, has been the one that has finally provided the instruments to demonstrate in formal terms the inconsistency of its own infinitist foundational axiom. Among such instruments, the ω-order that will be used in the proof of inconsistency (Hilbert's machine) reproduced in paper 3 in this series of articles.

Hilbert's machine argument, inspired by the emblematic Hilbert's Hotel, was written by the author more than twenty years ago. By that time, I was already convinced of the impossibility of publishing arguments about the inconsistency of the Hypothesis of the Actual Infinity (at least by unknown authors in the field of logic and set theory, as was my case). And Hilbert's machine argument, written in a very cautious and conciliatory way, was the last one I sent to a scientific journal (not with the intention that it be published but with the intention of being informed of any possible error). As expected, the article was rejected. The editor sent me the report of its unique referee, which basically said: "the author proves one thing and later the opposite." I wrote to the editor (mainly to check his reaction) saying that, indeed, I demonstrated a thing and then the opposite because I was demonstrating a contradiction. His response was also what was expected: no response. The reader of Part 3 will be able to decide if that argument deserves, or not, a little more consideration and respect.

The inconsistency of the actual infinity will have enormous consequences in physics (and naturally in mathematics itself) because

physics has been written with infinitist mathematics for more than a century. One of these consequences is that all physical concepts and quantities (including space and time) must be discrete, with indivisible minima, as, on the other hand, it is already assumed for quantum magnitudes, although quantum physics, mainly the physics of discrete entities, is also expressed with the indiscrete mathematics of the infinitist continuum $\mathbb{R}$ of the real numbers. The same can be said of the mathematical language based on Hilbert space on which quantum field theory is built.

It does not seem an exaggeration to state that the pre-Socratic discovery of the irrational numbers and the proposal and acceptance of the Axiom of Infinity have had dire consequences in the development of mathematics and of the physical theories most committed to mathematics. Nowadays, practically nobody within the academic orthodoxy has stopped to think about the consequences of taking the continuum of real numbers as a model. Infinity theories have been growing for more than a century and a half. And nobody knows how and when they will stop growing. Let us hope that Henry Poincaré was right and the day will soon come when we will be able to say that the actual infinity was a disease of which we have already been cured [22, p. 121], [5, p. 1].

Fortunately, all experimental sciences are finitist in their results (there are no real measurements with an infinite number of decimals), and experimental results prevail over theoretical constructs. We have physical models and theories that explain the physical world reasonably well (even very well), but they are not complete theories precisely because they are founded on the infinitist hypothesis of the spacetime continuum, in which both space and time have a continuous, non-discrete nature. In this series of articles we will have the opportunity to demonstrate that neither space nor time can be divided infinitely. Therefore, they must be discrete, with indivisible minima. And this conclusion changes everything.

And not only in mathematics and physics, the inconsistency of the Hypothesis of the Actual Infinity also has consequences in logic and philosophy. In particular, the article 16 in this series of articles demonstrates how the inconsistency of that hypothesis is sufficient to resolve three of Kant's four antinomies. In the same article the solution of the fourth antinomy is also demonstrated, although

now using the concept of physical information (from my original scientific research on organic evolution [12, 13]).

1.3 The problem of change

Also of pre-Socratic origin, the problem of change is surely the most difficult problem ever posed by man. So difficult that it remains unsolved more than twenty-seven centuries after it was posed. So difficult that some classic and modern authors, such as Parmenides, Zeno, McTaggart or Hegel, have defended that it could be an inconsistent process, despite its overwhelming evidence: change is the most pervasive characteristic of our continuously evolving universe [23, 30, 1, 2, 20, 25, 21, 9, 10, 33].

So, without solving the problem of change it will be impossible to explain the physical world. It is therefore surprising how little interest modern physics, the science of change, takes in the problem of change. I have the impression that many physicists ignore that the problem of change is still an unsolved problem, ignore that until now nobody has been able to explain how a simple change of position occurs.

Naturally, physics has explained a great variety of changes of all kinds (mechanical, electrical, magnetic, thermal, etc.). It has explained which constants and which variable magnitudes are involved in each of these changes, and what mathematical relations exist between these variables and these constants, but none of these changes has been explained, as such a process of change. In paper 6 of this series of articles it will be proved that change is indeed inconsistent in the spacetime continuum. But it will also be proved there that the problem of change can be solved within certain theoretical frameworks (similar to cellular automata) in which space and time are of a discrete nature.

In addition, in these discrete and finite frameworks, most of the oddities of relativity and quantum physics could be explained. Rarities that surely appear because of the insistence of physics to explain the discrete physical world by means of inappropriate continuous mathematics, the same mathematics that makes it impossible to solve the problem of change. As an anticipated conclusion, we could say that if the Hypothesis of the Actual Infinity were in-

consistent (and it will be proved that it is), then space and time can only be discrete. And in a discrete space and time the problem of change can be finally solved.

1.4 Infinite regress of arguments, definitions and causes

Since statements do not prove themselves, to prove a statement, say S_1, we need at least another different statement S_2 justifying S_1 in formal terms; for the same reason, to prove S_2, we need at least another different statement S_3 justifying S_2 in formal terms; for the same reason again, to prove S_3, we need at least another different statement S_4 justifying S_3 in formal terms; and so on and on. This is the Aristotelian infinite regress of arguments, the reason for which we need axioms in formal sciences and inductive principles (or fundamental laws) in experimental sciences.

We have the same problems with definitions: since concepts do not define themselves, to define a concept, say C_1, we need at least another different concept C_2 defining C_1 in semantic terms; for the same reason, to define C_2, we need at least another different concept C_3 defining C_2 in semantic terms; and so on. This is the reason for which primitive (undefined) concepts are inevitable and necessary in all languages, either formal or ordinary.

Most basic scientific concepts are primitive: set, number, point, force, mass, energy, time, instant etc. Axioms, principles, fundamental laws and primitive concepts drastically limit human knowledge, although we tend to pay little attention to these inevitable restrictions, at least in comparison with the attention we pay to certain famous theorems of logic based on another idiotic concept: self-referent statements ("this sentence is false", and the like) [15, 16].

The need to use primitive concepts is surely behind the lack of rigor in the use of ordinary language and also of formal language that will be analyzed in the article 2. A very serious case, as will be seen in this series of articles, is the misuse of successiveness in the case of points in space and instants in time, in which there is no immediate successiveness (adjacency): the natural number 5, for example, is the immediate successor of the natural number 4, but no point (instant) can be the immediate successor of another

point (instant) in the spacetime continuum.

In spite of this, the (primary and secondary) physics literature are replete with dramatically erroneous expressions such as:

... it propagates through the adjacent points...
... is distributed point to point...
... through each of the contiguous points...
... in the next instant...
... in the previous instant...
etc.

Always forgetting that in the spacetime continuum there is no adjacency: between every two points (instants), whatever they are, there are always other $2^{\aleph_o}$ different points (instants). So the description of all supposedly continuous physical phenomena can only be discontinuous, in jumps. And the reason is the same reason why it is impossible to solve the problem of change or Zeno's paradoxes: the infinitist topology of the spacetime continuum.

But the infinite regress that interests us most here is that of origins (causes). Indeed, things do not originate by themselves either. The reader will be able to guess what will be found in paper 15 devoted to this subject, a subject unattended by modern science surely because of certain religious prejudices.

1.5 The foundation of science

Although the infinite regress of arguments, definitions and causes is assumed implicitly, it should be made explicit (as will be done in this series of articles, particularly in the article 8). The consequences will be very positive, making it possible to sort out the certain disorder that still exists today in the foundations of the different sciences, whether formal or experimental. Two levels of foundation should be distinguished, a general logical level that only affects the consistency of the arguments, and a particular level of each science.

At the general level, the laws that every argument must comply with have been established since the time of Aristotle [18]: at least the Law of Identity and the Law of Non-Contradiction (the Law

of the Excluded Middle could also be included). As is known, these laws are assumed by all sciences and allow to establish the basic modes of inference (Modus Ponens and Modus Tollens). At the particular level, it is necessary to establish the basic laws of each science. In the case of experimental sciences, we will call them principles, they should be inductive in nature, based on observation and experimentation.

It is interesting to note at this point that the nineteenth-century naturalistic Principle of Actualism Uniformism says the same thing as the principle of relativity, although without reference to reference frames.

> Natural laws are the same in all places and times.

Well, the foundation that will be proposed in this series of articles includes an inductive principle valid for all sciences and which is even more general and basic than the above mentioned Principle of Actualism Uniformism. We will call it the Principle of Directional Evolution:

> The observable universe evolves independently of its human observers and always in the same direction of increasing its global entropy.

From this principle, which is explained and empirically justified in the article 5, several theorems will be proved (including the Principle of Relativity) that can be used in a new foundation of experimental sciences. Among them the following:

1. **Theorem of the Consistent Universe**: The universe evolves under the control of a unique set of invariant and consistent physical laws.
2. **Theorem of the Formal Dependence**: No concept defines itself; no statement proves itself; no physical object is the cause of itself; and no cause is the cause of itself.
3. **Theorem of Reference Frames**: The laws of physics are the same in all reference frames.
4. **Theorem of the Extensive Intervals**: The intervals of space and time within which the physical laws apply must al-

ways be greater than zero.

5. **Theorem of the Indivisible Units**: There is an indivisible minimum of space (time) intervals of which all space (time) intervals are an integer multiple.
6. **Theorem of the Discrete Threshold**: Physical laws do not apply in length intervals less than the quantum of length nor in time intervals less than the quantum of time.
7. **Theorem of Adjacency**: No space exist between any two successive quanta of space, and no time elapses between two successive quanta of time.
8. **Theorem of Preinertia**: Every physical object inherits in one of its vector components the relative velocity vector of the reference system where it is set in motion, provided that the resulting speed does not exceed the possible maximum limit.
9. **Theorem of the Discrete Motion**: The continuum densely ordered spacetime cannot be used to model uniform motion.
10. **Theorem of Physical Space and Time**: The indivisible units of space and time are physical, and then real and absolute.
11. **Theorem of the Arrow of Time**: In a consistent universe the joint evolution of any system and its environment is always in the same direction of increasing its entropy.

1.6 Preinertia

In this case, preinertia it is not an inconsistency, nor an unsolved and forgotten problem, nor a formal impossibility. It is a concept assumed by modern physics, although only implicitly, surely because no relevant physicist has ever thought of the real existence of this universal property of all physical objects. And here is the only inconvenience, that concept should be explicitly declared in order to use it in the construction of models and theories on the physical world. Preinertia is the ability of all physical objects to inherit the *relative* velocity vector of the inertial reference frame in which they are set in motion, including photons created and set in motion in any reference frame.

Special relativity, for example, would be impossible without prein-

ertia. Indeed, assume a photon a^* is emitted by its source S in the direction parallel to the axis Y_o of the proper inertial reference frame RF_o of the source S. If RF_v is another inertial reference frame that coincided with RF_o at a certain instant and from whose perspective RF_o moves with a velocity v in the direction of the increasing axis X_v of RF_v, the photon a^* will be observed moving along a trajectory inclined by an angle α respect to the axis Y_v of RF_v. It can be easily proved that the vector components of its velocity are:

$$c_y = c \sin \alpha \tag{1}$$

$$c_x = c \cos \alpha = v \tag{2}$$

So that for RF_v-observers, the photon a^* inherits the relative vector velocity $\vec{v}$ of the inertial reference frame of its emitting source. In a more general sense, it can also be proved that all physical objects, including massless objects as photons, are preinertial (paper 7 in this series, and [17]).

Being a universal and significant attribute of all physical objects supported by the highest empirical evidence (confirmed, for example, every time an object falls on the surface of the Earth: it falls vertically from the point at which the fall begins), preinertia could be included in the statement of the Principle of Inertia as we will do in paper 7:

> Every physical object is preinertial and remains at rest or moves at a constant uniform velocity, unless an external force acts upon it.

It remains to analyze the role of this universal preinertia in gravitational attraction. And it could be the case that such a role would greatly simplify the explanation of the observed geodesic curvatures, even without the need for the cumbersome deformation of the spacetime continuum, an infinitist concept that could be inconsistent (paper 3 in this series proves that is the case), and is usually considered in modern physics as unreal. By the way, how could an unreal object vibrate -gravitational waves- and deform? (more on this subject in the following articles of this series).

1.7 The real/unreal nature of space and time

A major (though ignored) problem of contemporary physics is the division between the supporters of the unreal nature of space and time and the supporters of the real nature of both physical entities. This is indeed a serious problem, but a problem that is not even posed, as if it did not matter in the slightest. In this series of articles, particularly in the articles 9 and 10, we will have the occasion to see that it does matter, which will become evident as soon as the right questions are asked.

The supporters of the illusory character of space and time are in the majority with respect to the supporters of their physical reality. And both use as their basic model the set of real numbers, which is densely ordered, an ordering that does not allow immediate successiveness (adjacency): between any two real numbers there is always a non-numerable infinity of other, different real numbers. Under these conditions it is impossible to describe most physical events, for the same reason that it is impossible to solve Zeno's Dichotomy or the problem of change. In this sense, physical theories, as such theories, would be incomplete. But the vast majority of physicists do not even consider these questions.

The proponents of the non-real nature of space and time would have to explain, in addition, how something that does not exist, that is only an illusion, can extend, deform vibrate and transmit its own vibrations (gravitational waves). They would have to explain how it is possible that physical objects can move following non-existent geodesics, non-existent because the mathematical space in which they are defined does not physically exist. Do they move through equations or through some kind of reality?

1.8 The irreversible nature of time

Although each of us has a personal experience of time, and the nature of time has always been an important object of scientific discussion, until now it has not been possible to define neither the concept of time nor that of instant. They are, therefore, primitive concepts. St. Augustine's well-known phrase about what time is

sums up our experience with it very well [6, XI, 14, 7, p. 560]:

> If no one asks me, I know; if someone asks me and I want to explain it, I don't know.

In his Critique of Pure Reason, Kant deeply analyzed the concept of time and his analysis has been very influential in modern science. For Kant, time is not a property of matter, it does not exist as an independent physical object. It is an instrument of human perception; a relational instrument that our mind uses to explain the world in which it operates [11, B 54, p.165]:

> Time, therefore, is not to be regarded as an object, but as the mode of representation of myself as an object.

And according to Boltzmann's statistical interpretation, time flows in one direction (arrow of time) because it is vastly more likely to do so: the sense of time is a statistical property. Therefore, the fact that time flows backwards is very unlikely, but not impossible. Time would therefore be reversible. Physical theories, almost without exception, are compatible with a reversible time, which in addition is usually interpreted as relational, devoid of absolute existence. Nor does absolute time exist in the theory of relativity. And Gödel, who in the last years of his life also dealt with time, defended ideas very similar to those of Kant [8].

For many contemporary authors time does not exist, it is a mere consequence of covariant quantum fields [26]. But the stratigraphic series of sedimentary rocks are there, arranged from top to bottom so that each layer was deposited before the one above it (Law of Superposition); and its enormous macro and (above all) micropaleontological content is indicating an indisputable arrow of time that has been operative on this planet for at least the last 3600 million years. These stratigraphic series and their organic content are the indelible mark of the irreversible passage of time on Earth, in a reality that already existed as such a reality billions of years before there were human observers to observe it. A reality, then, objective and independent of human observers that has left billions of proofs for those who want to analyze them. By the way, the gasoline that moves the cars of time-denying physicists is also

a proof of the irreversible passage of time.

In the article 10 of this series of articles it will be proved that, if the universe is consistent (an it will also be proved it is), time cannot be divided infinitely. Therefore, there must be indivisible minimums of time. And, as we shall see, this changes everything.

Bibliographic references

[1] Henri Bergson. *Creative Evolution.* Dover Publications Inc., New York, 1998.

[2] Henri Bergson. The Cinematographic View of Becoming. In Wesley C. Salmon, editor, *Zeno's Paradoxes*, pages 59 – 66. Hackett Publishing Company, Inc, Indianapolis/Cambridge, 2001.

[3] Alberto Bernabé. Introducción y notas. In Alberto Bernabé, editor, *Fragmentos presocráticos.* Alianza, Madrid, 1988.

[4] Giorgio Colli. *Zenón de Elea.* Sexto Piso, Madrid, 2006.

[5] Josep W. Dauben. *Georg Cantor. His mathematics and Philosophy of the Infinite.* Princeton University Press, Princeton, N. J., 1990.

[6] Agustín de Hipona. *Confesiones.* Editorial Gredos, Madrid, 2010 (397-398).

[7] Adolf Grünbaum. *Modern Science and Zeno's Paradoxes.* George Allen And Unwin Ltd, London, 1967.

[8] Shahen Hacyan. *Física y metafísica del espacio y el tiempo.* Fondo de Cultura Económica, Mexico, 2004.

[9] Charles Hamblin. Starting and Stopping. *The Monist*, 53:410 –425, 1969.

[10] Georg Wilhelm Frederich Hegel. *Lógica.* Folio, Barcelona, 2003.

[11] Immanuel Kant. *Critique of pure reason*. Cambridge University Press, 1998.

[12] A. León. Coevolution: New Thermodynamic Theorems. *J. Theor. Biol.*, 147(2):205 – 212, 1990.

[13] A. León. Living beings as informed systems: towards a physical theory of information. *Journal of Biological Systems*, 4(4):565 – 584, 1996.

[14] A. León. The aleph-zero or zero dichotomy. *Cogprints*, pages 1–7, September 2006. https://arxiv.org/abs/0804.2934.

[15] A. León. A critique of selfreference: what Gödel theorem really proves. *The General Science Journal*, pages 1–9, 2021.

[16] A. León. *Paradoxes and theorems*. Self edition. Printed at amazon.com, 2021.

[17] A. León. *Apparent relativity*. Self edition in KDP. Printed at Amazon.com. Free pdf, 2022.

[18] John Losee. *Introducción histórica a la filosofía de la ciencia*. Alianza, Madrid, 1987.

[19] Joseph Mazur. *The Motion Paradox*. Dutton, 2007.

[20] J. E. McTaggart. The unreality of time. *Mind*, 17:457 – 474, 1908.

[21] Brian Medlin. The Origin of Motion. *Mind*, 72:155 – 175, 1963.

[22] Andreas W. Moore. *The Infinite*. Routledge, New York, 2001.

[23] Chris Mortensen. Change. In E. N. Zalta, editor, *Stanford Encyclopaedia of Philosophy*. Stanford University, URL = http://plato.stanford.edu, 2020.

[24] Javier Ordoñez, Victor Navarro, and José Manuel Sánchez Ron. *Historia de la Ciencia*. Espasa Calpe, Madrid, 2004.

[25] Parménides. Acerca de la naturaleza. In Alberto Bernabé, editor, *De Tales a Demócrito. Fragmentos presocráticos*, pages 159 – 167. Alianza, Madrid, 1988.

[26] Carlo Rovelli. *La realidad no es lo que parece. La estructura elemental de las cosas.* Tusquets, 2015.

[27] Bertrand Russell. *Historia de la Filosofía Occidental.* Espasa Calpe, Madrid, 1997.

[28] W. C. Salmon. *Zeno's Paradoxes.* Hackett Publishing Company, Inc, Indianapolis, Cambridge, 2001.

[29] Wesley C. Salmon. Introduction. In Wesley C. Salmon, editor, *Zeno's Paradoxes*, pages 5 – 44. Hackett Publishing Company, Inc, Indianapolis, Cambridge, 2001.

[30] Steven Savitt. Being and Becoming in Modern Physics. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy.* The Stanford Encyclopedia of Philosophy, 2008.

[31] Erwin Schrödinger. *La naturaleza y los griegos.* Tusquets, Barcelona, 1996.

[32] Carlos Solís and Luis Sellés. *Historia de la ciencia.* Espasa Calpe, Madrid, 2005.

[33] G. H. Von Wright. *Time, Change and Contradiction.* Cambridge University Press, Cambridge, 1968.