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The Minimum Speed of a Free Massive Particle

Pavle I. Premović

Laboratory for Geochemistry, Cosmochemistry and Astrochemistry,
University of Niš, P.O. Box 224, 18000 Niš, Serbia

Abstract. The equations for the minimum speed and kinetic energy of a free massive particle are derived within the non-relativistic and special relativistic frameworks. These equations are based on de Broglie's relation between momentum and wavelength of this particle.

Keywords: particle, de Broglie's wavelength, speed, kinetic energy, relativistic.

The particle-wave duality of matter is one of the biggest mysteries in science because a massive particle and a wave are completely opposite each other in every way. Indeed, this particle is a discrete entity enclosed to a relatively very small volume of space, while a wave propagates over a large region of space.

Introduction. In modern physics, it is now widely accepted that light (or in more general terms, electromagnetic radiation) has a dual nature. The wave-like nature of light explains most of its properties: reflection, refraction, diffraction, interference and Doppler effect. On the other hand, the photoelectric effect and Compton effect can be only explained based particle-photon nature of light. The wavelength of this photon λ can be expressed as follows

$$\lambda = h/p \quad \ldots (1)$$

where h (= 6.62×10^{-34} J sec^{-1}) is Planck's constant and p is the momentum of photon.

Elementary physics shows that the momentum of a free massive particle p is

$$p = m\upsilon$$

where m is the relativistic mass of this particle and υ is its speed relative to an observer at rest. De-Broglie postulated that a free massive particle shows the dualistic wave-particle character just like light and the eqn. (1) can be also applied to this particle

$$\lambda = h/m\upsilon \quad \ldots (2).$$

In the first part of this paper, we will deal with a free[1] "non-relativistic" massive particle (hereinafter "non-relativistic" particle) and in the second part we will deal, in general,

[1] It is a free particle in the sense that it is experiencing no net force.

with a free massive particle (hereinafter a free particle).

In general, the “non-relativistic” particles are those whose speed υ is far less than the speed of light or $\upsilon << c$ (= 2.99792×10^5 km sec^{-1}). Physicists usually assume that the massive particles with $\upsilon/c \leq 0.1$ (or $\upsilon \leq 0.1c$) are “non-relativistic”. De Broglie’s particle-wave duality has been also verified experimentally for the “non-relativistic” (e. g. electron or neutron diffraction) and relativistic (e. g. electron diffraction) massive particles.

Derivation and Discussion. Suppose now that the “non-relativistic” particle is spherical and has a diameter and a mass at rest: D_0 and m_0. Obviously, de Broglie’s wave needs a medium through which to spread and that is of course - matter. From analogy with light[2], we postulate that the maximum de Broglies’s wavelength λ_{max} of a “non-relativistic particle” must be equal to its diameter.

Mathematically speaking

$$\lambda_{max} = D_0.$$

Applying the formula (2), we get

$$\lambda_{max} = h/m_0\upsilon_{min}$$

where υ_{min} is the minimum speed of a “non-relativistic” particle. Substituting λ_{max} of this equation with D_0 and after some rearrangement we get

$$\upsilon_{min} = h/m_0D_0 = \beta \quad \ldots (3).$$

According to this equation, in order for a massive “non-relativistic” particle to have a minimum velocity β equals zero its diameter must be infinitely large, which is, of course, impossible.

The minimum kinetic energy of a “non-relativistic” particle is

$$KE_{min} = 1/2(m_0\beta^2) \quad \ldots (4).$$

In sum, eqn. (3) appear also to set a lower limit for its speed, momentum and kinetic energy.

Examples:

(1) Alpha (α)-particle consists of two protons and two neutrons. The mass of the α-particle is about 6.65×10^{-27} kg and its diameter is about 3.6×10^{-15} m. Employing eqn. (3) we calculate that

[2] We can go a step further postulating that the wavelength of a “non-relativistic particle” is equal to its diameter divided by an integer n = 1, 2, 3,… Mathematically speaking, $\lambda = D_0/n$. Alternatively, from analogy with the standing wave in a spring we can also postulate that $\lambda = D_0/n$ but where n = 1/2, 1, 3/2,… so that $\lambda_{max} = 2D_0$. Further considerations of these two approaches are beyond the scope of this paper.

the minimum speed of this particle is about 2.8×10^7 m sec^{-1} (or about 0.093c). In other words, this is roughly the lowest possible speed of the free α-particle.

The kinetic energy of the free α-particles emitted by the uranium isotopes (^{226}U - ^{238}U) ranges from about 4.2 MeV (= 6.7×10^{-13} J) up to about 7.6 MeV (= 12.1×10^{-13} J). Using the expression for the "non-relativistic" kinetic energy KE = $1/2(m\upsilon_\alpha^2)$ we estimated that the speed of their α-particles υ_α range is about 1.42×10^7 m sec^{-1} up to about 1.60×10^7 m sec^{-1}. These values agree in order of magnitude with are the above rough estimate of the lowest possible speed of the free α-particle.

Alpha particles are relatively big and heavy and are not able to penetrate very far through a medium. As a result of scattering collisions with various nuclei of the medium through which the free α-particle passes, its kinetic energy is reduced and usually becomes a helium atom capturing two electrons from its surroundings.

(2) Macroscopic non-relativistic objects would have a minimum speed extremely low. For the golf ball with a mass of about 0.05 kg and a diameter of 0.05 m we estimate [using eqn. (3)] its minimum speed would be about 2.7×10^{-31} m. The average speed of a golf ball is about 50 m sec^{-1}.

In the next part of this paper, we will consider a relativistic particle and the consequences of that consideration on the case of a "non-relativistic" particle.

Strictly speaking, all free massive particles are relativistic. Even if their speed is much less than the speed of light (or $\upsilon \ll c$), they are still relativistic. So, there is only a relativistic free massive particle or better to say a free massive particle, in general. This is why the term non-relativistic is put under quotation marks.

Special theory of relativity sets the light speed c as an upper limit to the speed of a massive particle. According to this theory, the mass of a relativistic particle

$$m = m_0/\sqrt{(1 - \upsilon^2/c^2)} \quad \dots (5).$$

However, Special relativity states that the above spherical "non-relativistic" particle traveling at a relativistic speed would contract in the direction of motion becoming the prolate spheroid-shaped relativistic particle, Fig 1. Its diameter at rest D_0 would be shortened in the direction of its motion, by the factor $(1- \upsilon^2/c^2)^{1/2}$.
In equation form,

$$L = D_0\sqrt{(1 - \upsilon^2/c^2)} \quad \dots (6)$$

where L is the length of a relativistic particle[3] (hereinafter length) along its direction of motion Fig. 1.

Multiplying eqns. (5) and (6) we have

[3] In fact, its equatorial axis.

$$mL = m_0D_0.$$

Eqn. (6) shows that in contrast to the previous non-relativistic particle whose diameter D_0 is assumed to be constant; the length of a relativistic particle L depends on the speed of this particle υ. Obviously, its wavelength λ cannot be larger than the length L. In equation form,

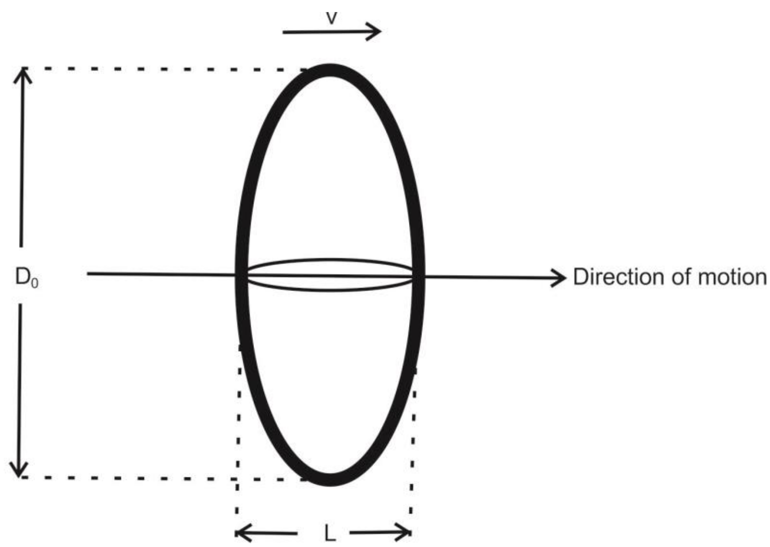

Fig. 1. The shape and dimensions of the free spherical particle at relativistic speed.

$$\lambda \leq L = D_0\sqrt{(1 - \upsilon^2/c^2)}.$$

Combining eqn. (2) and the left end of this equation we have

$$\upsilon \geq h/mL.$$

Substituting into this equation m_0D_0 instead mL we obtain

$$\upsilon \geq h/m_0D_0$$

having a minimum value

$$\upsilon_{min} = h/m_0D_0 = \beta.$$

So, its minimum kinetic energy

$$KE_{min} [= (1/2m\upsilon_{min}^2)] = 1/2(m_0\beta^2)$$

These equations are identical to eqn. (3) and eqn. (4) derived for υ_{min} and KE_{min} of a "non-relativistic" particle. In other words, our approach to the minimum speed and kinetic energy of "non-relativistic" massive particle in the first part of this work sounds reasonable.

Conclusion. The equations for the minimum speed and kinetic energy of a free massive particle are derived. These equations are based on the non-relativistic and special relativistic formulations using de Broglie's relation between linear momentum and wavelength of this particle.