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The LLR Experiment:

An Analysis of the Lunar Laser Ranging Test of the Invariance of c

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Abstract:

In the following investigation, theoretical predictions, related to the reported result of the Lunar-Laser-Ranging Experiment, are computed and analyzed, within the framework of several physical theories. And the similarities as well the differences, in each case, are pointed out.

Keywords:

Relative speed of light; lunar laser ranging experiment; corner-cube reflectors; Lorentz invariance; ballistic speed of light; classical wave theory; Doppler effect; emission theories; modeled distances; relativity; Larmor-Lorentz theory; computed predictions; anomalous variations; earth-moon system; reference frame; barycenter of the solar system; elastic-impact theory.

Introduction:

In the LLR experiment, under discussion, the observer and the source of the laser light are at rest relative to each other; while, at the same time, they're moving with the same speed in the same direction with respect to retro-reflectors on the moon.

Laser light pulses are sent from the earth and reflected by retro-reflectors on the moon. The speed of laser light pulses is calculated from round-trip time-of-flight measurements and modeled distances.

The measured speed of light c in the moving observer's rest frame is found to exceed the canonical value ($c = 299{,}792{,}458$ m/s) by 200 ± 10 m/s, which is just the speed of the observatory along the line of sight due to the rotation of the earth during the LLR experiment.

This result, according to the experimental report, is a first-order violation of local Lorentz invariance; since the speed of light seems to depend on the motion of the observer after all, as in the classical wave theory, which assumes that a preferred reference frame exists for the propagation of light [*Ref. #1.a*].

The main problem with the local Lorentz invariance, in this regard, lies, of course, in the fact that Einstein's special theory, from the very beginning, has not acknowledged explicitly the validity of the following equation for calculating the speed of light relative to moving observers and light sources:

$$c' = \sqrt{c^2 + v^2 + 2vc\cos\theta}$$

where c' is the relative speed of light; and θ is the angle between the direction of the speed of light c and the direction of the speed of the observer, or the speed of the light source, v.

But why does special relativity fail to acknowledge clearly the concept of relative speeds of light?

Special relativity cannot acknowledge verbally the validity of relative speeds of light, because its theoretical framework is built upon the assumption that the speed of light is independent of the speed of the observer; otherwise relative speeds of light with respect to moving observers will turn the null result of the Michelson-Morley experiment into a major anomaly.

But once again, why doesn't the null result of the Michelson-Morley experiment prevent the Larmor-Lorentz theory, which is quite similar to special relativity in many respects, from explicitly acknowledging the validity of relative speeds of light?

Obviously, that is because, in the Larmor-Lorentz theory, the Lorentz-FitzGerald length contraction is assumed as a postulate first; and the null result of the Michelson-Morley experiment is deduced as a consequence second. While, in Einstein's special relativity, by contrast, the null result of the Michelson-Morley experiment is taken as evidence for the nonexistence of relative speeds of light and assumed as a postulate first; and the Lorentz-FitzGerald length contraction is deduced as a consequence second.

Notwithstanding its verbal denial, the theory of special relativity, nonetheless, has no other option but to acknowledge implicitly the notion of relative speeds of light; and to use it mathematically, in exactly the same way like every other physical theory generally and the Larmor-Lorentz theory in particular, throughout its calculations.

So, now, in the final analysis, can Einstein's special relativity predict that, in the LLR experiment, the measured speed of light *c* in the moving observer's rest frame must exceed the "*canonical value (c = 299,792,458 m/s)* by *200 ±10 m/s*"?

If, as in the treatment of Doppler effect, Sagnac effect, light aberration, and similar phenomena, the verbal prohibition on relative speeds of light is quietly dropped, then the theory of special relativity should make, through the mathematical treatment of the LLR case, the same theoretical prediction as the one made by the classical wave theory.

In any case, the following points can be made with regard to the current LLR experiment:

- The reported result of the LLR experiment demonstrates very convincingly that the velocity of light is dependent on the velocity of the measuring observer.

- The reported result of the LLR experiment implies, necessarily, that observers moving with different speeds measure and obtain different numerical values for the speed of the same light from the same light source.

- The result of the LLR experiment is in agreement with the result of the Sagnac experiment. And both experiments are consistent with the notion that the relative speed of light is, necessarily, dependent on the motion of the measuring observer.

- The result of the LLR experiment is in total agreement with Doppler-shift measurements in all cases of approaching as well as receding observers.

- Although the distance to the moon, at any moment of time, can be determined by several methods, none of those methods, except the method of lunar laser ranging, can determine the distance and position of the moon with the high degree of precision required for carrying out the current LLR experiment.

- All of the modeled distances to the moon, in the LLR data table, are based firmly, by definition, upon these two equations of the classical wave theory:

$$d = \frac{t_A\left(c+v\right)}{2}$$

where t_A is the total travel time of reflected light, in the case of approach; and:

$$d = \frac{t_R\left(c-v\right)}{2}$$

where t_R is the total travel time of reflected light, in the case of recession.

- Since the above method, for obtaining the modeled distances to the moon, is related to the framework of the classical wave theory, the computed predictions, on the basis of this theory, should be the closest numerically to the reported result of the LLR experiment.

- Accordingly, the general procedure, employed in the current investigation, is to calculate the predictions of various physical theories; and then to compare those predictions to the computed predictions of the classical wave theory.

- Because the aforementioned modeled distances to the moon are based upon the assumption of the independence of the speed of light of the speed of the light source, the reported result of the LLR experiment cannot be used directly to check whether or not the speed of light and the speed of the source are additive in accordance with various ballistic theories of light.

- Nonetheless, the LLR experimental result can be used indirectly to check for the additive speed of light by assuming first that the speed of light is independent of the speed of the light source; and then to search for the main anomalies that should appear, if the speed of light is, in fact, dependent, in the ballistic sense, on the speed of the light source.

In the following calculations, the prediction of each physical theory will be computed separately. Also, in order to avoid any potential ambiguity, algebraic signs will be inserted into the equations explicitly; and the absolute value of the velocity vector of the LLR laboratory $v\cos\theta$ will be used throughout those calculations.

1. The Prediction of the Classical Wave Theory:

Let v denote the rotational velocity of the earth around its geometrical axis; and d denote the distance to the moon.

Because the transverse and the radial components of the rotational velocity of the earth, during the LLR experiment, are greater than zero, the light path from the earth to the moon and the light path from the moon to the earth, in this experiment, must form two similar isosceles triangles; instead of just one isosceles triangle as, for example, in the case of the Michelson-Morley experiment:

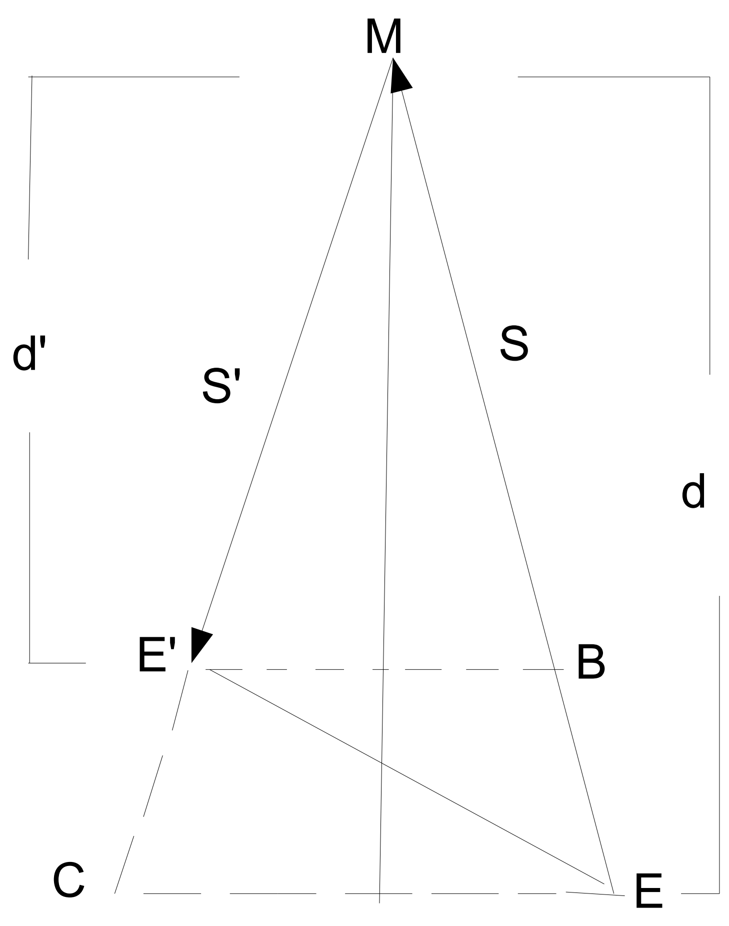

Illustration: Laser Light Path

- During the first leg of its journey from the earth to the moon, the laser light travels along the side S of the larger isosceles triangle *EMC* whose height is *d*:

$$S = \sqrt{d^2 + (t_1 v \sin\theta)^2}$$

where t_1 is the travel time of the later light during the first leg; and θ is the angle between the rotational velocity vector of the earth and the line of sight to the moon.

- During the second leg of its journey from the moon to the earth, the laser light travels along the side S' of the smaller isosceles triangle *BME'* whose height is *d'*:

$$d' = d - t_1 v \cos\theta - t_2 v \cos\theta$$

where t_2 is the travel time of the later light during the second leg of its journey, in the case of approach; and:

$$d' = d + t_1 v \cos\theta + t_2 v \cos\theta$$

in the case of recession.

- And since these two isosceles triangles are similar, the two sides are related by this equation:

$$S' = S\left(\frac{d'}{d}\right)$$

A. In the Case of Approach:

In order to implement the plus and minus signs explicitly in all the equations to follow, let the laboratory along with the observer approach the moon at a radial speed equals to the absolute value of *$v\cos\theta$;* i.e, the magnitude of:

$$v\cos\theta = |v\cos\theta|$$

According to the classical wave theory, therefore, in the reference frame of the laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{S}{c} = \frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2}} \qquad 1.1$$

where c is the speed of light.

And the travel time of the laser light, during its journey from the moon to the earth, t_2 is:

$$t_2 = \frac{S'}{c} = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2}}\right)\left(\frac{c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2}-v\cos\theta}{c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2}+v\cos\theta}\right) \qquad 1.2$$

And therefore, the classical wave theory predicts that the total travel time of the laser beam, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2}+v\cos\theta} \qquad 1.3$$

And if, as in the case of the LLR experiment:

$$v << c$$

then the total travel time is approximately:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

B. In the Case of Recession:

Let the laboratory along with the observer recede from the moon with a velocity whose magnitude equals to the absolute value of *vcosθ*.

And therefore, the travel time of the laser light, during its journey from the earth to the moon t_1 is:

$$t_1 = \frac{S}{c} = \frac{d}{c\sqrt{1-\left(\frac{v}{s}\sin\theta\right)^2}} \qquad 1.4$$

where d is the distance to the moon; and c is the speed of light.

And the travel time of the laser light, during its journey from the moon to the earth, t_2 is:

$$t_2 = \frac{S'}{c} = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{s}\sin\theta\right)^2} - v\cos\theta} \right) \left(1 + \frac{v\cos\theta}{c\sqrt{1-\left(\frac{v}{s}\sin\theta\right)^2}} \right) \qquad 1.5$$

And hence, in the reference frame of the laboratory, the classical wave theory predicts that the total travel time of the laser beam, in the case of recession, t_R is:

$$t_R = t_1 + t_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2} - v\cos\theta} \qquad 1.6$$

And since in the LLR experiment:

$$v << c$$

the total travel time, in the receding case, is about:

$$t_R \approx \frac{2d}{c - v\cos\theta}$$

2. The Prediction of Larmor-Lorentz Theory:

Since, according to the Larmor-Lorentz theory, the velocity of light is independent of the velocity of the light source, its LLR prediction is the same as that of the classical wave theory.

A. In the Case of Approach:

Let the laboratory along with the observer approach the moon at a speed equals to the absolute value of *vcosθ*.

Accordingly, in the reference frame of the laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \qquad 2.1$$

where c is the speed of light; and v is the rotational speed of the earth.

And the travel time of the laser light, during its journey from the moon to the earth, t_2 is:

$$t_2 = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}+v\cos\theta}\right)\left(1-\frac{v\cos\theta}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}}\right) \qquad 2.2$$

And therefore, the Larmor-Lorentz theory predicts that the total travel time of the laser light, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}+v\cos\theta} \qquad 2.3$$

And because in the LLR experiment:

$$v << c$$

the total travel time, in this case, is approximately:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

B. In the Case of Recession:

Let the laboratory along with the observer recede from the moon at a speed equals to the absolute value of $v\cos\theta$:

According to the Larmor-Lorentz theory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \qquad 2.4$$

where d is the distance to the moon; and c is the speed of light.

And the travel time of the laser light, during its journey from the moon to the earth, t_2 is:

$$t_2 = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} - v\cos\theta}\right)\left(1 + \frac{v\cos\theta}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}}\right) \qquad 2.5$$

And so, , in the reference frame of the laboratory, the Larmor-Lorentz theory predicts that the total travel time of the laser beam, in the case of receding earth, t_R is:

$$t_R = t_1 + t_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} - v\cos\theta} \qquad 2.6$$

And if, as in the case of the earth's rotation:

$$v << c$$

then the total travel time is approximately:

$$t_R \approx \frac{2d}{c - v\cos\theta}$$

<u>3. The Prediction of the Special Theory of Relativity:</u>

The only problem, here, is that Einstein's special theory does not acknowledge explicitly the notion of relative speeds of light such as:

$$c' = c + v$$

and:

$$c' = c - v$$

and generally:

$$c' = \sqrt{c^2 + v^2 + 2vc\cos\theta}$$

as a valid concept in physics.

And that is because the notion of relative speeds of light implies, necessarily, that the speed of light depends on the speed of the observer; which implies, in turn, that observers, moving with different speeds relative to each other, have to measure different numerical values for the speed of light from the one and the same source of light, in a stark contradiction with one of the two main assumptions of this theory.

Nonetheless, the theory, under discussion, acknowledges implicitly the idea of relative speeds of light; and uses it, in the same way like all physical theories generally and the Larmor-Lorentz theory in particular, throughout its calculations.

And since, according to the special theory, the velocity of light and the velocity of the light source are not additive, its computed predictions, in the context of LLR experiment, are the same as those of the classical wave theory.

A. In the Case of Approach:

Let the laboratory along with the observer approach the moon with a velocity whose magnitude equals to the absolute value of *$v\cos\theta$*.

In the reference frame of the moving laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1, according to special relativity, therefore, is:

$$t_1 = \frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \qquad 3.1$$

where c is the speed of light.

Likewise, the travel time of the laser light, during its journey from the moon to the earth, t_2 is:

$$t_2 = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} + v\cos\theta} \right)\left(1\text{-}\frac{v\cos\theta}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \right) \qquad 3.2$$

And therefore, the special theory predicts that the total travel time of the LLR beam, in the case of approach, t_A is:

$$t_A = t_1 + \mathrm{t}_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} + v\cos\theta} \qquad 3.3$$

And since in this experiment:

$$v << c$$

the total travel time is about:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

B. In the Case of Recession:

Let the laboratory along with the observer recede from the moon with a speed equals to the absolute value of *vcosθ*:

Accordingly, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \qquad 3.4$$

where d is the distance to the moon; and c is the speed of light.

And the travel time of the laser light, during its journey from the moon to the earth, t_2 is:

$$t_2 = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} - v\cos\theta}\right)\left(1+\frac{v\cos\theta}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}}\right) \qquad 3.5$$

And hence, , in the reference frame of the laboratory, special relativity predicts that the total travel time of the laser beam, in the case of recession, t_R is:

$$t_R = t_1 + t_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} - v\cos\theta} \qquad 3.6$$

And because in the case of the LLR experiment:

$$v << c$$

the total travel time is approximately:

$$t_R \approx \frac{2d}{c - v\cos\theta}$$

4. The Prediction of the New-Source Emission Theory:

The new-source emission theory of light makes, with regard to the current experiment, two different predictions, depending on whether or not the so-called '*extinction theorem*' is applied to this case:

- ***With Extinction Theorem:***

If the extinction theorem is applied, then the velocity of light is de facto independent of the velocity of the light source; and hence, the LLR prediction given by the modified new-source emission theory is the same as the one given by the classical wave theory.

A. In the Case of Approach:

If the light source and the observer approach the moon at a speed of $|v\cos\theta|$, then, in the reference frame of the laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is obtained by using this equation:

$$t_1 = \frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \qquad 4.1$$

where c is the speed of light.

And the travel time of the laser light, during its journey from the moon to the earth, t_2 can be obtained from this equation:

$$t_2 = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} + v\cos\theta} \right) \left(1 - \frac{v\cos\theta}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \right) \qquad 4.2$$

And therefore, the modified new-source emission theory predicts that the total travel time of the laser beam, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} + v\cos\theta} \qquad 4.3$$

And if, as in the case of this LLR experiment:

$$v << c$$

then the total travel time is approximately:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

B. In the Case of Recession:

Let the laboratory along with the observer recede from the moon with a velocity of a magnitude equals to the the absolute value of $v\cos\theta$:

And subsequently, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}} \qquad 4.4$$

where d is the distance to the moon; and c is the speed of light.

And the travel time of the laser light, during its journey from the moon to the earth, t_2 is:

$$t_2 = \left(\frac{d}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} - v\cos\theta}\right)\left(1+\frac{v\cos\theta}{c\sqrt{1-\left(\frac{v}{c}\sin\right)^2}}\right) \qquad 4.5$$

And it follows, therefore, that in the reference frame of the laboratory, the modified new-source theory predicts that the total travel time of the laser light, in the case of recession, t_R is:

$$t_A = t_1 + t_2 = \frac{2d}{c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2} - v\cos\theta} \qquad 4.6$$

And since in the LLR experiment:

$$v << c$$

the total travel time, in this case, is about:

$$t_R \approx \frac{2d}{c - v\cos\theta}$$

- ***Without Extinction Theorem:***

If the extinction theorem is not applied to the current case, then, according to the original new-source theory, the velocity of light is dependent on the velocity of the light source; i.e:

$$c' = \sqrt{c^2 + v^2 + 2vc\cos\theta}$$

where c' is the resultant velocity of light; and θ is the angle between the velocity vector of the earth's rotation and the velocity vector of light c.

However, in the LLR experiment, θ is defined as the angle between the direction of the earth's rotation and the line of sight to the moon. And hence, according to the unmodified new-source theory, the angle θ, in this case, is the direction of the velocity resultant of light; and the above equation takes the following form:

$$c' = c\sqrt{1 - \left(\frac{v}{c}\sin\theta\right)^2} + v\cos\theta$$

where the headlight effect is included in this equation; and subsequently no further correction is required for the bending of light rays in the forward direction due to the motion of the light source.

A. In the Case of Approach:

Let the laboratory along with the observer and the light source approach the moon with a speed $|v\cos\theta|$.

And therefore, according to the original new-source theory, in the reference frame of the laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{S}{c'} = \frac{d}{c'\sqrt{1 - \left(\frac{v}{c'}\sin\theta\right)^2}} \qquad 4.7$$

where c' is obtained by using this equation:

$$c' = c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2} + v\cos\theta$$

and S is calculated by using this equation:

$$S = \sqrt{d^2 + \left(t_1 v \sin\theta\right)^2}$$

Since the orbital velocity vector of the moon, around the center of the earth-moon system, is at right angles to the LLR line of sight, the incident laser light, according to the new-source emission theory, is re-emitted back to the earth, along the same line of sight, at the resultant of the muzzle velocity of the laser light c and the orbital velocity of the moon v_m:

$$c'' = c\sqrt{1 - v_m{}^2 / c^2}$$

where c'' is the velocity resultant of light, during its journey from the moon to the earth.

And accordingly, the travel time of the laser light, during the second leg of its journey, t_2 is:

$$t_2 = \frac{S'}{c''} = \left(\frac{d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}}\right)\left(\frac{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2} - v\cos\theta}{c''\sqrt{1-\left(\frac{v}{c''}\sin\theta\right)^2} + v\cos\theta}\right) \qquad 4.8$$

where S' is obtained from the following equation:

$$S' = S\left(\frac{d - t_1 v\cos\theta - t_2 v\cos\theta}{d}\right)$$

And therefore, the unmodified new-source emission theory predicts that the total travel time of the laser, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \left(\frac{d}{c'' \sqrt{1 - \left(\frac{v}{c''} \sin \right)^2} + v \cos\theta} \right) \left(1 + \frac{c'' \sqrt{1 - \left(\frac{v}{c''} \sin \right)^2}}{c' \sqrt{1 - \left(\frac{v}{c'} \sin \right)^2}} \right) \qquad 4.9$$

And if, as in the case of the LLR experiment:

$$v << c$$

then the total travel time is approximately:

$$t_A \approx \left(\frac{2d}{c + v\cos\theta} \right) \left(1 - \frac{v\cos\theta}{2(c + v\cos\theta)} \right)$$

B. In the Case of Recession:

Let's assume that the laser source and the observer are receding from the moon at a speed equals to the absolute value of *vcosθ*:

And therefore, according to the new-source theory, in the reference frame of the laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{S'}{c'} = \frac{d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}} \qquad 4.10$$

where c' is:

$$c' = c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} - v\cos\theta$$

and S is:

$$S = \sqrt{d^2 + \left(t_1 v \sin\theta\right)^2}$$

Since the orbital velocity vector of the moon, around the center of the earth-moon system, is at right angles to the LLR line of sight, the incident laser light, according to the original new-source emission theory, is re-emitted back to the earth, along the same line of sight, at the resultant of the muzzle velocity of light c and the orbital velocity of the moon v_m:

$$c'' = c\sqrt{1 - v_m^{\,2}/c^2}$$

where c'' is the velocity resultant of light, during its journey from the moon to the earth.

And consequently, the travel time of the laser light, during the second leg of its journey, t_2 is:

$$t_2 = \frac{S'}{c''} = \left(\frac{d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}}\right)\left(\frac{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}+v\cos\theta}{c''\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}-v\cos\theta}\right) \qquad 4.11$$

where S' is defined as:

$$S' = S\left(\frac{d + t_1 v\cos\theta + t_2 v\cos\theta}{d}\right)$$

And it follows, therefore, that the unmodified new-source emission theory predicts that the total travel time of the laser beam, in the case of receding LLR laboratory, t_R is:

$$t_R = t_1 + t_2 = \left(\frac{d}{c''\sqrt{1-\left(\frac{v}{c''}\sin\right)^2} - v\cos\theta}\right)\left(1 + \frac{c''\sqrt{1-\left(\frac{v}{c''}\sin\right)^2}}{c'\sqrt{1-\left(\frac{v}{c'}\sin\right)^2}}\right) \quad 4.12$$

But since in the case of the LLR experiment:

$$v << c$$

the total travel time, in the case of recession, is about:

$$t_R \approx \left(\frac{2d}{c - v\cos\theta}\right)\left(1 + \frac{v\cos\theta}{2\left(c - v\cos\theta\right)}\right)$$

**5. The Prediction of the Elastic-Impact Emission Theory:**

Within the framework of the theory, under discussion, the muzzle velocity of light c and the velocity of the light source v are additive.

Let the source of the laser light and the observer approach the moon with a speed equals to the absolute value of the radial velocity $v\cos\theta$.

Therefore, according to the elastic-impact emission theory, in the reference frame of the laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{S}{c'} = \frac{d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}} \qquad 5.1$$

where c' is obtained by using the following equation:

$$c' = c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2} + v\cos\theta$$

and S is defined in accordance with this equation:

$$S = \sqrt{d^2 + \left(t_1 v \sin\theta\right)^2}$$

Since the orbital velocity vector of the moon, around the center of the earth-moon system, is at right angles to the laboratory's line of sight, the incident laser light, according to the elastic-impact emission theory, is reflected back to the earth, along the same line of sight, at its velocity of incidence c':

$$c' = c\sqrt{1-\left(\frac{v}{c}\sin\theta\right)^2} + v\cos\theta$$

And accordingly, the travel time of the laser light, during the second leg of its journey, t_2 is:

$$t_2 = \frac{S'}{c'} = \left(\frac{d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}} \right) \left(\frac{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2} - v\cos\theta}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2} + v\cos\theta} \right) \qquad 5.2$$

where S' is:

$$S' = S\left(\frac{d - t_1 v\cos\theta - t_2 v\cos\theta}{d} \right)$$

And therefore, the elastic-impact emission theory predicts that the total travel time of the laser beam, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \frac{2d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2} + v\cos\theta} \qquad 5.3$$

And because in the LLR experiment:

$$v << c$$

the total travel time is approximately:

$$t_A \approx \frac{2d}{c + 2v\cos\theta}$$

B. In the Case of Recession:

Let the laboratory along with the observer and the light source recede from the moon with a velocity whose magnitude is equal to the absolute value of $v\cos\theta$:

And accordingly, in the reference frame of the laboratory, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}} \qquad 5.4$$

where c' is:

$$c' = c\sqrt{1-\left(\frac{v}{c}\sin\right)^2} - v\cos\theta$$

and S is:

$$S = \sqrt{d^2 + \left(t_1 v \sin\theta\right)^2}$$

And because the orbital velocity vector of the moon, around the center of the earth-moon system, is at right angles to the LLR line of sight, the incident laser light, according to the elastic-impact emission theory, is reflected back to the earth, along the same line of sight, at its velocity of incidence.

And hence, the travel time of the laser light, during the second leg of its journey, t_2 is:

$$t_2 = \frac{S'}{c'} = \left(\frac{d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}} \right) \left(\frac{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}+v\cos}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}-v\cos} \right) \qquad 5.5$$

where S' is:

$$S' = S\left(\frac{d + t_1 v\cos\theta + t_2 v\cos\theta}{d}\right)$$

And therefore, the elastic-impact emission theory predicts that the total travel time of the laser beam, in the case of receding earth, t_R is:

$$t_R = t_1 + t_2 = \frac{2d}{c'\sqrt{1-\left(\frac{v}{c'}\sin\theta\right)^2}-v\cos\theta} \qquad 5.6$$

And if, as in the case of the current LLR experiment:

$$v << c$$

then the total travel time is about:

$$t_R \approx \frac{2d}{c - 2v\cos\theta}$$

6. A Brief Evaluation of Computed Predictions:

Let's, now, take a closer look at the LLR predictions of the above theories.

I. The Calculated Prediction of the Classical Wave Theory:

All of the modeled distances to the moon, in the LLR data table, are obtained by the lunar-laser-ranging method, to begin with, and based firmly upon these two equations of the classical wave theory:

$$d = \frac{t_A\left(c + v\cos\theta\right)}{2}$$

where t_A is the total travel time of reflected light, in the case of approach; and:

$$d = \frac{t_R\left(c - v\cos\theta\right)}{2}$$

where t_R is the total travel time of reflected light, in the case of recession

It's expected all along, therefore, that the computed prediction above, with respect to the reference frame of the laboratory, in accordance with the classical wave theory, produces, in most cases, numerical results very close to the observed result of the LLR experiment.

In addition, this same prediction can be easily transformed to the reference frame of the gravitational center of the earth-moon system to produce the same numerical results without any significant changes.

Of course, the second-order effects, due to the orbital velocity of the earth-moon system around the sun, are still present, just as in the days of Michelson and Morley; but the universal medium, as postulated by this theory, can be redefined in a number of ways to effectively hide those second-order effects, in calculations done in the reference frame of the LLR laboratory and the reference frame of the gravitational center of the earth-moon system.

However, if the same computations are carried out, in the reference frame of the gravitational center of the solar system, then the classical wave theory cannot produce the same numerical results; and instead it will give new numerical results significantly different from the computed results in the reference frame of the gravitational center of the earth-moon system and in the reference frame of the moving LLR laboratory.

Furthermore, no published redefinition of the universal medium, postulated by the classical wave theory, so far, can make the first-order effects, due to the orbital velocity of the earth-moon system around the sun, go away or somehow disappear in calculations done, on the basis of this theory, in the reference frame of the gravitational center of the solar system .

Let's, for example, recalculate the above prediction of the classical wave theory, in the case of approach, in the reference frame of the gravitational center of the solar system.

To simplify the calculations, let the moon's orbital velocity vector be ahead of the earth's orbital velocity vector along a straight line, in their orbit around the sun.

And let the laboratory along with the observer, as observed in the reference frame of the gravitational center of the solar system, approach the moon with a velocity v.

And therefore, according to the classical wave theory, in the reference frame of the gravitational center of the solar system, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d + v_O t_1}{c} = \frac{d}{c - v_O}$$

where v_O is the orbital speed of the earth-moon system around the gravitational center of the solar system.

During the second leg, the displacement $v_O t_1$ made by the receding moon and the displacement $v_O t_1$ made by the approaching earth, during the first leg, cancel each other out.

And therefore, the travel time of the laser light, during its journey from the moon to the earth t_2, as computed, on the basis of this theory, in the reference frame of the gravitational center of the solar system, is:

$$t_2 = \frac{d - vt_1 - vt_2 - v_O t_2}{c} = \left(\frac{d}{c - v_O}\right)\left(\frac{c - v_O - v}{c + v_O + v}\right)$$

where v is the velocity of the LLR laboratory relative to the moon.

And so finally, in the reference frame of the gravitational center of the solar system, the classical wave theory predicts that the total travel time of the laser beam, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \left(\frac{2d}{c - v_O}\right)\left(\frac{1}{1 + v_O/c + v/c}\right)$$

which is quite different from this Equation #1.3:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

as computed earlier with respect to the reference frame of the LLR laboratory.

It follows, therefore, that the classical wave theory does not transform correctly its computed prediction from the reference frame of the LLR laboratory to the reference frame of the barycenter of the solar system.

II. The Calculated Prediction of the Larmor-Lorentz Theory:

In the reference frame of the moving laboratory and in the reference frame of the barycenter of the earth-moon system as well, the Larmor-Lorentz theory gives the same numerical results as those of the classical wave theory with regard to the reported result of the LLR experiment.

Furthermore, its Lorentz-FitzGerald length contraction effectively hides the second-order effects, due to the orbital velocity of the earth-moon system around the center of the solar system.

Nevertheless, if the same computations are performed, in the reference frame of the gravitational center of the solar system, then the Larmor-Lorentz theory will give different numerical results from the calculated numerical results in the reference frame of the gravitational center of the earth-moon system and in the reference frame of the moving LLR laboratory.

Furthermore, the Lorentz-FitzGerald length contraction, postulated by the Larmor-Lorentz theory, cannot conceal, in any conceivable way, the first-order effects, due to the orbital velocity of the earth-moon system around the sun, in calculations done, on the basis of this theory, in the reference frame of the barycenter of the solar system .

To demonstrate that is indeed the case, let's recalculate the above prediction of the Larmor-Lorentz theory, in the case of approach, for instance, in the reference frame of the gravitational center of the solar system.

And in order to simplify the following calculations, let the earth's orbital velocity vector be trailing the moon's orbital velocity vector along a straight line, in their orbit around the barycenter of the solar system.

And let the laboratory along with the observer, as observed in the reference frame of the gravitational center of the solar system, approach the moon with a velocity v.

And so, in the reference frame of the gravitational center of the solar system, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d + v_O t_1}{c} = \frac{d}{c - v_O}$$

where v_O is the orbital speed of the earth-moon system around the barycenter of the solar system.

During the second leg of the laser beam's journey, the displacement $v_O t_1$ made by the receding moon and the displacement $v_O t_1$ made by the approaching earth, during the first leg, cancel each other out.

And accordingly, the travel time of the laser light, during its journey from the moon to the earth t_2, as computed, on the basis of the Larmor-Lorentz theory, in the reference frame of the gravitational center of the solar system, is:

$$t_2 = \frac{d - vt_1 - vt_2 - v_O t_2}{c} = \left(\frac{d}{c - v_O}\right)\left(\frac{c - v_O - v}{c + v_O + v}\right)$$

where v is the velocity of the LLR laboratory relative to the moon.

And therefore, in the reference frame of the gravitational center of the solar system, the Larmor-Lorentz theory predicts that the total travel time of the laser light, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \left(\frac{2d}{c - v_O}\right)\left(\frac{1}{1 + v_O/c + v/c}\right)$$

which is very different from this Equation #2.3:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

as calculated earlier with respect to the reference frame of the moving laboratory.

It follows, therefore, that the Larmor-Lorentz theory does not transform correctly its computed prediction from the reference frame of the LLR laboratory to the reference frame of the barycenter of the solar system.

III. The Calculated Prediction of Special Relativity:

In the reference frame of the LLR laboratory, as well as in the reference frame of the barycenter of the earth-moon system, special relativity produces the same numerical results as those of the classical wave theory with regard to the observed result of the LLR experiment.

In addition, its length contraction and time dilation effectively conceal the second-order effects, due to the orbital velocity of the earth-moon system around the center of the solar system.

However, if the same computations are performed, in the reference frame of the gravitational center of the solar system, then the special theory of relativity will give different numerical results from the numerical results computed in the reference frame of the gravitational center of the earth-moon system and in the reference frame of the moving LLR laboratory.

Moreover, neither the length contraction, nor the time dilation, nor the constancy postulate of Einstein's special theory, can hide, in any conceivable way, the first-order effects, due to the orbital velocity of the earth-moon system around the sun, in any calculations done, on the basis of this theory, in the reference frame of the barycenter of the solar system .

Let's, for instance, recalculate the above prediction of the special theory, in the case of approach, in the reference frame of the gravitational center of the solar system.

To get rid of trigonometric functions, let the moon's orbital velocity vector be leading the earth's orbital velocity vector along a straight line, in their orbit around the sun.

And let the laboratory along with the observer, as observed in the reference frame of the gravitational center of the solar system, approach the moon with a velocity v.

And so, in the reference frame of the gravitational center of the solar system, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d + v_O t_1}{c} = \frac{d}{c - v_O}$$

where v_O is the orbital speed of the earth-moon system around the barycenter of the solar system.

During the second leg, the displacement $v_O t_1$ made by the receding moon and the displacement $v_O t_1$ made by the approaching earth, during the first leg, cancel each other out.

And consequently, the travel time of the laser light, during its journey from the moon to the earth t_2, as computed, on the basis of this theory, in the reference frame of the gravitational center of the solar system, is:

$$t_2 = \frac{d - vt_1 - vt_2 - v_O t_2}{c} = \left(\frac{d}{c - v_O}\right)\left(\frac{c - v_O - v}{c + v_O + v}\right)$$

where v is the velocity of the LLR laboratory relative to the moon.

And therefore, in the reference frame of the gravitational center of the solar system, the special theory of relativity predicts that the total travel time of the laser light, in the case of approach, t_A is:

$$t_A \approx t_1 + t_2 \approx \left(\frac{2d}{c - v_O}\right)\left(\frac{1}{1 + v_O/c + v/c}\right)$$

which is very different from its Equation #3.3:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

as computed earlier with respect to the reference frame of the moving laboratory.

And it follows, therefore, that special relativity does not transform correctly its computed prediction from the reference frame of the LLR laboratory to the reference frame of the barycenter of the solar system.

IV. The Calculated Prediction of the New-Source Emission Theory:

- ***The New-Source Theory With Extinction Theorem:***

If the extinction theorem is applied to the case under discussion, then the velocity of light is practically independent of the velocity of the light source; and hence, the LLR prediction given by the modified new-source emission theory is identical to the one given by the classical wave theory.

In the reference frame of the moving LLR laboratory, as well as in the reference frame of the barycenter of the earth-moon system, the new-source theory gives the same numerical results as those of the classical wave theory with regard to the reported result of the LLR experiment.

But, if the same computations are worked out, in the reference frame of the gravitational center of the solar system, then the modified new-source theory will give different numerical results from the numerical results computed, in accordance with this theory, in the reference frame of the gravitational center of the earth-moon system and in the reference frame of the moving LLR laboratory.

Furthermore, the extinction theorem, as applied within the framework of the new-source emission theory, cannot make the orbital velocity of the earth-moon system around the sun disappear or somehow go away, in any calculations carried out, on the basis of this theory, in the reference frame of the barycenter of the solar system.

To make sure that is indeed the case, let's recalculate the above prediction of the modified new-source emission theory, in the case of approach, for example, in the reference frame of the gravitational center of the solar system.

Also, in order to simplify the computations, let's choose the instant, at which the moon's orbital velocity vector is ahead of the earth's orbital velocity vector along a straight line, in their orbit around the barycenter of the solar system.

And let the laboratory along with the observer, as observed in the reference frame of the gravitational center of the solar system, approach the moon with a velocity v.

And consequently, in the reference frame of the gravitational center of the solar system, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 \approx \frac{d + v_O t_1}{c} \approx \frac{d}{c - v_O}$$

where v_O is the orbital speed of the earth-moon system around the barycenter of the solar system.

During the second leg of the round trip, the displacement $v_O t_1$ made by the receding moon and the displacement $v_O t_1$ made by the approaching earth, during the first leg, cancel each other out.

And therefore, the travel time of the laser light, during its journey from the moon to the earth t_2, as computed in the reference frame of the gravitational center of the solar system, is:

$$t_2 \approx \frac{d - v_O t_2 - v t_1 - v t_2}{c - v_O} \approx \left(\frac{d}{c+v}\right)\left(1 - \frac{v}{c - v_O}\right)$$

where v is the velocity of the LLR laboratory relative to the moon.

And hence, in the reference frame of the gravitational center of the solar system, the modified new-source theory predicts that the total travel time of the laser light, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \left(\frac{2d}{c+v}\right)\left(\frac{1-\frac{1}{2}v_O/c}{1-v_O/c}\right)$$

which is clearly different from its Equation #4.3:

$$t_A \approx \frac{2d}{c + v\cos\theta}$$

as calculated earlier with respect to the reference frame of the moving laboratory.

And it follows, therefore, that the modified new-source emission theory does not transform correctly its computed prediction from the reference frame of the moving LLR laboratory to the reference frame of the barycenter of the solar system.

- ***The New-Source Theory Without Extinction Theorem:***

If the extinction theorem is not applied, then the velocity of light is dependent on the velocity of its source, according to the original new-source emission theory.

In the reference frame of the LLR laboratory, as well as in the reference frame of the barycenter of the earth-moon system, the new-source theory gives numerical results less than the numerical results given by the classical wave theory with regard to the observed result of the LLR experiment, in the case of approach:

$$\frac{t_{nst}}{t_{cwt}} = \frac{1+\frac{1}{2}v\cos\theta/c}{1+v\cos\theta/c}$$

and numerical results greater than the results given by the classical wave theory, in the case of recession:

$$\frac{t_{nst}}{t_{cwt}} = \frac{1 - \frac{1}{2} v \cos\theta / c}{1 - v \cos\theta / c}$$

where t_{nst} is the total travel time of the laser beam as predicted by the unmodified new-source emission theory; and t_{cwt} is the total travel time as predicted by the classical wave theory.

In addition, if the same computations are done, in the reference frame of the gravitational center of the solar system, then the original new-source emission theory will give the same numerical results as the numerical results computed in the reference frame of the gravitational center of the earth-moon system and in the reference frame of the moving LLR laboratory.

To demonstrate that is indeed the case, let's recalculate the above prediction of the original new-source theory, in the case of approach, for instance, in the reference frame of the gravitational center of the solar system.

And in order to make the calculations simple, let the moon's orbital velocity vector be ahead of the earth's orbital velocity vector along a straight line, in their orbit around the sun.

And let the laboratory along with the observer and the light source, as observed in the reference frame of the barycenter of the solar system, approach the moon at a velocity v.

According to the original new-source emission theory, the laser beam travels from the earth to the moon at the velocity resultant c':

$$c' = c + v_O + v$$

where v_O is the orbital speed of the earth-moon system around the barycenter of the solar system.

And therefore, in the reference frame of the gravitational center of the solar system, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d + v_O t_1}{c + v_O + v} = \frac{d}{c + v}$$

During the second leg of the journey of the laser light, the displacement $v_O t_1$ made by the receding moon and the displacement $v_O t_1$ made by the approaching earth, during the first leg of the same round trip, balance each other out.

And therefore, the travel time of the laser light, during its journey from the moon to the earth t_2, as computed in the reference frame of the gravitational center of the solar system, is:

$$t_2 = \frac{d - v_O t_2 - v t_1 - v t_2}{c - v_O} = \left(\frac{d}{c+v}\right)\left(\frac{1}{1+v/c}\right)$$

where v is the velocity of the moving LLR laboratory relative to the moon.

And hence, in the reference frame of the gravitational center of the solar system, the unmodified new-source theory predicts that the total travel time of the laser beam, in the case of approach, t_A is:

$$t_A = t_1 + t_2 = \left(\frac{2d}{c+v}\right)\left(1 - \frac{v/c}{2(1+v/c)}\right)$$

which is the same as its Equation #4.9:

$$t_A \approx \left(\frac{2d}{c + v\cos\theta}\right)\left(1 - \frac{v\cos\theta}{2(c + v\cos\theta)}\right)$$

as computed earlier with respect to the reference frame of the laboratory.

It follows, therefore, that the original new-source emission theory transforms correctly its computed prediction from the reference frame of the moving LLR laboratory to the reference frame of the barycenter of the solar system.

V. The Calculated Prediction of the Elastic-Impact Emission Theory:

In the reference frame of the moving LLR laboratory, as well as in the reference frame of the gravitational center of the earth-moon system, the elastic-impact emission theory gives numerical results less than the numerical results given by the classical wave theory, with regard to the reported result of the LLR experiment, in the case of approach:

$$\frac{t_{eit}}{t_{cwt}} = \frac{1 + v\cos\theta/c}{1 + 2v\cos\theta/c}$$

and it produces numerical results greater than the numerical results given by the classical wave theory, in the case of recession:

$$\frac{t_{eit}}{t_{cwt}} = \frac{1 - v\cos\theta/c}{1 - 2v\cos\theta/c}$$

where t_{eit} is the total travel time of the laser beam as predicted by the elastic-impact emission theory; and t_{cwt} is the total travel time as predicted by the classical wave theory.

In addition, if the same computations are carried out, in the reference frame of the gravitational center of the solar system, then the elastic-impact theory will give the same numerical results as the computed results in the reference frame of the gravitational center of the earth-moon system and in the reference frame of the moving LLR laboratory.

To demonstrate that is indeed the case, let's recalculate the above prediction of the elastic-impact emission theory, in the case of approach, for instance, in the reference frame of the gravitational center of the solar system.

And to dispose of trigonometric terms in the following calculations, let the moon's orbital velocity vector be ahead of the earth's orbital velocity vector along a straight line, in their orbit around the sun.

And let the light source and the observer, as observed in the reference frame of the barycenter of the solar system, approach the moon with a velocity v.

According to the elastic-impact emission theory, the laser beam travels from the earth to the moon at the velocity resultant c':

$$c' = c + v_O + v$$

where v_O is the orbital speed of the earth-moon system around the barycenter of the solar system.

And therefore, in the reference frame of the gravitational center of the solar system, the travel time of the laser light, during its journey from the earth to the moon, t_1 is:

$$t_1 = \frac{d + v_O t_1}{c + v_O + v} = \frac{d}{c + v}$$

Upon reflection from the moon, the laser beam travels back to the earth at the velocity resultant c':

$$c' = c - v_O + v$$

where v_O is the orbital speed of the earth-moon system around the barycenter of the solar system.

During the second leg of the round trip, the displacement $v_O t_1$ made by the receding moon and the displacement $v_O t_1$ made by the approaching earth, during the first leg, cancel each other out.

And accordingly, the travel time of the laser light, during its journey from the moon to the earth t_2, as computed, on the basis of this theory, in the reference frame of the gravitational center of the solar system, is:

$$t_2 = \frac{d - v_O t_2 - v t_1 - v t_2}{c - v_O + v} = \left(\frac{d}{c + v}\right)\left(\frac{1}{1 + 2v/c}\right)$$

where v is the velocity of the LLR laboratory relative to the moon.

And so, in the reference frame of the gravitational center of the solar system, the elastic-impact theory predicts that the total travel time of the laser light, in the case of approach, t_A is:

$$t_A = \frac{2d}{c + 2v}$$

which is clearly the same as this Equation #5.3:

$$t_A \approx \frac{2d}{c + 2v\cos\theta}$$

as computed earlier with respect to the reference frame of the moving laboratory.

It follows, therefore, that the elastic-impact emission theory transforms correctly its computed prediction from the reference frame of the moving LLR laboratory to the reference frame of the barycenter of the solar system.

7. Concluding Remarks:

As mentioned earlier, the reported result of the LLR experiment strongly supports the notion that the measured values of the velocity of light depends on the actual values of the velocity of the measuring observer.

And it should be obvious, therefore, that the LLR experiment provides clear experimental evidence against the official statements of Einstein's special relativity concerning the relative speed of light. But, at the same time, it's consistent with the calculations of this theory. In other words, the first-order violation of the Lorentz invariance pointed out in the LLR experimental report, is basically the longstanding inconsistency between the physical statements and the mathematical equations of special relativity with regard to relative speeds of light.

The relative speed of light, due to the motion of the observer, the motion of the light source, or both, make the computed predictions of the classical wave theory, and other similar theories as well, appear strikingly ballistic. For instance, the distance D traveled by the light beam, during the round trip, is not obtained by multiplying the measured time of the round trip by the speed of light c, as might be expected; but, instead, that distance is obtained through the multiplication of the measured time of the round trip by the relative speed of light ($c + v\cos\theta$), in the case of direct approach; and by the relative speed of light ($c - v\cos\theta$), in the case of direct recession; i.e:

$$D_{total} = t_{total}\left(c + v\cos\theta\right)$$

and:

$$D_{total} = t_{total}\left(c - v\cos\theta\right)$$

in the two cases respectively.

Consequently, the computed predictions of the classical wave theory, Larmor-Lorentz theory, Einstein's special theory, and the modified new-source emission theory, regarding the LLR experiment, are essentially the same.

It should be noted, however, that when the relative speed of light, due to the motion of the observer, the motion of the light source, or both, is combined with the ballistic speed of light, within the framework of the elastic-impact emission theory, the distance D traveled by the light beam, during the round trip, is obtained through the multiplication of the measured time of the round trip by the combined speed of light ($c + 2v\cos\theta$):

$$D_{total} = t_{total}\left(c + 2v\cos\theta\right)$$

in the case of direct approach; and by the combined speed of light ($c - 2v\cos\theta$):

$$D_{total} = t_{total}\left(c - 2v\cos\theta\right)$$

in the case of direct recession.

In the general case, therefore, the computed distance to the moon, within the framework of the elastic-impact emission theory, varies with the cosine of the angle θ between the rotational velocity vector of the earth and the position of the moon, as compared to the same computed distance, on the basis of the classical wave theory.

Let Δd denote the difference between the distance to the moon d_e, as calculated on the basis of the elastic-impact emission theory, and the distance to the moon d_w, as calculated on the basis of the classical wave theory; i.e.,

$$\Delta d = d_e - d_w$$

For θ:

$$\theta = 0^o$$

the difference between the two computed distances is about:

$$\Delta d \approx 260 \text{ m}$$

And for θ:

$$\theta = 90^o$$

the difference is:

$$\Delta d = 0$$

And generally for θ:

$$0^o \leq \theta \leq 90^o$$

the difference in the computed distance is:

$$260 \geq \Delta d \geq 0 \quad \text{m}$$

when the LLR laboratory is approaching the moon.

And for for θ:

$$90^o \leq \theta \leq 180^o$$

the difference in the computed distance is:

$$0 \geq \Delta d \geq -260 \quad \text{m}$$

when the LLR laboratory is receding from the moon.

Now, let's assume, for a moment, that the ballistic principle of additive velocities is physically true; and that light travels, in free space, at the resultant of the muzzle velocity of light c and the velocity of the light source v: What are the main anomalies that can be observed in the LLR case?

If it's true that light travels, in vacuum, at the aforementioned combined velocity, then the following anomalies, in principle, should show up, in lunar-laser- ranging experiments:

- Since the distance to the moon d_w, within the framework of the classical wave theory, and the distance to the moon d_e, as computed on the basis of the elastic-impact emission theory, are related in accordance with these two equations:

$$d_e = d_w \left(1 + \frac{v \cos \theta}{c} \right)$$

in the case of approaching earth; and:

$$d_e = d_w \left(1 - \frac{v \cos \theta}{c} \right)$$

in the case of receding earth;
if the distance d_e is the true distance to the moon, then very small, but still statistically significant, anomalous variations, in the total travel time of the laser beam, with varying distance between the moon and the LLR laboratory, must be present. For example, if the distance to the moon is shortened by 10%, the total travel time of the laser beam will be shortened by less than 9.5%. And, by contrast, if the distance becomes longer by 15%, then the total travel time of the laser beam will increase by less than 14.3%; and so on. In short, the total travel time, in all cases, would not vary precisely, in a linear manner, with distance, as expected on the assumption that the computed distance on the basis of the classical wave theory is the correct distance to the moon.

- If the distance to the moon d_w, as computed on the basis of the classical wave theory, is incorrect, and if the distance d_e of the elastic-impact theory is the correct distance to the moon, then the above anomalous variations, in the total travel time of the laser light, will tend to make the orbit of the moon appear to expand by a tiny amount with time.

- The apparent expansion in the orbit of the moon, with time, is one of two possible interpretations to explain away the above anomalous variations, in the total travel time of the laser light, within the framework of the classical wave theory.

- It's also possible, within the framework of the classical wave theory, to explain away the anomalous variations, in the total travel time of the laser beam, as being due to a very minute but continuous deceleration of the earth's rotation around its geometrical axis.

In conclusion, therefore, the LLR experiment demonstrates convincingly that the speed of light is dependent on the relative speed of the observer and the light source. But as long as the modeled distances to the moon are obtained exclusively by the method of lunar laser ranging, this experiment cannot verify, in a straightforward manner, whether or not the muzzle speed of light and the speed of the light source are independent of each other, as assumed in the classical wave theory; or the two speeds are additive, in accordance with the elastic-impact emission theory.

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