Orbital Precession without GR

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Abstract

The anomalous 43 arc second per earth century precession of the planet Mercury's orbit puzzled scientist for decades. Einstein developed a curved space model of gravitation (General Relativity) and showed that the precession of the planets could be explained very accurately by this model. This fact combined with the prediction and subsequent observation of the gravitational bending of light rays, made GR one of, if not the most, and highly accepted theories in science.
In this paper I will show that the precession of the planets can be explained with nothing more then Special Relativity, Newtonian derived formulas and simple mathematical relationships. The result is extremely accurate.

Keywords: Planetary Precession, General Relativity, Orbital Precession of mercury, Flat Space, Lagrangian operators

History

Prior to the advent of Special Relativity [1] attempts to explain the anomalous precession of the perihelion of the planet Mercury were generally discarded for various reasons. Attempts to explain the anomalous precession after the advent of SR and prior to the development of General Relativity appear absent in the literature. The present consensus among "mainstream" physicist seems to be that it simply cannot be explained with Special relativity [9]. Recently a few papers have appeared in the literature attempting to explain the anomalous precession using some modifications/combinations of SR and Newtonian gravity. Biswas [2] has shown that a Lorentz covariant modification of the classic Newtonian gravitational potential can yield the correct precession. Others [4] have made more radical assumptions such as modifying the gravitation field equations to parallel the field equations of Electromagnetism [3,8] or assuming a relativistic Lagrangian that looks surprisingly like a Schwarzschild metric [4].
The fact that the relativistic effects are very small in planetary motion would lead one to believe that some simple approximation involving Newtonian mechanics combined with SR should at least give a good approximation to orbital precession. It turns out that this can in fact be done with simple mathematics and a couple of well known and universally accepted physical formulas. No assumptions regarding the nature of the gravitational field or the geometry of space are required.

Background physics

The energy of a relativistic particle in motion is given by:

$$E = m_0c^2 + {m_0v^2}/{2} + {3m_0v^4}/{8c^2} + \ldots\ldots\ldots\ldots \qquad 1.10$$

These terms are all we will use as additional terms only become relevant at extreme velocities. In the discussion to follow, the kinetic energy of the particle will be assumed to be:

$$T = {m_0v^2}/{2} + {3m_0v^4}/{8c^2} \qquad 1.20$$

For the remainder of this discussion, unless specifically stated otherwise the symbol m will be equal to the rest mass m_0.

From Newtonian mechanics the total velocity of an object moving under the influence of a central force is given by [6]:

$$v^2(r) = (L^2/m^2P^2)[(e^2 - 1) + 2P/r] \qquad 1.30$$

Where $P = {L^2}/{GMm^2}$ and L is the total constant angular momentum and e is the eccentricity of the orbit.

Equations 1.20 and 1.30 are all the physics that is needed. The rest is mathematics.

The Math

To develop the necessary mathematical relationships needed we first explore the relationship between the Lagrangian operators on general power functions.
The Lagrangian operators are:

$$L_t = \frac{d}{dt}\frac{\partial}{\partial v} \text{ and } L_x = \frac{\partial}{\partial x}$$

In one dimensional mechanics, given the initial conditions one can determine both the position as a function of the time $x(t)$ and the velocity as a function of time $v(t)$. Similarly time can be written (by taking the inverse function) as a function of position $t(x)$, and hence velocity can be written as a function of position $v(x)$.

Consider a general function of the form $a + kv^n$, where n is a positive integer ≥ 2 and a is a constant. The function can be written as:

$f(t) = a + kv^n(t) = f(x) = a + kv^n(x)$. Where $x = x(t)$

Applying L_t to $f(t)$ we find:

$$\frac{d}{dt}\frac{\partial}{\partial v}(a + kv^n) = \frac{d}{dt}(nkv^{n-1}) = n(n-1)kv^{n-2}\frac{dv}{dt} \qquad 1.40$$

Applying L_x to $f(x)$ we find:

$$\frac{\partial}{\partial x}\left(kv^n(x)\right) = nkv^{n-1}(x)\frac{\partial v}{\partial x} = nkv^{n-2}\frac{dx}{dt}\frac{\partial v}{\partial x} = nkv^{n-2}\frac{dv}{dt} \qquad 1.50$$

In 1.50, kv^{n-1} was written as $kv^{n-2}\frac{dx}{dt}$ and $\frac{dx}{dt}\frac{\partial v}{\partial x}$ was replaced with $\frac{dv}{dt}$

From 1.40 and 1.50 it is clear that:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(kv^n(t)\right) = (n-1)\frac{\partial}{\partial x}\left(kv^n(x)\right) \qquad 1.60$$

Equation 1.60 is not an equation of motion nor a "law of physics", but a simple mathematical equality valid for any functions where the inverse of $x(t)$ exist and the functions are $v(t)$ and $v(x)$ are differentiable and $v = \frac{dx}{dt}$. In any real world physics problem both $kv^n(t)$ and $kv^n(x)$ will be kinetic energy functions and:

$kv^n(t) = kv^n(x)$. Where $x = x(t)$. 1.65

We can derive the classical Lagrangian equation of motion from 1.60 by assuming:

a. The total energy of a body moving is the sum of its kinetic energy T, and its potential energy V
b. The total energy is a constant

From 1.60 with $n = 2$:

$$\frac{d}{dt}\frac{\partial}{\partial v}\big(T(t)\big) = \frac{\partial}{\partial x}\big(T(x)\big) \qquad 1.70$$

1.70 is valid regardless of any constants added to T so we can write:

$\frac{d}{dt}\frac{\partial}{\partial v}\big(T(t)\big) = \frac{\partial}{\partial x}\big(-E_T + T(x)\big)$. Where E_T is the total energy which is assumed constant.

$-E_T + T(x)$ is just $-V(x)$, the potential energy which is assumed to be a function only of x.

Since L_t operating on $V(x)$ is zero and L_x operating on $T(t)$ is zero we can write 1.70 as:

$$\frac{d}{dt}\frac{\partial}{\partial v}\big(T(t) - V(x)\big) = \frac{\partial}{\partial x}\big(T(t) - V(x)\big) \qquad 1.80$$

1.80 is in the form of the classical Lagrangian "equation of motion".

In a one dimensional system, when the kinetic energy can be written as a function of $v(t)$ only and the potential energy is a function of x only and the total energy is constant we conclude that:

$$\frac{d}{dt}\frac{\partial}{\partial v}\big(T(t)\big) = (n-1)\frac{\partial}{\partial x}\big(T(x)\big) = (n-1)\frac{\partial}{\partial x}\big(-E_T + T(x)\big) = (n-1)\frac{\partial}{\partial x}\big(-V(x)\big) \qquad 1.82$$

We can use 1.70 to generate the gravitational force on an orbiting from 1.30.

Using:

$T(t) = \frac{1}{2}mv^2(t)$ and $T(r) = \frac{1}{2}mv^2(r)$ and substituting 1.30 for $v^2(r)$ then differentiating we get:

$$ma = -{}^{GMm}/_{r^2} \qquad 1.85$$

1.85 is the classic Newtonian formula.

We just showed that, knowing the velocity as a function of r 1.70 allows one to generate the potential energy function.

In most real problems $v(x)$ is not known, rather some potential field or potential energy function is known. Although the potential energy must be equal to $-(-E_T + T(x))$, the functional form may not resemble $v^n(t)$ and equation 1.60 could be written as:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(kv^n(t)\right) = (n-1)\frac{\partial}{\partial x}k_1\emptyset_n(x) \qquad 1.90$$

k_1 *in* 1.90 could represent any constant and in the case of gravitational potential it is the mass or reduced mass of the orbiting body.

$k_1\emptyset_n(x)$ *is* a spatially distributed energy function and the force it exerts on any classical Newtonian particle (a particle whose kinetic energy is $\frac{1}{2}mv^2$) would be determined by 1.90 with $n = 2$ regardless of the functional form of $\emptyset_n(x)$.

In particular if $k_1\emptyset_n(x) = E_T - V(x) = T(x) = {3mv^4}/{8c^2}$ a classical Newtonian particle would behave according to:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{1}{2}mv^2\right) = \frac{\partial}{\partial x}\left({3mv^4(x)}/{8c^2}\right) \qquad 2.00$$

In simple Newtonian terms if some strange particle, whose kinetic energy was equal to ${3mv^4(t)}/{8c^2}$, wondered into a potential energy field of $E_T - V(x) = {3mv^4(x)}/{8c^2}$, Newton's law of motion would have to be modified to something like:
$mA = 3F$, where $A = \frac{1}{m}\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{3mv^4(t)}{8c^2}\right)$

Any real particle whose kinetic energy is given by ${1}/{2}mv^2$ would of course obey Newton's law $(ma = F)$ regardless of the functional form of the potential energy function.

The above reasoning applies to a one dimensional system and by inference it could be applied to any multidimensional Cartesian coordinate system. In polar coordinates the square of the velocity is not a function of a single variable and conventional Lagrangian methods must be used, however because the potential field is a function only of r and 1.30 is a function only of r , the above reasoning can be used to determine an effective potential field.

We now have the necessary tools to solve the problem of orbital precession. Using 1.20 and 1.60 we get the equalities:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{mv^2(t)}{2}\right) = \frac{\partial}{\partial r}\left(\frac{mv^2(r)}{2}\right) \text{ and } \frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{3mv^4(t)}{8c^2}\right) = 3\frac{\partial}{\partial r}\left(\frac{3mv^4(r)}{8c^2}\right)$$

Combining terms we get the equality:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{mv^2(t)}{2}\right) + \frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{3mv^4(t)}{8c^2}\right) = \frac{\partial}{\partial r}\left(\frac{mv^2(r)}{2}\right) + 3\frac{\partial}{\partial r}\left(\frac{3mv^4(r)}{8c^2}\right) \qquad 2.10$$

We don't know $v(r)$ exactly but for most problems such as the orbit of the Planet Mercury we know that it very closely approximates the classical $v(r)$. We will then use 1.30 on the right side. Squaring 1.30 to get $v^4(r)$ and differentiating we get.

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{mv^2(t)}{2}\right)+\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{3mv^4(t)}{8c^2}\right)=-\frac{GMm}{r^2}-\frac{3GMmk}{2r^2}-9\frac{G^2M^2m}{r^3c^2} \qquad 2.20$$

Where: $$k=\frac{3G^2M^2m^2(e^2-1)}{L^2c^2} \qquad 2.30$$

$k\ is$ very small compared with the first term and has no effect on the orbital precession but may have some interesting consequences that we shall discuss later.

At this point 2.20 is still just a mathematical identity. To make it into a solvable physic problem we note that it can be considered to represent a simple Newtonian particle, with an "extra" acceleration term $\frac{1}{m}\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{3mv^4(t)}{8c^2}\right)$, being acted upon by a force equal to: $-\frac{GMm}{r^2}-\frac{3GMmk}{2r^2}-9\frac{G^2M^2m}{r^3c^2}$

The $v(t)$ function in both terms on the left must be the same function and hence we will seek a solution for the first term. The second term on the left is very small compared to the first and we can convert it to a force by again using 1.30 as an approximation. From 1.82 we determine an equivalent force to be:

$$(n-1)\frac{\partial}{\partial r}\left(\frac{3mv^4(r)}{8c^2}\right)=(n-1)\left(-\frac{GMmk}{2r^2}-3\frac{G^2M^2m}{r^3c^2}\right) \qquad 2.40$$

To determine n in 2.40 we note that after this substitution into 2.20 the only acceleration term left is that of the simple Newtonian particle: $\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{mv^2(t)}{2}\right)$.

This force will be acting on this simple Newtonian particle and therefore, from the previous reasoning, n must be 2.

We can then make the substitution, rearrange terms and arrive at a solvable equation:

$$\frac{d}{dt}\frac{\partial}{\partial v}\left(\frac{mv^2(t)}{2}\right)=-\frac{GMm}{r^2}-\frac{GMmk}{r^2}-6\frac{G^2M^2m}{r^3c^2} \qquad 2.50$$

We could either ignore the second term on the right as it is very small, or combine it with the first effectively skewing the Gravitational constant. Interesting, it is a function of eccentricity and depending on the orbit, it can be either positive negative or zero.

We can proceed to solve 2.50 in polar coordinates using conventional Lagrangian methods. It is obvious that $T=\frac{1}{2}mv^2$ (where $v^2=\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\theta}{dt}\right)^2$) and V can be determined from $\frac{\partial V}{\partial r}=\frac{GMm}{r^2}+\frac{GMmk}{r^2}+6\frac{G^2M^2m}{r^3c^2}$.

The conversion results in two equations, one from the r coordinate and one from the θ coordinate.

Since we want to determine orbital precession we would like to solve 2.50 for $r(\theta)$. Fortunately the solution is rather simple and straight forward and can be found in many text books and many online sites [6].

In summary the solution of the θ equation is independent of the nature of the potential field and results in conservation of angular momentum designated as L. The following conversions are then used on the radial equation:

$$\frac{d}{dt} = \frac{d\theta}{dt}\frac{d}{d\theta} = \frac{L}{mr^2}\frac{d}{d\theta}$$

And $$\frac{1}{r^2}\frac{dr}{d\theta} = -\frac{d(1/r)}{d\theta}$$

And $$u \equiv \frac{1}{r}$$

Applying these conversions yields:

$$u^2\left(\frac{d^2u}{d\theta^2} + u\right) = \frac{u^2}{l} + 6\left(\frac{h}{cl}\right)^2 u^3 \qquad 2.55$$

Where, $h = L/m$, the angular momentum per unit mass and $l = h^2/GM$. c is of course the speed of light. l is a measurable property of an orbit and is designated the semi-latus rectum.
Simplifying 2.55 yields:

$$\frac{d^2u}{d\theta^2} + \left(1 - 6\left(\frac{h}{cl}\right)^2\right)u = \frac{1}{l} \qquad 2.60$$

2.60 can be solved exactly by letting $u = A + B\cos\Delta\theta$

Doing the math gives $\Delta = \sqrt{1 - 6(h/cl)^2}$, from which we can directly determine the orbital precession.

$$\left(\frac{h}{cl}\right)^2 = \frac{GM}{c^2 l}$$

For the planet mercury $l = 55.443 \times 10^6\ km$

$$\frac{GM}{c^2} = 1.475\ km$$

Giving $\Delta = 0.999999920188298314$

The perihelion precession per revolution in radians is: $\frac{2\pi}{\Delta} - 2\pi = 5.014717513978 \times 10^{-7}\ radians$

Converting to arc seconds the precession becomes: 0.1034359736404089

Mercury orbits the sun 414.9378 times in one earth century, so the precession per century is: 42.9195 arc seconds per century in excellent agreement with observation and GR.

Conclusions

The precession observed in the orbit of Mercury can be explained with nothing more than well accepted physical formulas, mathematical identities and relatively simple mathematics. No assumptions about the nature of the gravitational field are required and space is treated as “flat”. Although the above discussion is based entirely on approximations the small relativistic components should and do make it very accurate. In Lagrangian terms setting the kinetic energy equal to the total energy minus the rest energy and minus the potential energy is all that is necessary to explain orbital precession. Using this same reasoning, a photon's kinetic energy would then be ($E/_{c^2}$), since it's rest energy is zero. It is then easy to develop a "light bending" formula. The well known Newtonian bending formula using $\frac{1}{2}mv^2$ as the kinetic energy yields a value of half that of GR. Following the same procedure for developing that formula [10], but using $mc^2$for the kinetic energy yields a value of .707 times the GR value. A complete analysis of experimental data collected to date indicates a very inaccurate value for light deflection [11], [12]. One cannot rule out the possibility that .707 times the GR value is accurate. The gravitational red shift formula was original developed by Einstein using SR equations alone [13].

One could then infer the plausibility that simply modifying Newtonian gravity to act on an objects total energy divided by the speed of light squared ($E/_{c^2}$) may be a reasonable alternative to General Relativity.

A curious term appears in the calculations that deserve some exploration. The k term is orbital path related. It skews the gravitational constant and this skewing can be positive or negative or zero depending on the eccentricity of the orbit. For the Pioneer space probe $e = 1.7372$ and k is positive resulting in an increase in the gravitational force. Prior to being subjected to the "sling shot effect" its eccentricity was most likely much smaller. This would have resulted in an effective increase in G after it reached its escape velocity.

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