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Stars and Black Holes

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Abstract: This paper discusses the concepts of stars and black holes. In this article, linear density Model plays a central role.

Keywords: Stars; Black Holes; gravitational binding energy; Schwarzschild radius; pressure; density; Temperature.

For a spherical star of uniform density, the gravitational binding energy U_B is given by the formula:

$$U_B = -\frac{3GM^2}{5R}$$

where G is the gravitational constant, M is the mass of the star, and R is its radius.

$$\frac{U_B}{Mc^2} = -\frac{3r_s}{10R}$$

$$-\frac{U_B}{0.3Mc^2} = \frac{r_s}{R}$$

where $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius of the star. Any star with Radius smaller than its **Schwarzschild radius** will form a black hole.

If $R < r_s$:

$$-U_B > 0.3Mc^2$$

The star will form a **black hole**.

The core pressure of a star of mass M and radius R is given by:

$$P_c = \frac{5GM^2}{4\pi R^4}$$

$$P_c = \frac{5}{4\pi R^3} \times \frac{GM^2}{R}$$

$$P_c = -\frac{25}{9}\frac{U_B}{V} = -\frac{25}{9}\,\rho_B$$

where ρ_B is the gravitational binding energy density of the star.

$$\frac{10U_B}{3Mc^2} = -\frac{r_s}{R}$$

Since $-\frac{9P_cV}{25} = U_B$:

$$\frac{90P_cV}{75Mc^2} = \frac{r_s}{R}$$

$$\frac{90P_c}{75\rho_E} = \frac{r_s}{R}$$

where $\rho_E = \frac{Mc^2}{V}$ is the mass energy density of the star.

$$\frac{P_c}{0.833\rho_E} = \frac{r_s}{R}$$

If $R < r_s$:

$$P_c > 0.833\ \rho_E$$

The star will form a **black hole**.

The core density of the star is given by:

$$\rho_c = \frac{3M}{\pi R^3}$$

The core temperature of the star is given by:

$$T_c = \frac{5\mu m_H}{3k_B}\frac{GM}{R}$$

where k_B is the Boltzmann constant, μ denotes mean molecular weight of the matter inside the star and m_H is the mass of hydrogen nucleus.

$$\rho_c \times T_c = \frac{4\mu m_H P_c}{k_B}$$

$$P_c = \frac{\rho_c T_c\, k_B}{4\mu m_H}$$

Since $-\frac{9P_c}{25} = \frac{U_B}{V} = \rho_B$:

$$\rho_B = -\frac{9\rho_c T_c\, k_B}{100\mu m_H}$$

The ideal gas equation $PV = Nk_BT$ does not hold good for the matter present inside a star. Because, most stars are made up of more than one kind of particle and the gas inside the star is ionized. There is no indication of these facts in the above equation. We need to change the ideal gas equation, so that it holds good for the material present inside the star. It can be shown that the required equation can be written as $PV = \frac{M}{\mu m_H} k_B T$ where μ denotes mean molecular weight of the matter inside the star, M is the mass of the star and m_H is the mass of hydrogen nucleus.

Since $\frac{k_B}{\mu m_H} = \frac{4P_c}{\rho_c T_c}$:

$$\frac{P}{P_c} = 4 \times \frac{\rho}{\rho_c} \times \frac{T}{T_c}$$

If there were no outward force holding a star up against gravity, how long would it take to collapse? This quantity is called the **"free-fall"** time scale, t_{ff}:

$$t_{ff} = \frac{1}{2}\sqrt{\frac{R^3}{GM}} = \frac{1}{2}\sqrt{\frac{R^2}{c^2}} \times \sqrt{\frac{2R}{r_s}}$$

The **Einstein time scale**, t_E, is given by:

$$t_E = \frac{Mc^2}{L}$$

The **Kelvin-Helmholtz time-scale**, t_{KH}, is given by:

$$t_{KH} = \frac{GM^2}{RL}$$

where G is the gravitational constant, M is the mass of the star, R is the radius of the star, and L is the star's luminosity.

$$\frac{t_E}{t_{KH}} = \frac{2R}{r_s}$$

$$t_{ff} = \frac{R}{2c} \times \sqrt{\frac{t_E}{t_{KH}}}$$

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