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Solar and Lunar Tides

Revised, Corrected, Simplified

solah@cox.net

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Abstract

Solar and lunar tides were part of life since there was a Sun and a Moon and Oceans. Water must have a love-hate relationship with the Sun and the Moon; water is attracted to them and repulsed by them. The exact cause of tides was always mysterious. Why is there a lunar high tide when the Moon is on the other side of Earth? Why is there a solar high tide when the Sun is on the other side of the Earth? Why is there a high tide when both the Sun and the Moon are on the other side of the Earth? Sun's gravitational pull on Earth is 180 times greater than Moon's, but observed lunar tides are twice the size of solar tides.

In some papers it is claimed that tidal forces are inversely proportional to the cube of the distance from the Sun or the Moon. (http://en.wikipedia.org/wiki/Tide) That is incorrect. Gravitational forces are inversely proportional to the square of distance. This is rather elementary. The general frustration with tide theories is expressed in a paper by Miles Mathis. (http://www.wbabin.net/mathis/mathis21.htm) The purpose of this paper is to clear up the confusion.

Tidal forces are not unique to Earth, but we are here to observe the effects. Tidal forces are generated in every binary system; each object will cause tides on its partner. For simplicity we shall concentrate on the Earth-Moon binary system. The equations are valid to all binary systems.

Tidal forces have two components caused by the same gravitational forces of the binary partner. The larger component is the differential gravitational acceleration at the center of Earth and at points of Earth nearest and farthest to the Moon. These components are pointing away from the center of Earth, causing high tides.
The smaller component is caused by the point source nature of the gravitational forces of the Moon having a radial component at the terminator of Earth. These forces are pointing towards the center of Earth causing low tides at the Terminator. This second component has a Radius of Earth / Distance to the Moon multiplier causing the confusion about the cube of distance mentioned earlier.

We are able to observe the result of tidal forces, because Earth rotates and we have oceans of water. The equations are simple. The numerical results are plausible. A summary of results is presented.

Definitions

Both Solar and Lunar orbiting systems are quite complex. Orbits are elliptic. Earth has an axial tilt. Moon's orbital plane is inclined to Earth's orbital plane. Earth is not homogeneous and is not a perfect sphere. To ease the task at hand a simplified version of the systems will be used. While the Sun-Earth binary system is different from the Earth-Moon binary system, the equations describing tidal forces are identical to all binary systems.

In this paper gravitational acceleration ***g*** and radial acceleration ***a*** will be used. This will eliminate any possible confusion about forces and acceleration.

We are familiar with equations: $F = m \times a$ and $F = m \times g$

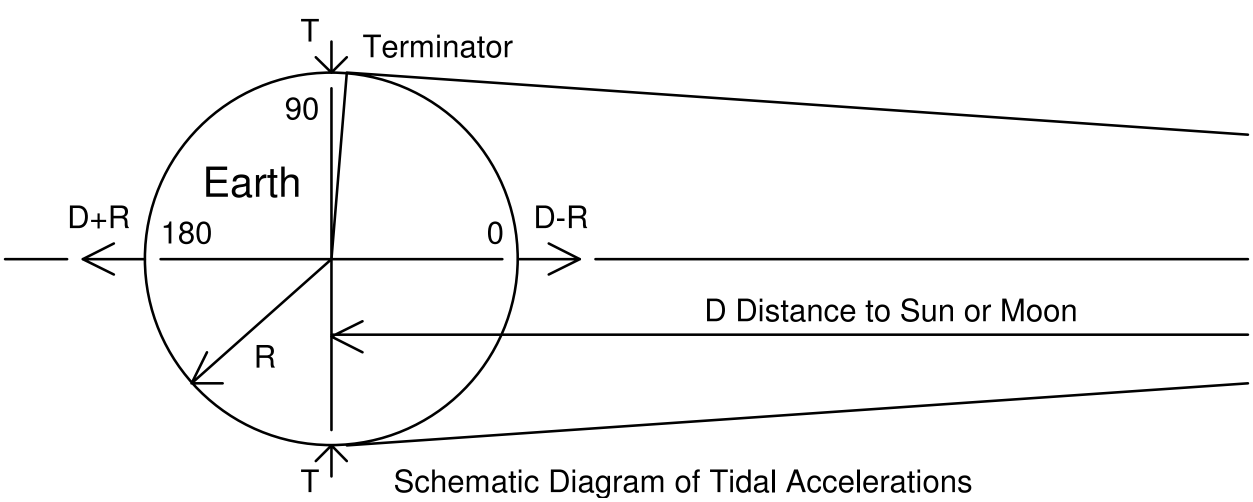
Equations we will rely on are: $a = \varpi^2 \times B$ and $g = \dfrac{G \times M}{D^2}$

B = Distance to Barycenter
D = Center to center Distance from Earth to Moon or Sun

$X = 0°$ is the point on Earth closest to the Moon or Sun. Here the gravitational acceleration of the Moon or Sun is larger than radial acceleration caused by the orbit around the barycenter. The result is high tide.

$X = 180°$ is the point on Earth farthest from the Moon or Sun. Here the gravitational acceleration of the Moon or Sun is smaller than radial acceleration caused by the orbit around the barycenter. The result is high tide

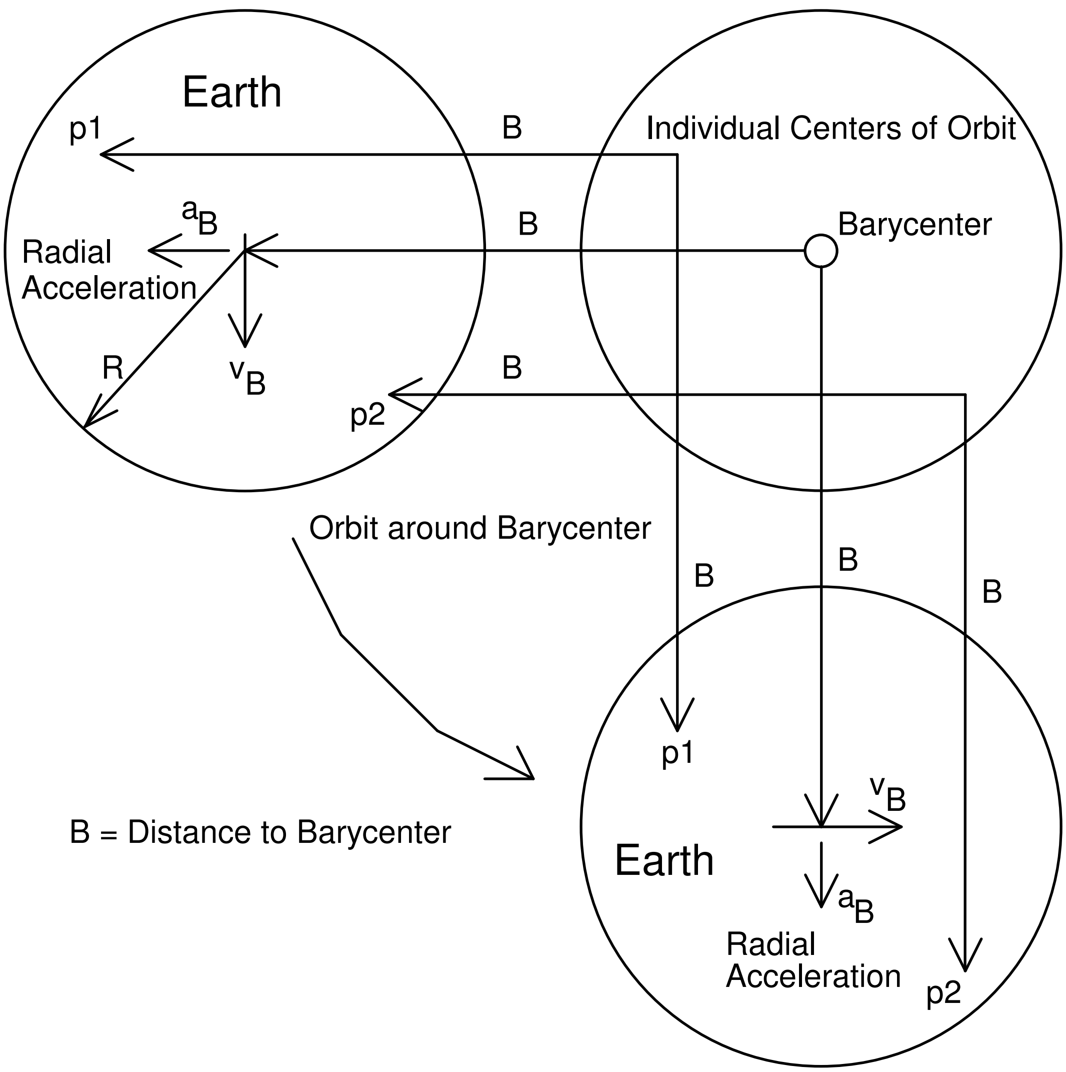
The terminator is between $X = 89°$ to $90°$ all around. Most of the gravitational acceleration from the Moon or the Sun is cancelled by radial acceleration caused by the orbit around the barycenter. The perpendicular component is an R/D fraction of the gravitational acceleration. This perpendicular component creates low tide at the terminator. Tides are differential. Low tide at the terminator causes high tide at the $X = 0°$ and $X = 180°$ points.



Schematic Diagram of Tidal Accelerations

Earth – Moon Binary

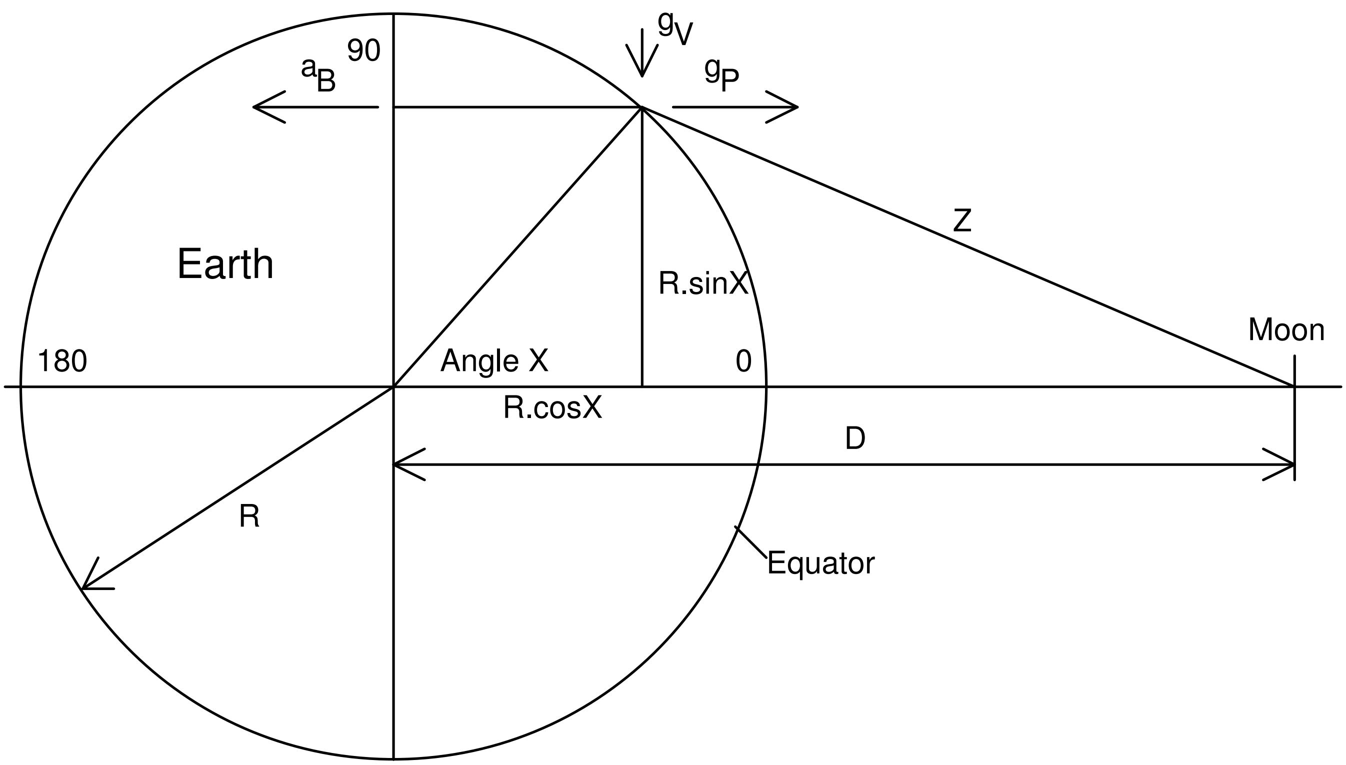
For simplicity let's talk about the Earth-Moon binary system. It will be shown that the equations can be extended to all binaries. In this model Earth is not rotating. Rotation causes only bulging at the equator and doesn't cause tides. Non-rotating means that a particular spot on Earth always looks at the same distant stars. The Moon rotates once every orbit. In the non-rotating model each point of Earth has its own center of orbit at the orbital radius B. All orbital centers are located in an Earth size space around the Barycenter. It doesn't matter if the Barycenter is far away or is within the body of Earth.
Orbital acceleration is identical for every point of Earth: $a = \varpi^2 \times B$.



Non-Rotating Earth in Orbit around Barycenter

Gravitational force exerted by the mass of the Moon is an inverse square function of distance from the center of the Moon. It is stronger closer to the Moon and weaker at far side, away from the Moon. This binary system is in a stable orbit. At the center of Earth, the gravitational acceleration caused by mass of the Moon equals the radial acceleration caused by the orbit around Barycenter. We have already seen that the radial acceleration is identical for every point in a non-rotating Earth.

At center of Earth: $$g_D = \frac{G \times M}{D^2} = a_B = \varpi^2 \cdot B$$



Gravitational Acceleration Caused by Mass of Moon at Equator of Earth

For points on the equator the gravitational acceleration is: $$g_Z = \frac{G \times M}{Z^2}$$

Each vector has a component parallel to line D and a component perpendicular to line D.
The parallel component is reduced by the radial acceleration that has the same value everywhere.

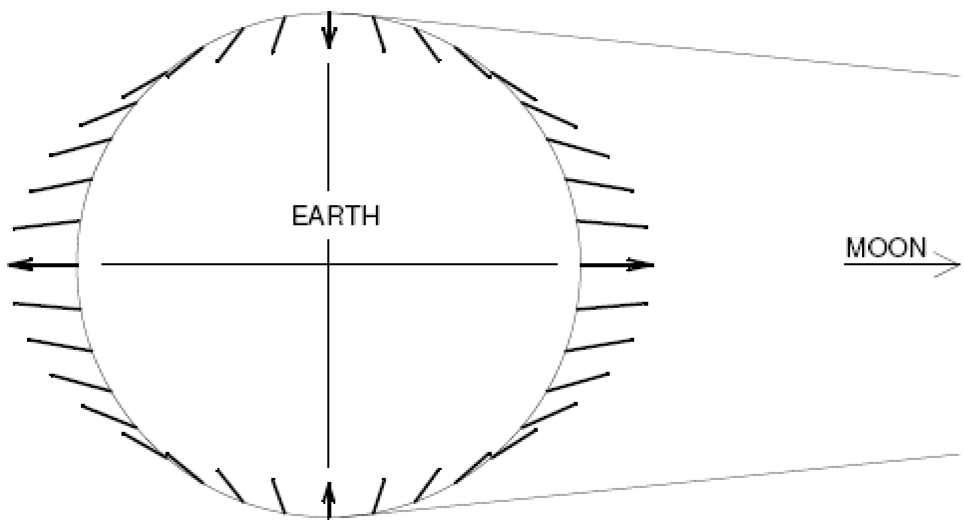
Parallel to line D reduced by $a_B = g_D = \frac{G \times M}{D^2}$: $$g_P = \frac{G \times M}{Z^2} \times \frac{D - R\cos X}{Z} - \frac{G \times M}{D^2}$$

Perpendicular to line D: $$g_V = \frac{G \times M}{Z^2} \times \frac{R \times \sin X}{Z}$$

The value of *Z* is: $$Z = \sqrt{(D - R \times \cos X)^2 + (R \times \sin X)^2}$$

The first equation is the difference of two large numbers very close in value. The second equation is a large number multiplied with $R \times \sin X/Z$; R/Z at $X = 90°$. Numerical values of the first equation at $X = 0°$ and at $X = 180°$ yield about twice the value of the second equation at $X = 90°$.

A map of the calculated gravitational vectors is shown in the Figure below. The map was created from the first table shown in Appendix A. The values in the tables were calculated to 16 digit accuracy for four cases: Lunar and Solar tides on Earth and Terrestrial and Solar Tides on the Moon.
This map is useful to understand the directions and relative sizes of the forces creating tides.
Angle X was defined on the previous page.



Gravitational Vectors Created by Moon

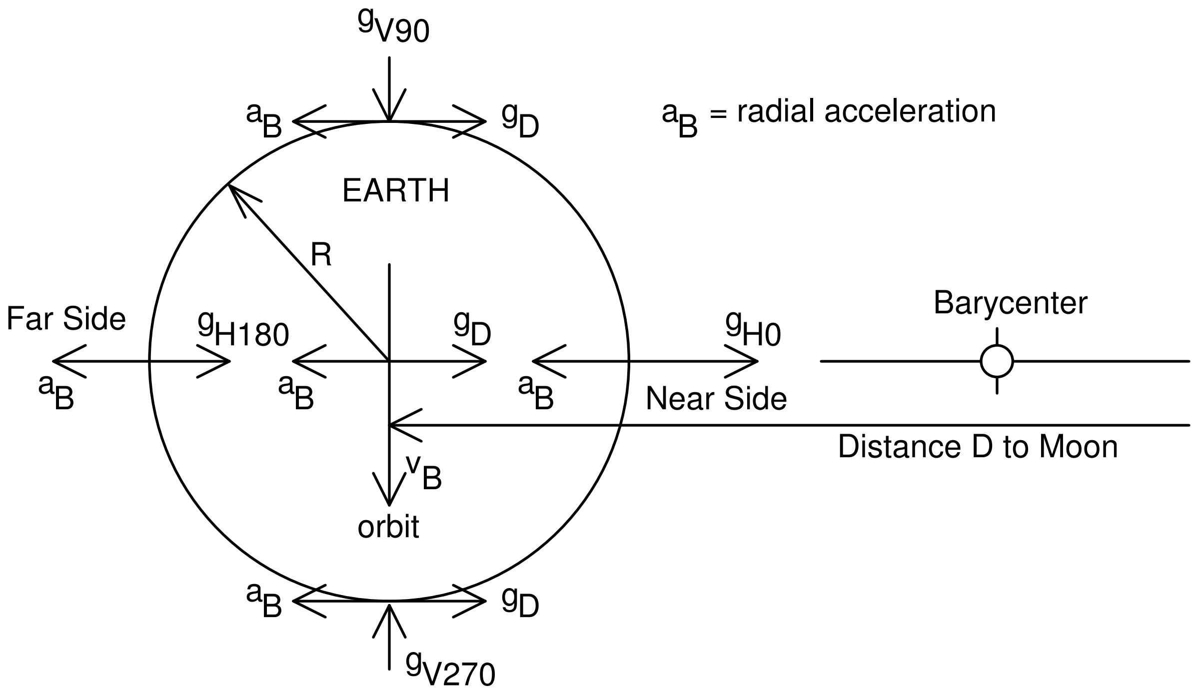
In real life there are multitudes of other contributing factors. Size and location of land masses greatly influence water flow. Accurate calculations are nice, but replacing *Z* with *D* will change the results less than one percent. In addition the general interest lies in measuring high tides and low tides. It is not an easy task to calculate what happens between the two end values. The simplified equations for these high and low tide points are as follow:

Earth – Moon binary: $D/R = 60$ and at the terminator $Z = 1.00014 \times D$

At $X = 0°$ $Z = D - R$ $$g_0 = \frac{G \times M}{D^2} \times \left(\left(\frac{D}{D-R} \right)^2 - 1 \right)$$ High Tide

At $X = 180°$ $Z = D + R$ $$g_{180} = \frac{G \times M}{D^2} \times \left(1 - \left(\frac{D}{D+R} \right)^2 \right)$$ High Tide

At $X = 90°$ $Z \approx D$ $$g_{V90} = \frac{G \times M}{D^2} \times \frac{R}{D}$$ Low Tide



Gravitational Acceleration on Non-rotating Earth In Orbit around Barycenter for D>>R

Tidal forces and their effects are differential; water supply of the oceans is given. At points closest to the Moon the tide causing acceleration is added to local gravity; at the terminator it is deducted from the local gravity. We can incorporate equation at $X = 90°$ into equations at $X = 0°$ and at $X = 180°$.

At near side:

$$a_{0+90} = \frac{G \times M}{D^2} \times \left(\left(\frac{D}{D-R} \right)^2 - 1 + \frac{R}{D} \right)$$

At far side:

$$a_{180+90} = \frac{G \times M}{D^2} \times \left(1 - \left(\frac{D}{D+R} \right)^2 + \frac{R}{D} \right)$$

Sample values – for Lunar tides on Earth:

Best estimate for G	$G = 6.673 \times 10^{-11} m^3 / kg \cdot s^2$
Mass of Moon	$M = 7.3477 \times 10^{22} kg$
Distance to center of Moon	$D = 3.8445635 \times 10^8 m$
Radius of Earth	$R = 6.378 \times 10^6 m$

Lunar Tides

Lunar tides occur at every 12 hours and 48 minutes, alternating between Moon side and far side high tides with lunar low tides in-between as reference. Water level highs and lows lag the tide causing forces by 1-3 hours. Large amounts of water must flow long distances around uneven land masses. Here are the numbers used to calculate the tide causing accelerations shown in Appendix A:

Best estimate for G	$G = 6.673\times10^{-11} m^3 / kg \cdot s^2$
Mass of Moon	$M = 7.3477\times10^{22} kg$
Distance to center of Moon	$L = 3.8445635\times10^{8} m$
Radius of Earth	$R = 6.378\times10^{6} m$
Calculated acceleration change	$a_M = 1.68\times10^{-6} m / s^2$

Solar Tides

Solar tides occur at every 12 hours at noon and midnight alternating between Sun side and far side high tides with solar low tides in-between as reference. Here are the numbers used to calculate the tide causing accelerations shown in Appendix A:

Best estimate for G	$G = 6.673\times10^{-11} m^3 / kg \cdot s^2$
Mass of Sun	$S = 1.9885\times10^{30} kg$
Distance to center of Sun	$D = 1.4959\times10^{11} m$
Radius of Earth	$R = 6.378\times10^{6} m$
Calculated acceleration change	$a_S = 0.76\times10^{-6} m / s^2$

Terrestrial Tides on the Moon

The Moon always presents the same face toward Earth; there are no terrestrial tides but a permanent bulge that we can't see from Earth. The equations used for terrestrial "tides" or bulges on the Moon are same as for lunar tides on Earth.

Best estimate for G	$G = 6.673\times10^{-11} m^3 / kg \cdot s^2$
Mass of Earth	$E = 5.9742\times10^{24} kg$
Distance to center of Moon	$L = 3.8445635\times10^{8} m$
Radius of Moon	$R_M = 1.73814\times10^{6} m$
Calculated acceleration change	$a_E = 3.67\times10^{-5} m / s^2$

Solar Tides on the Moon

One could consider solar tides on the Moon. Moon always presents the same face toward Earth therefore the Moon makes one revolution per orbit. For simplicity Moon orbit is ignored.

Best estimate for G	$G = 6.673 \times 10^{-11} m^3 / kg \cdot s^2$
Mass of Sun	$S = 1.9885 \times 10^{30} kg$
Average distance to center of Sun	$D = 1.4959 \times 10^{11} m$
Radius of Moon	$R_M = 1.73814 \times 10^6 m$
Calculated acceleration change	$a_S = 2.07 \times 10^{-7} m / s^2$

Size is Relative

The equations are identical for solar and lunar tidal accelerations. They don't contain the mass of Earth or the distance to a barycenter. The mass of Earth still plays a role when we compare these small tidal acceleration values to the gravitational acceleration of Earth:

$$g_E = 9.81 m / s^2$$

To calculate actual water level differences of tides is complex; it requires integration of gravitational differences to the depth of oceans and it is beyond the scope of this paper. In addition tides are greatly influenced by the size and location of land masses, the shape of ocean floors. Water is drawn away from all points of the terminator, be it at the equator or at the poles.
Solar and lunar tidal forces some times coincide and augment each other. Other times they are 6 hours out of phase and result in very small high-low tide differences.

Equatorial Bulge

Rotation of Earth causes radial acceleration and a bulge at the equator. In a simplified calculation:

Velocity at equator	$v_R = \dfrac{2 \times \pi \times R}{86400} = 463.6 m / s^2$
Radial acceleration at the equator	$a_R = \dfrac{v_R^2}{R} = 3.369786 \times 10^{-2} m / s^2$

Acceleration caused by the rotation of Earth is about 20,000 times the lunar tidal acceleration and it creates an equatorial bulge of 22,000 m (radius). The lunar tidal acceleration numbers presented in this paper could cause lunar tides of 1.1 meter are in agreement with measured numbers.

Summary

Tidal forces have two components caused by the same gravitational forces of the binary partner. The larger component is the differential gravitational acceleration at the center of Earth and at points of Earth nearest and farthest to the Moon. These components are pointing away from the center of Earth, causing high tides.
The smaller component is caused by the point source nature of the gravitational forces of the Moon having a radial component at the terminator of Earth. These forces are pointing towards the center of Earth causing low tides at the Terminator. Low tide at the Terminator is same as high tides at the near and far points.

We are able to observe the result of tidal forces, because Earth rotates and we have oceans of water. The equations are the same for all binaries.

R is the radius of the local body.
M is the mass of the tide causing body.
D is the distance between the centers of the two bodies.

Simplified Equations for High Tides with Low Tide as Base Line:

$$a_{D-R} = \frac{G \times M}{(D-R)^2} - \frac{G \times M}{D^2} + \frac{G \times M}{D^2} \times \frac{R}{D} \quad \Rightarrow \quad a_{D-R} = \frac{G \times M}{D^2}\left(\left(\frac{D}{D-R}\right)^2 - 1 + \frac{R}{D}\right)$$

$$a_{D+R} = \frac{G \times M}{D^2} - \frac{G \times M}{(D+R)^2} + \frac{G \times M}{D^2} \times \frac{R}{D} \quad \Rightarrow \quad a_{D+R} = \frac{G \times M}{D^2} \times \left(1 - \left(\frac{D}{D+R}\right)^2 + \frac{R}{D}\right)$$

Individual terms used above:

Gravitational acceleration at center of Earth: $g_D = \frac{G \times M}{D^2} = a_B = \varpi^2 \cdot B$

Gravitational acceleration at Moon side of Earth $g_{D-R} = \frac{G \times M}{(D-R)^2}$

Gravitational acceleration at far side of Earth $g_{D+R} = \frac{G \times M}{(D+R)^2}$

Gravitational acceleration at Terminator of Earth $g_V = \frac{G \times M}{D^2} \times \frac{R}{D}$ radial component

Numerical Results

Name of Item	Acceleration	Bulge or Tide
Earth Rotation	$a_R = 3.37\times10^{-2}\, m/s^2$	$22{,}000m$ radial, measured
Lunar tidal acceleration on Earth:	$a_M = 1.68\times10^{-6}\, m/s^2$	$\approx 1.1m$ expected
Solar tidal acceleration on Earth:	$a_S = 0.76\times10^{-6}\, m/s^2$	$\approx 0.5m$ expected
Terrestrial tidal acceleration on Moon:	$a_E = 3.67\times10^{-5}\, m/s^2$	$\approx 150m$ predicted
Solar tidal acceleration on Moon:	$a_S = 2.07\times10^{-7}\, m/s^2$	$\approx 0.85m$ predicted

Compare Tides on Earth and Moon

Gravity at Moon's equator is: $g_M = 1.622 m/s^2$

Gravity at Earth's equator is: $g_E = 9.81 m/s^2$

Let's compare lunar tidal forces on Earth with terrestrial tidal forces on Moon. Gravity at Earth's equator is about 6.05 times the gravity at Moon's equator. The size of terrestrial bulge and solar tides on the Moon can be estimated:

Terrestrial bulge on the Moon: $$Tide_E = 1.1\times\frac{g_E}{g_M}\times\frac{a_{L-R}}{a_{M-RM}} = 1.1\times6.05\times\frac{3.65\times10^{-5}}{1.62\times10^{-6}} = 150m$$

Solar tides on the Moon: $$Tide_S = 1.1\times\frac{g_E}{g_M}\times\frac{a_{S-R}}{a_{M-RM}} = 1.1\times6.05\times\frac{2.07\times10^{-7}}{1.62\times10^{-6}} = 0.85m$$

Terrestrial tidal acceleration on the Moon is 136 times larger than lunar tidal acceleration on Earth relative to the local gravity. Can you imagine 150 meter tides? There aren't any oceans on the Moon but this large influence could be the cause of Moon's synchronous rotation of one revolution per orbit. Gravitation stabilized spacecraft works the very same way.

Appendix A

Tidal acceleration values calculated for every 10 degrees around the “equator” of Earth

Lunar acceleration $m/s^2 \times 10^6$

Angle X	E-M	Y
0	1.12743	0.00000
10	1.10944	0.10029
20	1.05612	0.19708
30	0.96946	0.28703
40	0.85261	0.36712
50	0.70981	0.43477
60	0.54605	0.48797
70	0.36691	0.52528
80	0.17834	0.54587
89.17	0.00008	0.54980
90	-0.01367	0.54951
100	-0.20321	0.53652
110	-0.38467	0.50770
120	-0.55289	0.46429
130	-0.70327	0.40784
140	-0.83182	0.34017
150	-0.93527	0.26333
160	-1.01100	0.17949
170	-1.05719	0.09093
180	-1.07271	0.00000

Solar acceleration $m/s^2 \times 10^6$

Angle X	E-M	Y
0	0.50569	0.00000
10	0.49801	0.04391
20	0.47519	0.08648
30	0.43794	0.12643
40	0.38737	0.16253
50	0.32504	0.19369
60	0.25282	0.21897
70	0.17293	0.23759
80	0.08779	0.24899
89.59	0.00013	0.25283
90	-0.00002	0.25283
100	-0.08782	0.24898
110	-0.17296	0.23757
120	-0.25283	0.21894
130	-0.32503	0.19366
140	-0.38734	0.16250
150	-0.43789	0.12640
160	-0.47514	0.08646
170	-0.49795	0.04390
180	-0.50563	0.00000

Tidal acceleration values calculated for every 10 degrees around the “equator” of Moon

Terrestrial acceleration $m/s^2 \times 10^5$

Angle X	M-E	Y
0	2.45504	0.00000
10	2.41725	0.21457
20	2.30505	0.42235
30	2.12210	0.61681
40	1.87420	0.79188
50	1.56933	0.94213
60	1.21709	1.06303
70	0.82858	1.15097
80	0.41588	1.20348
89.48	0.00024	1.21921
90	-0.00827	1.21917
100	-0.43092	1.19782
110	-0.83931	1.14034
120	-1.22123	1.04871
130	-1.56537	0.92585
140	-1.86163	0.77560
150	-2.10143	0.60249
160	-2.27778	0.41172
170	-2.38567	0.20891
180	-2.42197	0.00000

Solar acceleration $m/s^2 \times 10^7$

Angle X	M-S	Y
0	1.37805	0.00000
10	1.35712	0.11965
20	1.29494	0.23566
30	1.19343	0.34452
40	1.05563	0.44290
50	0.88578	0.52782
60	0.68901	0.59672
70	0.47130	0.64747
80	0.23928	0.67855
89.59	0.00039	0.68901
90	-0.00001	0.68901
100	-0.23931	0.67854
110	-0.47132	0.64745
120	-0.68902	0.59670
130	-0.88578	0.52780
140	-1.05561	0.44288
150	-1.19340	0.34450
160	-1.29490	0.23565
170	-1.35707	0.11964
180	-1.37800	0.00000