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Irregularities in observing binary stars if the velocity of light depends on the velocity of its source

Filip Dambi
filipdambi1@gmail.com

Abstract: If the velocity of light depends on the velocity of its source, a binary star B of a binary system AB displays specific irregularities, which astronomers did not perceive in their observations. We consider that the velocity of light depends on the velocity of its source with the purpose of evaluating these irregularities. We calculate the irregularities according to the observation distance and then apply the result to a binary star B with the same geometry and circular velocity as Earth. The irregularities do not exclude the possibility that the velocity of light depends on the velocity of its source if the irregularities appear but cannot be visually detected. Astronomers have to present specific examples of binary stars in which the irregularities can be visually detected, and yet we do not see them.

1 Introduction

This paper studies the observation of a binary star B that belongs to a binary system AB. If the velocity of light depends on the velocity of its source, the observation of a binary stars B displays specific irregularities [1-2]. The binary star B revolves on a circular orbit with the binary star A at its center. This geometry is similar to Earth, as binary star B, which revolves on a circular orbit with the Sun, as binary star A, at its center.

We can call the observation line a line that goes through the center of star B's orbit. There is an infinite number of observation lines that depend on the inclination angle between the observation lines and the plane of star B's orbit. There is an infinite number of observation lines with the same inclination angle that are located on a double cone geometry with the common point of the cones at the center of star B's orbit, one cone on one side and another cone on another side of the orbit's plane. There is an infinite number of such double cone geometries, which depend on the inclination angle.

An observer at a point on one of these observation lines sees the star on an elliptical orbit except for two particular cases. When an observation line belongs to the orbit's plane, the observer sees star B's orbit as a line, as it is presented in section 2. For the single one observation line that is perpendicular to the orbit's plane, the observer sees star B's orbit as a circle, as it is presented in section 3. We approach the general case in section 4. Each of these sections exemplifies the magnitude of the irregularities for the Earth as binary star B.

2 Observation of a star B from an observation line within the plane of its orbit

2.1 Star B on its orbit at 0, π/2, π, and 3π/2 radians from the initial position

Figure 1 depicts star B revolving with a speed v, counterclockwise, on a circle with a radius R and center at O_s. The orbit of the star and point O are in the paper plane. Line OO_s is perpendicular to the line AC, and the observation distance from O to O_s is d. Point A is at the initial instance of the star location at 0 radians. Point A's position on the orbit depends on the observation line. An observer at point O sees the star moving back and force on a line.

The constant speed c is the speed of light in a vacuum from a source at rest.

At point A, there is a ray of light sent towards point A' with velocity c that is dragged with velocity v from point A in the direction $A'O$, and as a result, the ray travels toward point O. There is such a ray, as at point A, at points B, C, and D, and at any location of the star on its orbit. The directions of the rays from points A, B, C, and D are toward points A', B', C', and D', respectively. This pattern is typical when observing a star. The shape of the curve that includes points A', B', C', D' and again A' is imaginary and resembles an ellipse.

This section derives the times $t_{sa=0}$, $t_{sa=\pi/2}$, $t_{sa=\pi}$, and $t_{sa=3\pi/2}$ in which the light from points A, B, C, and D, respectively, travels to point O.

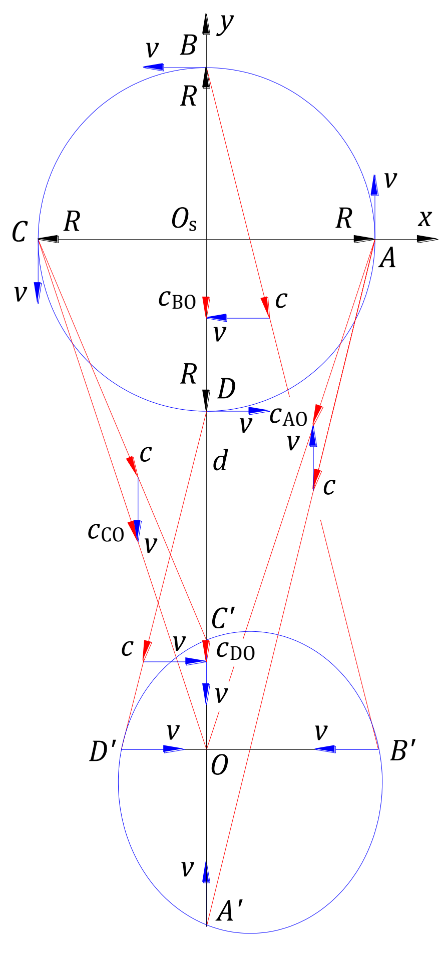

Figure 1. Star B on its orbit at 0, π/2, π, and 3π/2 radians.

The light from point A travels in the direction AA' with the velocity c. The velocity v from point A drags the velocity c in the direction $A'O$, and the light travels along the distance AO with the velocity c_{AO}, in time $t_{sa=0}$ calculated as follows:

$$AA'^2 = O_sA'^2 + O_sA^2 = (O_sO + OA')^2 + O_sA^2 \quad \Rightarrow$$

$$c^2 t_{sa=0}^2 = (d + vt_{sa=0})^2 + R^2 = d^2 + 2dvt_{sa=0} + v^2 t_{sa=0}^2 + R^2 \quad \Rightarrow$$

$(c^2 - v^2)t_{sa=0}^2 - 2dvt_{sa=0} - (d^2 + R^2) = 0$, with the convenient solution

$$t_{sa=0} = \frac{\sqrt{d^2 v^2 + (d^2 + R^2)(c^2 - v^2)} + dv}{c^2 - v^2}.$$

The light from point B travels in the direction BB' with the velocity c. The velocity v from point B drags the velocity c in the direction $B'O$, and the light travels along the distance BO with the velocity c_{BO}, in time $t_{sa=\pi/2}$ calculated as follows:

$$BB'^2 = OB^2 + OB'^2 = (0_s O + 0_s B)^2 + OB'^2 \quad \Rightarrow \quad c^2 t_{sa=\pi/2}^2 = (d + R)^2 + v^2 t_{sa=\pi/2}^2 \quad \Rightarrow$$

$(c^2 - v^2)t_{sa=\pi/2}^2 - (d + R)^2 = 0$, with the convenient solution

$$t_{sa=\pi/2} = \sqrt{\frac{(d + R)^2}{c^2 - v^2}} = \frac{d + R}{\sqrt{c^2 - v^2}}.$$

The light from point C travels in the direction CC' with the velocity c. The velocity v from point C drags the velocity c in the direction $C'O$, and the light travels along the distance CO with the velocity c_{CO}, in time $t_{sa=\pi}$ calculated as follows:

$$CC'^2 = 0_s C'^2 + 0_s C^2 = (0_s O - OC')^2 + 0_s C^2 \quad \Rightarrow$$

$$c^2 t_{sa=\pi}^2 = (d - vt_{sa=\pi})^2 + R^2 = d^2 - 2dvt_{sa=\pi} + v^2 t_{sa=\pi}^2 + R^2 \quad \Rightarrow$$

$(c^2 - v^2)t_{sa=\pi}^2 + 2dvt_{sa=\pi} - (d^2 + R^2) = 0$, with the convenient solution

$$t_{sa=\pi} = \frac{\sqrt{d^2 v^2 + (d^2 + R^2)(c^2 - v^2)} - dv}{c^2 - v^2}.$$

The light from point D travels in the direction DD' with the velocity c. The velocity v from point D drags the velocity c in the direction $D'O$, and the light travels along the distance DO with the velocity c_{DO}, in time $t_{sa=3\pi/2}$ calculated as follows:

$$DD'^2 = OD^2 + OD'^2 = (0_s O - 0_s D)^2 + OD'^2 \;\Rightarrow\; c^2 t_{sa=3\pi/2}^2 = (d - R)^2 + v^2 t_{sa=3\pi/2}^2 \;\Rightarrow$$

$(c^2 - v^2)t_{sa=3\pi/2}^2 - (d - R)^2 = 0$, with the convenient solution

$$t_{sa=3\pi/2} = \sqrt{\frac{(d - R)^2}{c^2 - v^2}} = \frac{d - R}{\sqrt{c^2 - v^2}}.$$

The observation angle of star AOC is $e = 2\arctan(R/d)$.

2.2 Star B on its orbit at a radians from the initial position

2.2.1 Derivation of the time in which the light travels from the star B to observer

Figure 2 illustrates the observation of star B located at point E. The radius R at point E makes an angle a with the coordinate $O_s x$.

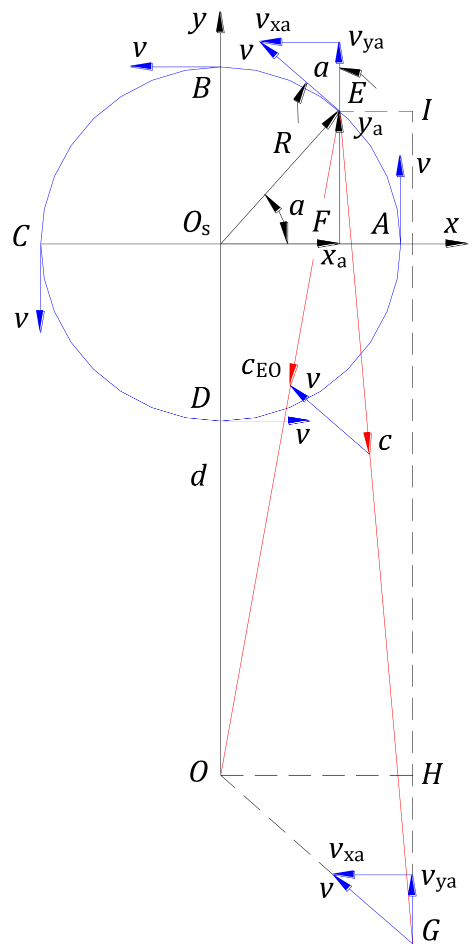

Figure 2. Star B on its orbit at a radians from the initial position.

The light from point E travels in the direction EG with the velocity c. The velocity v from point E drags the velocity c in the direction GO, and the light travels along the distance EO with the velocity c_{EO}, in time t_{sa} calculated as follows:

From figure 2, $x_a = R\cos a$, $y_a = R\sin a$, $v_{xa} = v\sin a$, and $v_{ya} = v\cos a$.

$$EG^2 = IG^2 + IE^2 = (O_sO + EF + GH)^2 + (OH - O_sF)^2 \quad \Rightarrow$$

$$c^2 t_{sa}^2 = \left(d + y_a + v_{ya} t_{sa}\right)^2 + (v_{xa} t_{sa} - x_a)^2$$
$$= (d + R\sin a + v t_{sa}\cos a)^2 + (v t_{sa}\sin a - R\cos a)^2 \quad \Rightarrow$$

$$c^2t_{sa}^2 = d^2 + R^2\sin^2 a + v^2t_{sa}^2\cos^2 a + 2dR\sin a + 2dvt_{sa}\cos a + 2Rvt_{sa}\sin a\cos a + v^2t_{sa}^2\sin^2 a - 2Rvt_{sa}\sin a\cos a + R^2\cos^2 a \quad \Rightarrow$$

$(c^2 - v^2)t_{sa}^2 - 2dvt_{sa}\cos a - (d^2 + 2dR\sin a + R^2) = 0$, with the convenient solution

$$t_{sa} = \frac{\sqrt{d^2v^2\cos^2 a + (d^2 + 2dR\sin a + R^2)(c^2 - v^2)} + dv\cos a}{c^2 - v^2}.$$

For angle a equal to 0, $\pi/2$, π and $3\pi/2$, the times $t_{sa=0}$, $t_{sa=\pi/2}$, $t_{sa=\pi}$ and $t_{sa=3\pi/2}$, respectively, have the same formula as in section 2.1.

2.2.2 Derivation of the distance $d = d_1$

In figure 2, the observation of the star depends on the time in which the light from the points on the star's orbit arrives at point O. The light from points B and D travels to point O with the same speed, $c_{BO} = c_{DO}$. Thus, the light from these points are at a constant distance between each other all the time, and cannot catch up to one another. The light from point C catches the light from point A. The distance $d = d_1$ at which the light from point C, for $a = \pi$, catches the light from point A, for $a = 0$, gives a significant magnitude of the irregularities. We calculate the distance d_1 for each example of this study.

The time t_{sa} is the time in which the light from the star at point E travels to point O. The star completes its full orbit in time $T = 2\pi R/v$. The star travels an angle of one radian on its orbit in time $t = T/2\pi = R/v$. The time in which the star travels from A to E is $t_{ra} = at = aR/v$. The time $t_a = t_{ra} + t_{sa}$.

The time $t_{ra=\pi} = \pi t = \pi R/v$ is the time in which the star travels along the orbit from point A to point C. The distance d_1 at which the light from point C for $a = \pi$ catches the light from point A for $a = 0$ is given by the equation $t_{sa=0} = t_{a=\pi} \;\Rightarrow\; t_{sa=0} = t_{ra=\pi} + t_{sa=\pi} \;\Rightarrow\; t_{sa=0} - t_{sa=\pi} = t_{ra=\pi}$. The equation $t_{sa=0} - t_{sa=\pi} = t_{ra=\pi}$, for time t_{sa} from section 2.2.1, and for $d = d_1$ becomes

$$\frac{\sqrt{d_1^2v^2 + (d_1^2 + R^2)(c^2 - v^2)} + d_1v}{c^2 - v^2} - \frac{\sqrt{d_1^2v^2 + (d_1^2 + R^2)(c^2 - v^2)} - d_1v}{c^2 - v^2} = \frac{\pi R}{v} \quad \Rightarrow$$

$$\frac{2d_1v}{c^2 - v^2} = \frac{\pi R}{v} \quad \Rightarrow \quad d_1 = \frac{\pi R(c^2 - v^2)}{2v^2}.$$

The distance d_{n+1} at which the light from point C, for $a = (2n + 1)\pi$ with $n = 0, 1, 2, 3, \ldots$, catches the light from point A, for $a = 0$, is

$$d_{n+1} = \frac{(2n + 1)\pi R(c^2 - v^2)}{2v^2} \quad \text{for} \quad n = 0, 1, 2, 3, \ldots.$$

2.2.3 Star is at the distance $\mathbf{d = d_1/2}$

The graphical representations are for the Earth as a star with the circular orbit of radius $R = 1.50573E + 11\ m$, period $T = 365\ days = 3.15360E + 07\ s$, and circular velocity $v = 3.0E + 4\ m/s$. The speed of light is $c = 3.0E + 8\ m/s$. The calculated distance d_1 is $d_1 =$

$2.3652E+19\ m$. The observer, located at the distance $d = d_1/2$ from the star, sees the light for different angles a with the corresponding times given in table 1 and illustrated in figure 3.

Table 1. Times when the star is at the distance $d = d_1/2$.

a [°]	0	30	45	60	90
t_a [year]	1250.125000	1250.191593	1250.213396	1250.229176	1250.250010
120	135	150	180	210	225
1250.270842	1250.286620	1250.308420	1250.375000	1250.475071	1250.536597
240	270	300	315	330	360
1250.604148	1250.749978	1250.895815	1250.963374	1251.024910	1251.125000

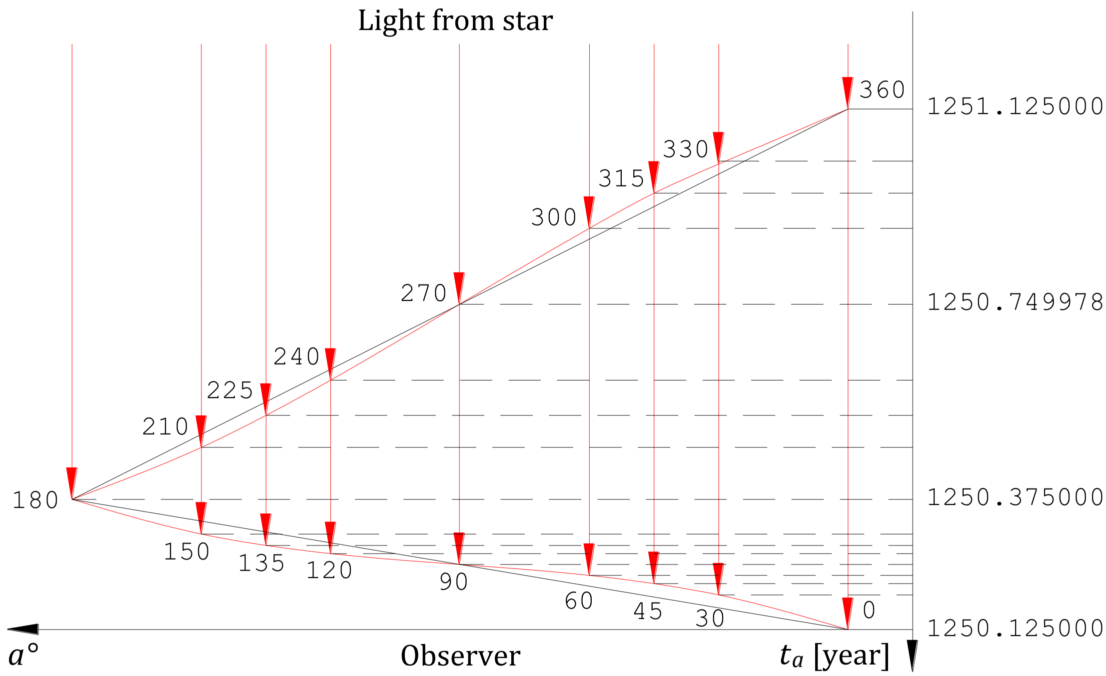

Figure 3. Star is at the distance $d = d_1/2$.

The light arrives at the observer according to the skewed sinusoidal functions when the star travels from 0^o to 180^o and 180^o to 360^o.

The observation angle of the star is $e = 2\arctan(R/d) = 2.54648E-08\ rad$.

The observer sees irregularities along with the directions AC and CA and between the two opposite directions that increase with the increasing of the observation distance d, which can be called time or circular velocity irregularities.

Table 2 gives the time differences observed in which the star travels in increments of 30° to fulfill a complete circular orbit.

Table 2. Time differences at 30° increments at the distance $d = d_1/2$.

Angles	30-0	60-30	90-60	120-90	150-120	180-150	180-0
Time [day]	24.3064	13.7177	7.6044	7.6040	13.7158	24.3017	91.2500

Angles	210-180	240-210	270-240	300-270	330-300	360-330	360-180
Time [day]	36.5258	47.1133	53.2278	53.2305	47.1198	36.5327	273.7500

The observed circular velocity is highest when the star is at 90^o and lowest when the star is at 270^o.

2.2.4 Star is at the distance $\mathbf{d} = \mathbf{d_1}$

The observer, located at the distance $d = d_1$ from the star, sees the light for different angles a with the corresponding times given in table 3 and illustrated in figure 4.

Table 3. Times when the star is at the distance $d = d_1$.

a [°]	0	30	45	60	90
t_a [year]	2500.250000	2500.299845	2500.301782	2500.291671	2500.250003
120	135	150	180	210	225
2500.208338	2500.198228	2500.200165	2500.250000	2500.366816	2500.448206
240	270	300	315	330	360
2500.541644	2500.749972	2500.958310	2501.051759	2501.133162	2501.250000

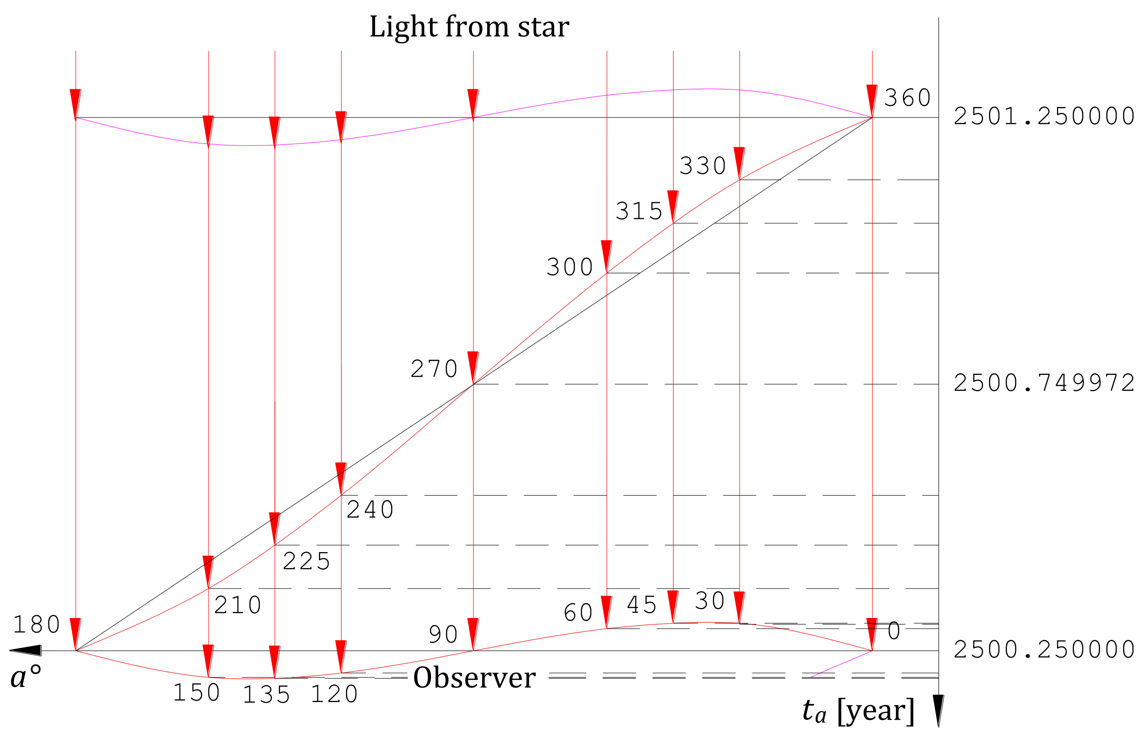

Figure 4. Star is at the distance $d = d_1$.

The observation angle of the star is $e = 2\arctan(R/d) = 1.27324E - 08\, rad$.

At the distance d_1, the observer sees multiple stars. The light from points B and C catche the light from point A simultaneously. The light from the star at $a = 135°$ arrives first at the observer. From this point, the observer starts to see two stars: one moving left and the other right. We can specifically call these time irregularities as multiple stars irregularities.

Figures 3 and 4 indicates that there is a distance between $d_1/2$ and d_1 at which the observer at point O starts to see multiple stars irregularities. This distance is $d_k = kd_1 = (2/3)d_1$. The factor $k = 2/3$ is a constant for any star. The data in table 4 are for $d = 0.6667d_1 = 1.57688\text{E} + 19\ m$. The angles that indicate the observation of the multiple stars are emphasized in gray.

Table 4. Times when the star is at the distance $d = 0.6667d_1$.

a [°]	0	30	45	60	90
t_a [year]	1666.916675	1666.977684	1666.992864	1667.000012	1667.000008
120	135	150	180	210	225
1667.000003	1667.007150	1667.022328	1667.083325	1667.188979	1667.257128
240	270	300	315	330	360
1667.333309	1667.499976	1667.666651	1667.742842	1667.811001	1667.916675

2.2.5 Star is at the distance $\mathbf{d = 3d_1}$

The observer, located at the distance $d = 3d_1$ from the star, sees the light for different angles a with the corresponding times given in table 5 and illustrated in figure 5.

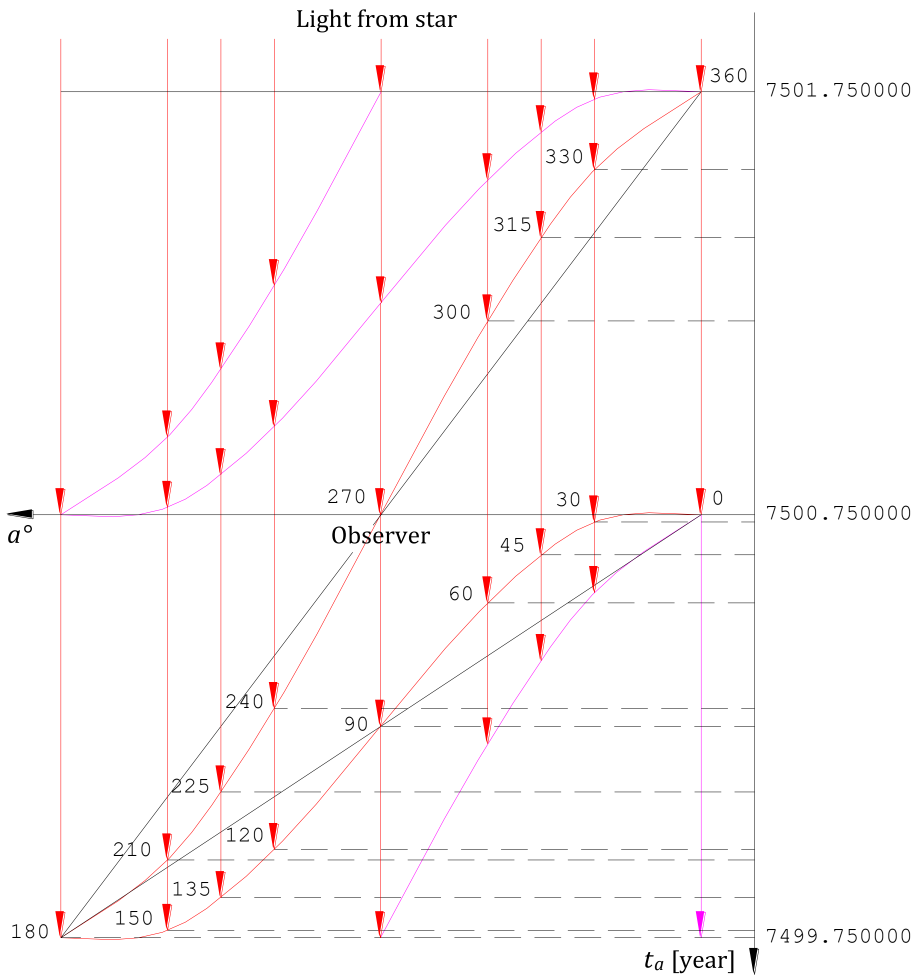

Figure 5. Star is at the distance $d = 3d_1$.

Table 5. Times when the star is at the distance $d = 3d_1$.

a [°]	0	30	45	60	90
t_a [year]	7500.750000	7500.732851	7500.655323	7500.541652	7500.249978
120	135	150	180	210	225
7499.958319	7499.844662	7499.767146	7499.750000	7499.933797	7500.094640
240	270	300	315	330	360
7500.291625	7500.749947	7501.208291	7501.405300	7501.566168	7501.750000

The observation angle of the star is $e = 2\arctan(R/d) = 4.24413E - 09\ rad$.

3 Observation of a star B from the observation line perpendicular to the star's orbit

In figure 6, the orbit of the star and the plane of the triangle OGH are parallel to each other and perpendicular to the paper plane. The triangle OGH belong to the plane of the circle that contains points A', G, B', C', and D'. The planes perpendicular to the paper plane are in blue. The lines AC and OH also belong to the paper plane and the distance O_sO is perpendicular to both perpendicular planes to the paper plane. All other lines in the paper plane are in green. The lines behind the paper are dashed lines.

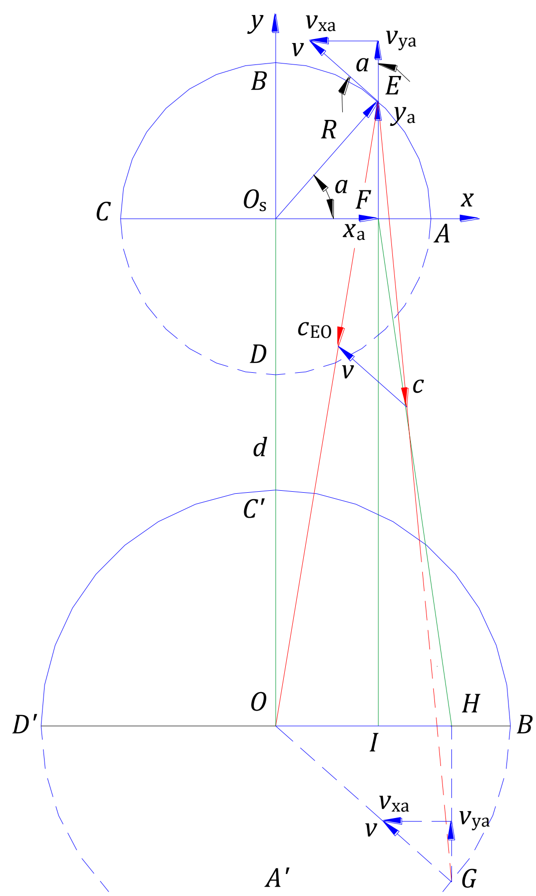

Figure 6. Observation of a star B from the observation line perpendicular to the star's orbit.

The light from point E travels in the direction EG with the velocity c. The projection of the distance EG on the paper plane is the distance FH. The velocity v from point E drags the velocity c in the direction GO, and the light travels along the distance EO with the velocity c_{EO}, in time t_{sa} calculated as follows:

From figure 6, $x_a = R\cos a$, $y_a = R\sin a$, $v_{xa} = v\sin a$, and $v_{ya} = v\cos a$.

$$EG^2 = FH^2 + (EF + GH)^2 = FI^2 + (\mathrm{OH} - \mathrm{OI})^2 + (EF + GH)^2 \quad \Rightarrow$$

$$c^2 t_{sa}^2 = d^2 + (v_{xa} t_{sa} - x_a)^2 + \left(y_a + v_{ya} t_{sa}\right)^2$$
$$= d^2 + (v t_{sa} \sin a - R\cos a)^2 + (R\sin a + v t_{sa}\cos a)^2 \quad \Rightarrow$$

$$c^2 t_{sa}^2 = d^2 + v^2 t_{sa}^2 \sin^2 a - 2Rv t_{sa} \sin a\cos a + R^2\cos^2 a + R^2 \sin^2 a$$
$$+ 2Rv t_{sa}\sin a\cos a + v^2 t_{sa}^2 \cos^2 a \quad \Rightarrow$$

$(c^2 - v^2)t_{sa}^2 - (d^2 + R^2) = 0$, with the convenient solution

$$t_{sa} = \sqrt{\frac{d^2 + R^2}{c^2 - v^2}}.$$

The time t_{sa} is independent of angle a for this geometry. Thus, the direction of the rays from the star's orbit that reach the point O is toward an imaginary circle with the center at point O. In figure 6, the imaginary circle is partially shown.

Observer O sees the star as one star without time irregularities for any distance d.

The ray of light from points A, B, C, and D to points A', B', C', and D' that are on the imaginary circle containing point G, have the same arrangement as in figure 1 but in space instead of in a plane. The distances AA', BB', CC' and DD' are equal.

4 Observation of a star B from an observation line that makes an angle *b* to the star's orbit

4.1 Derivation of the time in which the light travels from the star B to observer

Figure 7 depicts an observer at point O that is in the paper plane. He observed a star with its orbit making an angle $b \in [0 - 90^o]$ with the paper plane. On the backside of the orbit's plane, the circular velocity looks clockwise and on the front side counterclockwise. The star's orbit with its coordinates and all vectors that belong to it are drawn in blue. The paper plane contains the projections of the star orbit with its coordinates and the vectors that belong to it. The lines and vectors in the paper plane are drawn in green. The line AC also belongs to the paper plane. The coordinates $O_s y$ of the star's orbit make the angle b with the coordinate $O_s y'$ of the projection of the star's orbit on the paper plan. The line OG, drawn in blue, belongs to a plane parallel to the plane of the star's orbit.

In detail 1, the plane of the triangle v'_a, v'_{xa}, and v'_{ya} from the paper plane is brought up to point E, and the speed component v'_{za} is perpendicular to the paper plane. The line EK is perpendicular to the paper plane. Detail 2 shows the vectors v'_{ya} and v_{ya} that makes the angle b between them. Detail 3 illustrates the plane of points $E, K, G,$ and H in a plane perpendicular to the paper plane.

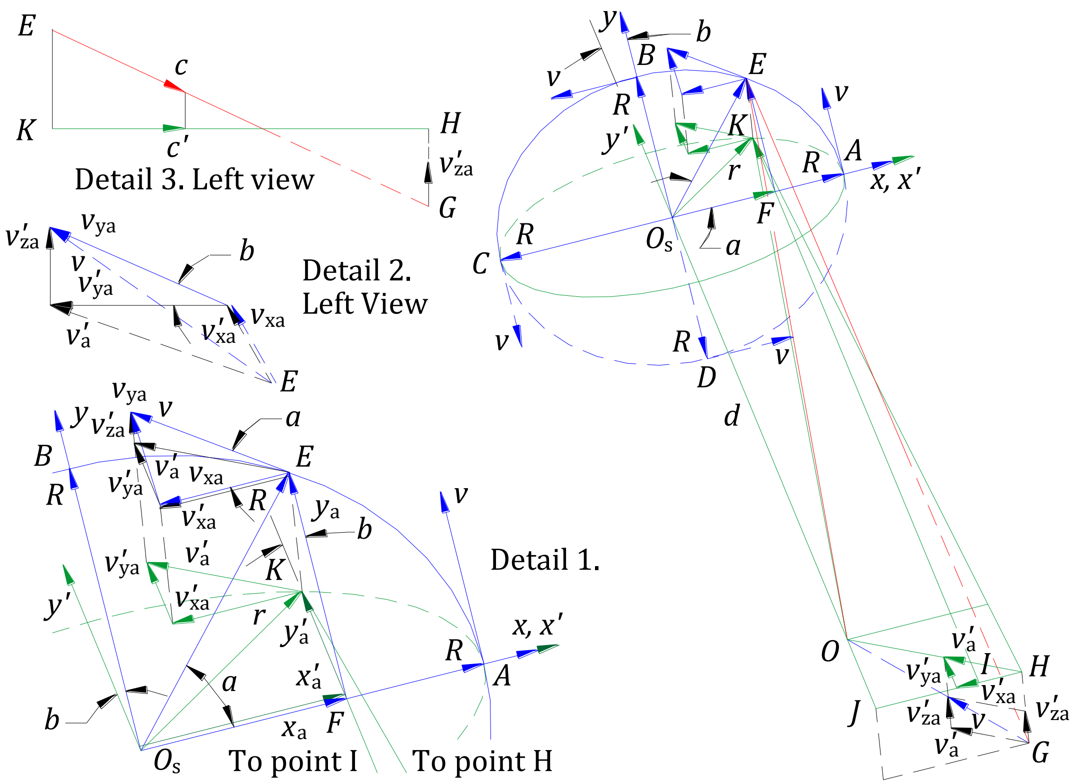

Figure 7. Observation of a star B from an observation line that makes an angle b to the star's orbit

The light from point E travels in the direction EG with the speed c. The velocity v from point E drags the speed c in the direction GO, and the light travels along the distance EO with the speed c_{EO} in time t_{sa}. The projection of the distance EG on the paper plane is the distance KH. The projection of light along KH travels from point K to point H with the speed c'. The velocity v_a' from point K drags the speed c' in the direction HO, and the light travels along the distance KO with the speed c_{KO} at the same time t_{sa}.

From figure 7, detail 1:
$x_a = R\cos a, \quad y_a = R\sin a,$
$v_{xa} = v\sin a, \text{ and } \quad v_{ya} = v\cos a.$
The projections of these vectors on the paper plane are
$x_a' = x_a = R\cos a, \quad y_a' = y_a\cos b = R\cos b\sin a,$
$v_{xa}' = v_{xa} = v\sin a, \text{ and } \quad v_{ya}' = v_{ya}\cos b = v\cos b\cos a.$

From figure 7, details 1, 2, and 3:
$EK = y_a\sin b = R\sin b\sin a,$
$v_{za}' = v_{ya}\sin b = v\sin b\cos a,$
$GH = v_{za}' t_{sa} = v t_{sa}\sin b\cos a.$

From figure 7, detail 3:

$$HK^2 = EG^2 - (EK + GH)^2 \quad \Rightarrow \quad c'^2 t_{sa}^2 = c^2 t_{sa}^2 - (R \sin b \sin a + v t_{sa} \sin b \cos a)^2$$
$$= c^2 t_{sa}^2 - R^2 \sin^2 b \sin^2 a - 2Rv t_{sa} \sin^2 b \sin a \cos a - v^2 t_{sa}^2 \sin^2 b \cos^2 a.$$

The distance HK depends on the quantities R, v, d, and b that are given and on the angle a that is variable. The distance HK can be calculated from triangle KIH of the paper plane.

$$HK^2 = IK^2 + HI^2 = IK^2 + (HJ - IJ)^2 = (OO_s + FK + OJ)^2 + (HJ - O_sF)^2 \quad \Rightarrow$$

$$c'^2 t_{sa}^2 = \left(d + \mathrm{y}'_a + v'_{ya} t_{sa}\right)^2 + (v'_{xa} t_{sa} - \mathrm{x}'_a)^2$$
$$= (d + R \cos b \sin a + v t_{sa} \cos b \cos a)^2 + (v t_{sa} \sin a - R \cos a)^2 \quad \Rightarrow$$

$$c'^2 t_{sa}^2 = d^2 + R^2 \cos^2 b \sin^2 a + v^2 t_{sa}^2 \cos^2 b \cos^2 a + 2dR \cos b \sin a$$
$$+ 2dv t_{sa} \cos b \cos a + 2Rv t_{sa} \cos^2 b \sin a \cos a + v^2 t_{sa}^2 \sin^2 a$$
$$- 2Rv t_{sa} \sin a \cos a + R^2 \cos^2 a.$$

Introducing the formula for $c'^2 t_{sa}^2$ obtained from detail 3 in the above equation,

$$c^2 t_{sa}^2 - R^2 \sin^2 b \sin^2 a - 2Rv t_{sa} \sin^2 b \sin a \cos a - v^2 t_{sa}^2 \sin^2 b \cos^2 a$$
$$= d^2 + R^2 \cos^2 b \sin^2 a + v^2 t_{sa}^2 \cos^2 b \cos^2 a + 2dR \cos b \sin a$$
$$+ 2dv t_{sa} \cos b \cos a + 2Rv t_{sa} \cos^2 b \sin a \cos a + v^2 t_{sa}^2 \sin^2 a$$
$$- 2Rv t_{sa} \sin a \cos a + R^2 \cos^2 a \quad \Rightarrow$$

$(c^2 - v^2) t_{sa}^2 - 2dv t_{sa} \cos b \cos a - (d^2 + 2dR \cos b \sin a + R^2) = 0$, with the convenient solution

$$t_{sa} = \frac{\sqrt{d^2 v^2 \cos^2 b \cos^2 a + (d^2 + 2dR \cos b \sin a + R^2)(c^2 - v^2)} + dv \cos b \cos a}{c^2 - v^2}.$$

For angle $b = 0$, the time t_{sa} is as in section 2.2.1. For angle $b = \pi/2$, the time t_{sa} is as in section 3.

4.2 Derivation of the distance $d = d_1$

In figure 7, the observation of the star depends on the time in which the light from the points of its orbit arrives at point O. Figure 7 suggests that the light from point C catches the light from point A.

The equation from section 2.2.2 for points A and C is $t_{sa=0} - t_{sa=\pi} = t_{ra=\pi}$. Thus, for the time t_{sa} from section 4.1, the equation $t_{sa=0} - t_{sa=\pi} = t_{ra=\pi}$ for $d = d_1$ becomes

$$\frac{\sqrt{d_1^2 v^2 \cos^2 b + (d_1^2 + R^2)(c^2 - v^2)} + d_1 v \cos b}{c^2 - v^2}$$
$$- \frac{\sqrt{d_1^2 v^2 \cos^2 b + (d_1^2 + R^2)(c^2 - v^2)} - d_1 v \cos b}{c^2 - v^2} = \frac{\pi R}{v} \quad \Rightarrow$$

$$\frac{2d_1 v\cos b}{c^2 - v^2} = \frac{\pi R}{v} \quad \Rightarrow \quad d_1 = \frac{\pi R(c^2 - v^2)}{2v^2 \cos b}.$$

For angle $b = 0$, the distance d_1 is as in section 2.2.2. For angle $b = \pi/2$, the distance d_1 is undefined and reflects the result of section 3 that the light from a point on the orbit does not catch the light from any other point on the orbit.

The distance d_{n+1} at which the light from point C, for $a = (2n+1)\pi$ with $n = 0, 1, 2, 3, \ldots$ catches the light from point A, for $a = 0$ is

$$d_{n+1} = \frac{(2n+1)\pi R(c^2 - v^2)}{2v^2 \cos b} \quad \text{for} \quad n = 0, 1, 2, 3, \ldots.$$

4.3 Star is at the distance $d = d_1/2$

The angle b is $b = \pi/4$ by choice, and the calculated distance d_1 is $d_1 = 3.3449E + 19\,m$. The observer, located at the distance $d = d_1/2$ from the star, sees the light for different angles a with the corresponding times given in table 6 and illustrated in figure 8.

Table 6. Times when the star is at the distance $d = d_1/2$.

a [°]	0	30	45	60	90
t_a [year]	1767.891949	1767.958540	1767.980343	1767.996122	1768.016955
120	135	150	180	210	225
1768.037788	1768.053566	1768.075367	1768.141949	1768.242022	1768.303550
240	270	300	315	330	360
1768.371102	1768.516933	1768.662769	1768.730327	1768.791862	1768.891949

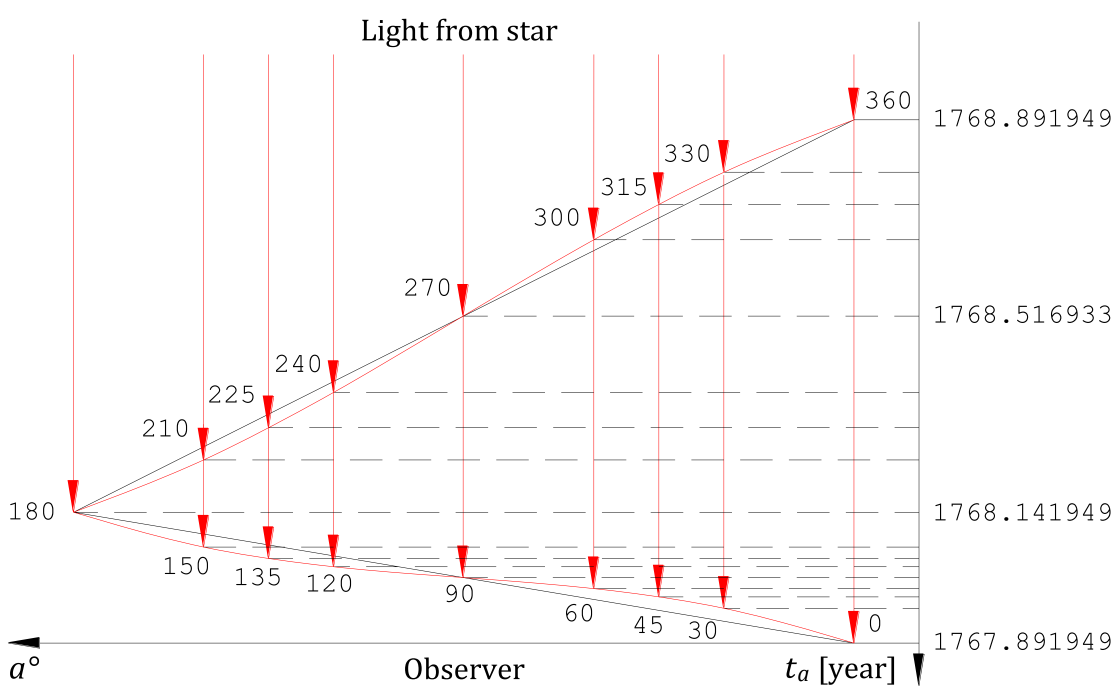

Figure 8. Star is at distance $d = d_1/2$.

4.4 Star is at the distance $d = d_1$

The observer, located at the distance $d = d_1$ from the star sees the light for different angles a with the corresponding times given in table 7 and illustrated in figure 9.

Table 7. Times when the star is at the distance $d = d_1$.

a [°]	0	30	45	60	90
t_a [year]	3535.783897	3535.833740	3535.835677	3535.825567	3535.783900
120	135	150	180	210	225
3535.742234	3535.732124	3535.734061	3535.783897	3535.900716	3535.982108
240	270	300	315	330	360
3536.075547	3536.283877	3536.492214	3536.585661	3536.667062	3536.783897

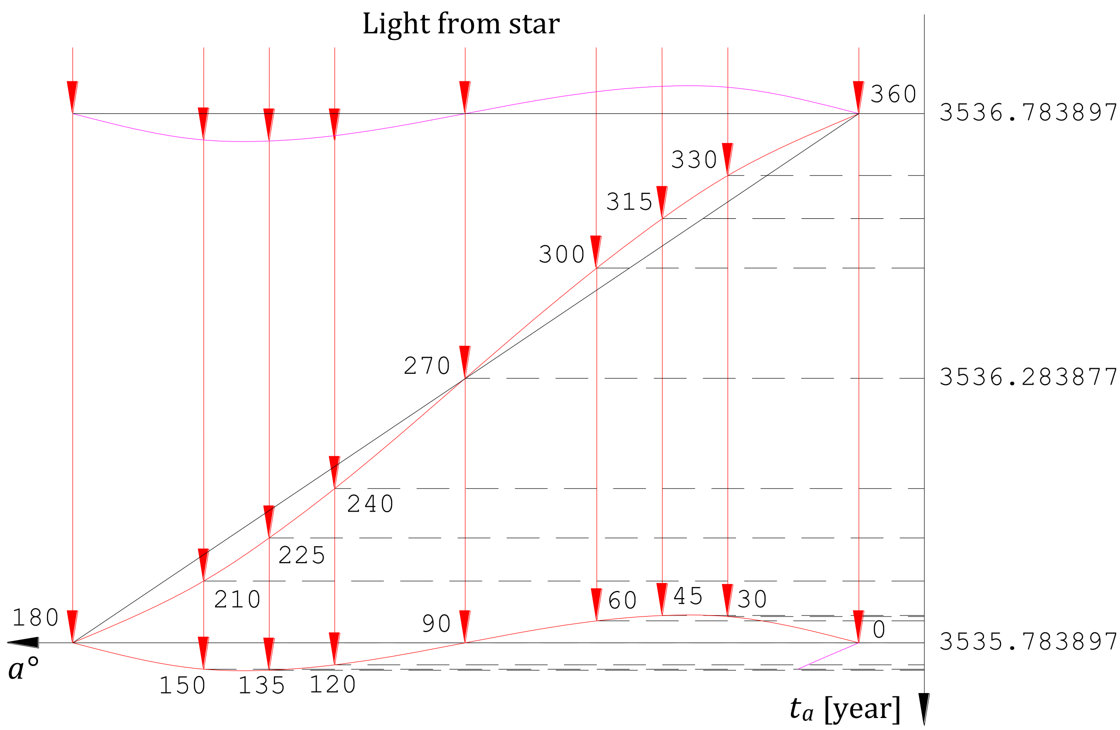

Figure 9. Star is at the distance $d = d_1$.

Figures 8 and 9 indicate that there is a distance between $d_1/2$ and d_1 at which the observer at point O starts to see multiple stars irregularities. This distance is $d_k = kd_1 = (2/3)d_1$. The data in the table 8 are for $d = 0.6667d_1 = 2.23004\text{E} + 19m$. The angles that indicate the observation of the multiple stars are emphasized in gray.

Table 8. Times of the star is at the distance $d = 0.6667d_1$.

a [°]	0	30	45	60	90
t_a [year]	2357.307124	2357.368131	2357.383311	2357.390459	2357.390455
120	135	150	180	210	225
2357.390450	2357.397597	2357.412775	2357.473774	2357.579431	2357.647581

240	270	300	315	330	360
2357.723764	2357.890432	2358.057106	2358.133295	2358.201454	2358.307124

4.5 Star is at the distance $d = 3d_1$

The observer, located at the distance $d = 3d_1$ from the star, sees the light for different angles a with the corresponding times given in table 9 and illustrated in figure 10.

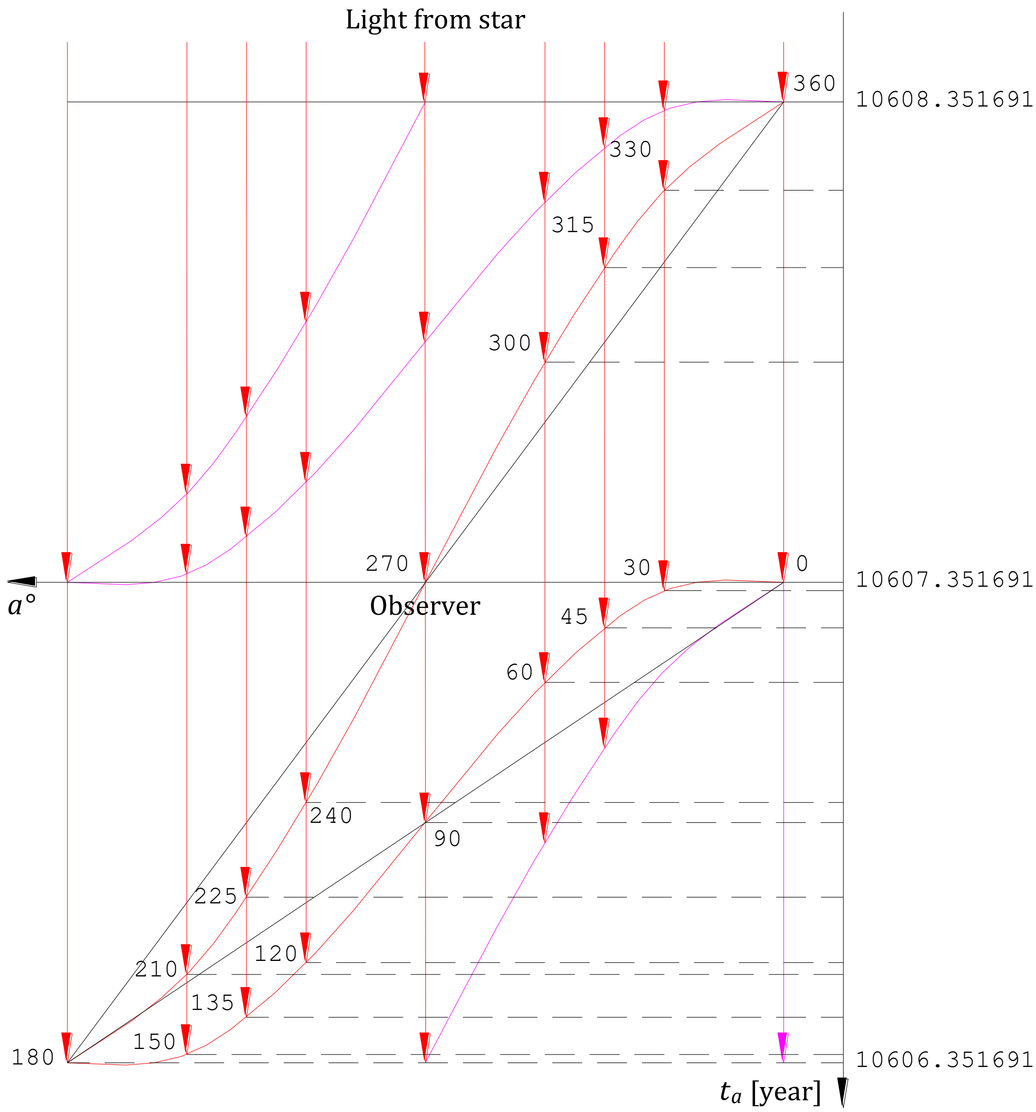

Figure 10. Star is at the distance $d = 3d_1$.

Table 9. Times when the star is at the distance $d = 3d_1$.

a [°]	0	30	45	60	90
t_a [year]	10607.351691	10607.334543	10607.257016	10607.143348	10606.851676
120	135	150	180	210	225
10606.560014	10606.446356	10606.368838	10606.351691	10606.535493	10606.696340

240	270	300	315	330	360
10606.893328	10607.351654	10607.809995	10608.007000	10608.167865	10608.351691

5 Conclusions

The multiple stars irregularities start to appear at the distance $d > kd = (2/3)d_1$, where d_1 depends on the orbit's radius of the star, the circular velocity of the star, and inclination angle of the observation of the star. The factor $k = 2/3$ looks to be a universal constant.

De Sitter presents the irregularities in a very general way. He does not make a distinction of the star observation according to the observation distance, or if the irregularities can be visually detectable. For the first example of the Earth as a star, the distance $d = kd = (2/3)d_1 = 1.57688E + 19\ m = 1666\ ly$. The observation angle at this distance is $e = 9.55025E - 09\ rad$. The observation of this star at this distance may look like a point at any time.

The astronomers affirm that they did not observe the irregularities as presented by De Sitter. The astronomers' observations may by partially right taking into account that the significant irregularities appear at distances at which the irregularities may not be detectable. The common double stars we know are at distances of up to 200-300 ly. At these distances, they do not display multiple stars irregularities. The observation of the circular velocities of these double stars cannot confirm if the velocity of light depends on the motion of its source or if the velocity of light is independent of the motion of its source.

For a complete confirmation, the astronomers have to present specific examples in which the multiple star irregularities can be visually detected, and yet we do not see them. The astronomers may have these examples, but the author of this paper is not aware of them.

References:

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[2] N. Bărbulescu, Bazele Fizice ale Relativității Einsteiniene (București: Editura Științifică și Enciclopedică, Enciclopedia de Buzunar, 1975).

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