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Ballistic Precession:

The Anomalous Advance of Planetary Perihelia

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***Abstract*:**

The primary objective of the current investigation is to calculate, on the basis of ballistic velocity of light in vacuum, and then to compare the computed numerical values of anomalous precession of planetary perihelia to the reported observational results. In particular, with regard to the anomalous advance of Mercury's perihelion, a value of *44.66* seconds of arc per century is obtained and deemed to be satisfactory and close and likely to be made much closer to Simon Newcomb's reported result of *43* seconds of arc per century, either by carrying out further more precise calculations, or by searching for any possibly overestimated perturbation effects by about *1.5* seconds of arc or so in the total value of planetary perturbations of about *532* seconds of arc per century for the anomalous precession of Mercury's orbit.

Keywords:

Precession; Mercury; Venus; perihelion; anomalous advance; ballistic speed of light; outer planets; sidereal period; transit; orbital velocity; perturbations; ingress; precession of the equinoxes; orbital period; egress; planetary orbits.

Introduction:

Invigorated and emboldened by the remarkable success of his previous calculations in the discovery of the planet Neptune, Urbain Le Verrier embarked, almost immediately, on carrying out a long series of similar calculations that, eventually, led him to predict the existence of a hypothetical planet named '*Vulcan*' between the Sun and the planet Mercury.

But, this time around, the predicted planet never materialized. And moreover, the portion of the sky, between the Sun and Mercury, is just too small and too clear to conceal anything as big and bright as the hypothetical planet Vulcan.

Nevertheless, from the standpoint of celestial mechanics, Le Verrier's calculations are quite robust and generally correct. And, in fact, their numerical result is off only by about *10%* from the currently accepted result with regard to the anomalous advance of Mercury's perihelion.

Initially, Le Verrier obtained, in *1859*, an anomalous precession of the orbit of the planet Mercury with an amount of about *39* seconds of arc per century. But that preliminary result was updated, in *1895*, by Simon Newcomb, to about *43* seconds of arc per century.

At first glance, an anomalous advance of *43* seconds of arc per century, in the perihelion of Mercury, appears incredibly insignificant. And it would have, probably, been written off as a completely insignificant telescopic or personal-equation error, if it was obtained through direct observation of that inner planet of the solar system.

However, Le Verrier inferred Mercury's anomalous precession, indirectly and statistically, from *400* meridian-circle observations, taken at the Paris Observatory between *1801* and *1842*; and from *24* timings of contact points during *12* transits of the planet Mercury between *1697* and 1832 [***Ref. #1***].

And precisely because it's based upon those two relatively high-quality sets of Mercury's observations, the above tiny amount of precession is extremely stable and difficult to explain away by adjusting perturbation parameters, for instance, or by merely playing around with orbital elements of known planets.

Undoubtedly, the anomalous precession of Mercury's perihelion, in spite of its extreme minuteness, is one of the most famous and inspiring anomalies in the history of celestial mechanics.

Since the publication of Le Verrier's report, many solutions have proposed, in order to explain away the anomalous precession of Mercury's perihelion; but here, for the sake of brevity, only a small but fairly representative sample of those published solutions, is included in the following list:

I. The Planet-Vulcan Solution: According to this solution, by Urbain Le Verrier, the anomalous advance of Mercury's perihelion is caused by the perturbation effect of a small planet called '*Vulcan*', which is assumed to be in orbit around the gravitational center of the solar system between the Sun and the planet Mercury. The main objection to Le Verrier's solution is that a celestial body with Vulcan's mass must be, at least, as bright as a star of the 4^{th} magnitude; and therefore, it can't remain hidden for too long from observers, here, on Earth.

II. The Zodiacal-Matter Solution: According to this solution, by Hugo von Seeliger, the materials that cause the '*zodiacal light*' can, in principle, have, collectively, enough perturbation effect to account for the anomalous precession of Mercury's perihelion [***Ref. #5.a***]. The published objection to Hugo von Seelinger's solution is that the total mass of zodiacal materials is just too small to cause the required perturbation effect.

III. The Hall-Newcomb Solution: According to this solution, in order to explain away the anomalous precession of Mercury's perihelion, the following Newtonian equation for computing the gravitational force F between two bodies with mass m_1 and mass m_2, respectively, and at a distance r from each other:

$$F = G\frac{m_1 m_2}{r^2}$$

should be replaced with this modified equation:

$$F = G\frac{m_1 m_2}{r^{(2+\delta)}}$$

where G is the gravitational constant; and δ has this numerical value:

$$\delta = 1.574 \times 10^{-7}$$

as calculated by S. Newcomb [***Ref. #12***]. The chief objection, to the Hall-Newcomb solution, is that it is too artificial, theoretically unjustified, and bound sooner or later to create many daunting and thorny problems in other areas of celestial mechanics.

IV. The Velocity-Dependent-Force Solution: According to this solution, by Paul Gerber, if the unknown constant k is set to *6*, then the Gerber's velocity-dependent-force equation should give, in the case of Mercury's perihelion, an amount of anomalous precession δ:

$$\delta = k\frac{4\pi^3 a^2}{T^2 c^2\left(1-\varepsilon^2\right)}$$

where T is the orbital period; a is the semi major axis of Mercury's orbit; ε is the orbital eccentricity; and k is assumed to be equal to *6*:

$$k = 6$$

and where c is the speed of light [***Ref. #5.b***]. The main objection to Gerber's solution, in the published literature, is that a velocity-dependent force of gravity is too contrived and ruled out, from the start, by the complete absence of any gravitational aberration analogous to that of electromagnetic radiation from sources located outside the reference frame of the Earth-Moon system.

V. The Curved-Space-Time Solution: According to this solution, by A. Einstein, the field equations of general relativity give an amount of anomalous precession of Mercury's orbit ε:

$$\varepsilon = \frac{24\pi^3 a^2}{T^2 c^2\left(1-e^2\right)}$$

where a is the semi major axis; T is orbital period; e is the orbital eccentricity; and c is the speed of light in vacuum [***Ref. #11***]. The primary objection to the curved-space-time solution is that it's too drastic; and that the conjecture of space-time curvature, upon which it's ultimately based, lacks any specific physical mechanism; and, in addition, it does not meet, epistemologically speaking, any of René Descartes' criteria of truth and clarity at all.

VI. The Dicke-Goldenberg Solution: According to this solution, an oblate sun can, in principle, account for the anomalous precession of Mercury's perihelion. The main objection to the Dicke-Goldenberg solution is that the Sun is almost perfectly spherical with little or no oblateness of its shape measured or observed at all.

VII. The Mass-Energy-Conservation Solution: According to this solution, by P. Marmet, the mass-energy-conservation equation gives, in the case of Mercury, an angle of precession per century $\Delta\phi$, as calculated in accordance with this relation:

$$\Delta\phi = \frac{6\pi G m'}{c^2 r\left(1-e^2\right)}$$

where c is the speed of light; G is the gravitational constant; e is the orbital eccentricity; m' is the Sun's mass; and r is Mercury's distance from the Sun [***Ref.*** *#7*]. The principal objection to the mass-energy-conservation solution is that the derivation of the mass-energy-conservation equation is somewhat arbitrary and seemingly devoid of any clear theoretical justification.

VIII. **The Mobile-Sun Solution:** According to this solution, by C. Tsolkas, the orbital revolution of the Sun, around the center of mass of our solar system, leads to the anomalous advance, in the perihelion of Mercury, by an amount equals to *43* arc-seconds per century [***Ref. #6***]. The anticipated objection to the Mobile-Sun solution is that although it might be possible, in principle, it's highly unlikely that Urbain Le Verrier and Simon Newcomb were unaware of the orbital motion of the Sun around the barycenter of the solar system; unless, of course, a detailed review of their works, in this particular area of celestial mechanics, shows otherwise.

IX. **The Co-Gravitational-Field Solution:** According to this solution, by C. J. de Matos and M. Tajmar, the anomalous precession of Mercury's perihelion precession is caused by the Sun's cogravitational field, which is assumed to be due to the Sun's spin [***Ref. #14***]. So far no objection has been advanced, in the published literature, against the co-gravitational-field solution, although it appears unlikely that the slow rotation of the Sun can produce any effect of this postulated sort.

X. **The Light-Carrying-Medium Solution:** According to this solution, by Tom Van Flandern, a light-carrying-medium surrounding the Sun, in which density varies with the Sun's gravitational potential, changes the motion of Mercury's perihelion, in accordance with this basic mathematical form:

$$BF = \frac{n\mu}{c^2 a\left(1-e^2\right)}$$

where BF stands for the basic form; μ is the product of the gravitational constant and the mass of the Sun; a is the semi-major axis of Mercury's orbit; e is orbital eccentricity; and n is defined as:

$$n = \frac{2\pi}{P}$$

and where P is the orbital period of the planet Mercury. The customary objection, in the published literature, to the light-carrying-medium solution is that the light-carrying medium, by its very defintion, can be always assumed theoretically; but it can never be detected by any practical means.

However, the surveyed literature on this subject, makes no reference at all to one more possible and seemingly very attractive solution, for the anomalous precession of Mercury's perihelion and planetary perihelia in general, which can be advanced and based entirely upon the assumption of ballistic velocity of light in vacuum.

The principal objective of the current investigation is to present a detailed analysis of this latter solution and to compare predictions, computed on the basis of the aforementioned ballistic assumption, with observations as well as with predictions of other solutions with regard to the anomalous precession of planetary perihelia in general, and the anomalous advance of Mercury's perihelion in particular.

1. An Outline of the Ballistic Solution:

As seen from the moving reference frame of the earth, the straight line between the point of maximum approach, at which the planet Mercury is approaching directly the earth, and the point of maximum recession, at which the planet Mercury is receding directly from the earth, divides the orbit of the planet Mercury into two equals parts:

1. The near-side part, which starts from the point of maximum velocity of approach and ends at the point of maximum velocity of recession.

2. And the far-side part, which starts from the point of maximum velocity of recession and ends at the point of maximum velocity of approach.

And therefore, if it's assumed, as within the framework of the elastic-impact emission theory, for example, that light travels in vacuum at the velocity resultant of its muzzle velocity c and the velocity of its source at the time of emission v, then the planet Mercury, as observed from Earth, must appear to spend more time in the near-side half of its orbit and less time in the far-side half of the same orbit.

That is because the near-side half of Mercury's orbit starts from the point of maximum velocity of approach and ends at the point of maximum velocity of recession; and accordingly, the value of the velocity resultant of light, at the start of the near-side half, is higher than the value of the velocity resultant of light, at the end of the near-side half of Mercury's orbit. And the reverse is true, in the case of the far-side half of Mercury's orbit, as seen by observers on Earth.

It follows, therefore, that all observations of the planet Mercury, in the near-side half of its orbit, necessarily lead to and always fit in with an apparent orbital period of Mercury longer than its actual orbital period. And the opposite is true, in the case of the far-side half of Mercury's orbit

And since all transits of the planet Mercury occur in the near-side half of its orbit, the times of transit events, such as the start, the ingress, the egress, the end, as well as the duration of every transit of Mercury across the Sun, fit ultimately into an apparent orbital period of Mercury, which is always longer than its actual orbital period by an amount of time that directly depends on how far the observer is located from the planet Mercury.

And so, now, the important question, here, is this:

How can the longer apparent orbital period, based on the timings of Mercury's transits, be employed, theoretically, to explain away Urbain Le Verrier's famous anomaly?

Qualitatively, the explanation of the anomalous precession of Mercury's perihelion, on the basis of the ballistic velocity of light, is quite easy and simple:

- Urbain Le Verrier deduced the anomalous advance of Mercury's perihelion from a sufficient sample of Mercury's transit data.

- The observational data of Mercury's transits lead to an apparent orbital period longer than the actual orbital period of the planet Mercury.

- And because the apparent orbital period, into which the various stages of Mercury's transits necessarily fit, is longer than the actual orbital period of the planet Mercury, Mercury's orbit must appear to advance anomalously by a certain amount in the same direction as that of the orbital revolution of the planet Mercury around the gravitational center of the solar system.

And so here, it is not, anymore, a question of whether or not the ballistic velocity of light can, in principle, explain away the anomalous precession of Mercury's perihelion in a qualitative manner; but it's, now, a question of whether or not it's capable of producing numerical values close enough to Simon Newcomb's numerical value of *42.9* arc-seconds per century for the anomalous advance of Mercury's perihelion.

However, before any attempt at calculating the aforementioned numerical value, it's, first and foremost, imperative, in this regard, to find out by how much exactly the apparent orbital time of the near-side half of Mercury's orbit is longer than its apparent orbital time of the far-side half of the same orbit.

2. The Two Apparent Periods of Mercury's Orbit:

As already pointed out, in the moving reference frame of the earth, the straight line between the point of maximum velocity of approach, at which the planet Mercury is approaching directly the earth, and the point of maximum velocity of recession, at which the planet Mercury is receding directly from the earth, divides the orbit of the planet Mercury into two equals parts: The near-side half and the far-side half of Mercury's orbit.

If P is the actual orbital period of the planet Mercury, throughout its entire orbit around the barycenter of the solar system, then the actual orbital time of the near-side half of Mercury's orbit is $\frac{1}{2}P$, and the actual orbital time of the far-side half of the same orbit is $\frac{1}{2}P$ as well.

A. The Apparent Orbital Period of the Near-Side Half:

The near-side half of Mercury's orbit starts from the point of maximum velocity of approach and ends at the point of maximum velocity of recession, as observed in the reference frame of the moving earth.

Let v denote the orbital velocity of Mercury.

Since sunlight is reflected from the planet Mercury, its incident velocity c, as computed in accordance with the ballistic assumption, is increased, upon reflection, by twice the orbital velocity of Mercury; i.e., $2v$, at the point of maximum velocity of approach:

$$c' = c + 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Mercury, at the point of maximum velocity of approach.

At the point of maximum velocity of recession, by contrast, the incident velocity of sunlight is decreased, upon reflection, by twice the orbital velocity of the planet Mercury:

$$c' = c - 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Mercury, at the point of maximum velocity of recession..

And therefore, if d is the distance between the planet Mercury and the observers on Earth, then the total travel time of sunlight, reflected from Mercury at the point of maximum velocity of approach, can be calculated, in accordance with the ballistic assumption, by using the following equation:

$$t_A = \frac{d}{c'} = \frac{d}{c+2v}$$

where t_A is the total travel time of reflected sunlight from the planet Mercury, at the point of maximum velocity of approach, to the observers on the earth.

And in the same way, the total travel time of sunlight, reflected from Mercury at the point of maximum velocity of recession, can be obtained by using this equation:

$$t_R = \frac{d}{c'} = \frac{d}{c-2v}$$

where t_R is the total travel time of reflected sunlight, from the planet Mercury, at the point of maximum velocity of recession, to the planet earth.

And subsequently, the apparent orbital time of the near-side half of Mercury's orbit, T_{nr}, is given by the following equation:

$$T_{nr} = \frac{1}{2}P + \left(t_R - t_A\right) = \frac{1}{2}P + \frac{4d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where P is the actual orbital period of the planet Mercury.

It follows, therefore, that the apparent orbital period of the near-side half of Mercury's orbit, P_{nr}, can be calculated in accordance with this formula:

$$P_{nr} = 2T_{nr} = P + \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the planet Mercury.

It should be pointed out, within this context, that, even though the number of all observable events of the near-side half of Mercury's orbit remains exactly the same regardless of whether the orbital period is apparent or actual, the duration of every event, in this half of the orbit, is longer during the apparent orbital period than the duration of the same event during the actual orbital period. And that is because the former period is longer than the latter one.

B. The Apparent Orbital Period of the Far-Side Half:

The far-side half of Mercury's orbit starts from the point of maximum velocity of recession and ends at the point of maximum velocity of approach, as observed from the moving reference frame of the earth.

Let v stand for the orbital velocity of Mercury.

Since sunlight is reflected from the planet Mercury, its incident velocity c, as calculated on the basis of the ballistic assumption, is decreased, upon reflection, by twice the orbital velocity of Mercury; i.e., *2v*, at the point of maximum velocity of recession

$$c' = c - 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Mercury, at the point of maximum velocity of recession.

At the point of maximum velocity of approach, by comparison, the incident velocity of sunlight is increased, upon reflection, by twice the orbital velocity of the planet Mercury:

$$c' = c + 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Mercury, at the point of maximum velocity of approach..

And consequently, if d is the distance between the planet Mercury and Earth, then the total travel time of sunlight, reflected from Mercury at the point of maximum velocity of recession, can be computed through the use of this equation:

$$t_R = \frac{d}{c'} = \frac{d}{c - 2v}$$

where t_R is the total travel time of reflected sunlight, from the planet Mercury, at the point of maximum velocity of recession, to the planet earth.

And similarly, the total travel time of sunlight, reflected from Mercury at the point of maximum velocity of approach, can be obtained by using this equation:

$$t_A = \frac{d}{c'} = \frac{d}{c+2v}$$

where t_A is the total travel time of reflected sunlight, from the planet Mercury, at the point of maximum velocity of approach, to observers on the earth.

And accordingly, the apparent orbital time of the far-side half of Mercury's orbit, T_{fr}, is given by the following equation:

$$T_{fr} = \frac{1}{2}P + \left(t_A - t_R\right) = \frac{1}{2}P - \frac{4d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where P is the actual orbital period of the planet Mercury.

And it follows, therefore, that the apparent orbital period of the far-side half of Mercury's orbit, P_{nr}, can be calculated in accordance with this formula:

$$P_{fr} = 2T_{fr} = P - \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the planet Mercury.

Once again, it should be mentioned that, although the number of all observable events of the far-side half of Mercury's orbit remains the same, the duration of every event, in this half of the orbit, is necessarily shorter during the apparent orbital period than the duration of the same event during the actual orbital period. And that is because the apparent period is shorter than the actual one.

3. The Ballistic Prediction of Le Verrier's Anomaly:

As mentioned earlier, in this discussion, Urbain Le Verrier deduced the anomalous advance of Mercury's perihelion from observational data gathered over several decades during Mercury's transits.

And since those transits occur only when the planet Mercury is in the near-side half of its orbit, their timing points and duration data can fit only in with this apparent orbital period of the near-side half of Mercury's orbit:

$$P_{nr} = P + \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the planet Mercury.

And because the apparent orbital period, into which the observations of Mercury's transits properly fit, is longer than the actual orbital period of the planet Mercury, Mercury's perihelion appears automatically to rotate anomalously in the same direction as that of the orbital revolution of the planet Mercury around the gravitational center of the solar system, by an amount of arc-seconds per orbital revolution directly proportional to a factor of ΔP:

$$\Delta P = \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where ΔP is the time difference between the apparent orbital period and the actual period of Mercury.

And accordingly, the numerical value of the anomalous precession of Mercury's perihelion per century $\Delta\phi$, as computed on the basis of the ballistic velocity of light in vacuum, can be obtained by using the following formula:

$$\Delta\phi = \Delta P \times N_{rev} \times \omega \times 100$$

where N_{rev} is the number of orbital revolutions per year; and ω is:

$$\omega = \frac{360}{P} \quad \text{arc-seconds per second}$$

and where P is the actual orbital period of the planet Mercury in hours; since the number of *3600* for converting degrees to seconds of arc and the number of *3600* for converting hours to seconds of time cancel each other out within the above numerical relation.

It follows, therefore, that if we insert the following observational data:

$$v = 47360 \quad \text{ms}^{-1}$$
$$c = 299792458 \quad \text{ms}^{-1}$$
$$P = 7600521.6 \quad \text{s}$$
$$\text{d} = 149597870700 \ \ \text{m}$$
$$\text{N}_{\text{rev}} = 4.149 \quad \text{rev.}$$
$$\omega = 0.1705 \ \ \text{arc-s/s}$$

into the following equation:

$$\Delta\phi = \left(\frac{8d}{c}\right)\left(\frac{v/c}{1-4v^2/c^2}\right)\left(N_{rev} \times \omega \times 100\right)$$

we obtain, for the anomalous precession of Mercury's perihelion, this numerical value:

$$\Delta\phi = 44.61 \quad \text{arc-seconds per century}$$

as calculated in accordance with the ballistic velocity of light in vacuum.

The above computed prediction of the anomalous advance of Mercury's perihelion, on the basis of the ballistic assumption, does not deviate significantly from Simon Newcomb's result of *43* seconds of arc per century. And it can be made much closer to the observed value by taking into account the eccentricity of Mercury's orbit and its inclination with respect to the plane of the earth's orbit.

Furthermore, the value of the anomalous precession of Mercury's perihelion, according to the ballistic assumption, varies directly with the distance from the point of maximum velocity of approach and the point of maximum velocity of recession.

For instance, at a distance of *1.524 AU*, on the planet Mars, the anomalous advance of Mercury's perihelion has, as predicted on the basis of ballistic speed of light, the following numerical value:

$$\Delta\phi = 67.99 \quad \text{arc-seconds per century}$$

and at a distance of *9.582 AU*, on the planet Saturn:

$$\Delta\phi = 427.45 \quad \text{arc-seconds per century}$$

and, of course, at a distance of *39.48 AU*, on Pluto:

$$\Delta\phi = 1761.2 \quad \text{arc-seconds per century}$$

where $\Delta\phi$ is the anomalous precession of Mercury's perihelion.

4. The Two Apparent Periods of Venus' Orbit:

As in the case of the planet Mercury, in the moving reference frame of the earth, the straight line between the point of maximum approach, at which the planet Venus is approaching directly the earth, and the point of maximum recession, at which the planet Venus is receding directly from the earth, divides the orbit of the planet Venus into two halves: The near-side half and the far-side half.

If P is the actual orbital period of the planet Venus, throughout its entire orbit around the barycenter of the solar system, then the actual orbital time of the near-side half of Venus' orbit is $\frac{1}{2}P$, and the actual orbital time of the far-side half of the same orbit is $\frac{1}{2}P$.

1. The Apparent Orbital Period of the Near-Side Half:

The near-side half of Venus' orbit starts from the point of maximum velocity of approach and ends at the point of maximum velocity of recession, as observed in the reference frame of the moving earth.

Let v denote the orbital velocity of the planet Venus.

Since sunlight is reflected from the planet Venus, its incident velocity c, as computed in accordance with the ballistic assumption, has to be increased, upon reflection, by twice the orbital velocity of Venus; i.e., $2v$, at the point of maximum velocity of approach

$$c' = c + 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Venus.

At the point of maximum velocity of recession, by comparison, the incident velocity of sunlight is decreased, upon reflection, by twice the orbital velocity of the planet Venus:

$$c' = c - 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Venus, at the point of maximum velocity of recession..

And therefore, if d is the distance between the planet Venus and the observers on Earth, then the total travel time of sunlight, reflected from the planet Venus at the point of maximum velocity approach, can be calculated by using this equation:

$$t_A = \frac{d}{c'} = \frac{d}{c + 2v}$$

where t_A is the total travel time of reflected sunlight, from the planet Venus, at the point of maximum velocity of approach, to observers on the earth.

And likewise, the total travel time of sunlight, reflected from Venus at the point of maximum velocity of recession, can be obtained through the use of the following equation:

$$t_R = \frac{d}{c'} = \frac{d}{c-2v}$$

where t_R is the total travel time of reflected, sunlight from the planet Venus, at the point of maximum velocity of recession, to the planet earth.

And subsequently, the apparent orbital time of the near-side half of Venus' orbit, T_{nr}, is given by the following equation:

$$T_{nr} = \frac{1}{2}P + \left(t_R - t_A\right) = \frac{1}{2}P + \frac{4d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where P is the actual orbital period of the planet Venus.

It follows, therefore, that the apparent orbital period of the near-side half of Venus' orbit, P_{nr}, can be calculated in accordance with this formula:

$$P_{nr} = 2T_{nr} = P + \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the planet Venus.

2. The Apparent Orbital Period of the Far-Side Half:

The far-side half of Venus' orbit starts from the point of maximum velocity of recession and ends at the point of maximum velocity of approach, as seen from the moving reference frame of the earth.

Let v be the orbital velocity of Venus.

Since sunlight is reflected from the planet Venus, its incident velocity c, as calculated on the basis of the ballistic assumption, is increased, upon reflection, by twice the orbital velocity of the planet Venus; i.e., $2v$, at the point of maximum velocity of approach

$$c' = c + 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Venus.

By contrast, at the point of maximum velocity of recession, the incident velocity of sunlight is decreased, upon reflection, by twice the orbital velocity of the planet Venus:

$$c' = c - 2v$$

where c' is the velocity resultant of sunlight, upon reflection from the planet Venus, at the point of maximum velocity of recession..

And therefore, if d is the distance between the planet Venus and Earth, then the total travel time of sunlight, reflected from Venus at the point of maximum velocity of approach, can be computed through the use of the following equation:

$$t_A = \frac{d}{c'} = \frac{d}{c + 2v}$$

where t_A is the total travel time of reflected sunlight from the planet Venus, at the point of maximum velocity of approach, to observers on the earth.

And similarly, the total travel time of sunlight, reflected from the planet Venus at the point of maximum velocity of recession, can be obtained by using this equation:

$$t_R = \frac{d}{c'} = \frac{d}{c - 2v}$$

where t_R is the total travel time of reflected sunlight, from the planet Venus, at the point of maximum velocity of recession, to the planet earth.

And hence, the apparent orbital time of the far-side half of Venus' orbit, T_{fs}, is given by the following equation:

$$T_{fr} = \frac{1}{2}P + (t_A - t_R) = \frac{1}{2}P - \frac{4d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where P is the actual orbital period of the planet Venus.

And it follows, therefore, that the apparent orbital period of the far-side half of Venus' orbit, P_{nr}, can be calculated in accordance with this formula:

$$P_{fr} = 2T_{fr} = P - \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the planet Venus.

5. The Ballistic Prediction of the Orbital Anomaly of Venus:

Like Urbain Le Verrier's anomaly of Mercury's orbit, the anomalous advance of Venus' orbit has to be deduced from observations gathered in the near-side half of its orbit.

And consequently, the timing data of those observations can fit only in with the apparent orbital period of the near-side half of Venus' orbit, as given by this equation:

$$P_{nr} = P + \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the planet Venus.

And because the apparent orbital period, into which the timings of observational data properly fit, is longer than the actual orbital period of the planet Venus, its orbit must appear to rotate anomalously in the same direction as that of the orbital revolution of the planet Venus around the barycenter of the solar system, by an amount of arc-seconds per revolution directly proportional to a factor of ΔP:

$$\Delta P = \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where ΔP is the time difference between the apparent orbital period and the actual period of Venus.

And thus, the numerical value of the anomalous precession of Venus' orbit per century $\Delta\phi$, as computed on the basis of the ballistic assumption, can be obtained by using this formula:

$$\Delta\phi = \Delta P \times N_{rev} \times \omega \times 100$$

where N_{rev} is the number of orbital revolutions per year; and ω is:

$$\omega = \frac{360}{P} \quad \text{arc-seconds per second}$$

and where P is actual orbital period of the planet Venus in hours.

And it follows, therefore, that if we insert the following observational data:

$$v = 35020 \quad \text{ms}^{-1}$$
$$c = 299792458 \quad \text{ms}^{-1}$$
$$P = 19414166.4 \quad \text{s}$$
$$\text{d} = 149597870700 \quad \text{m}$$
$$\text{N}_{\text{rev}} = 1.627 \quad \text{rev.}$$
$$\omega = 0.669 \quad \text{arc-s/s}$$

into the following equation:

$$\Delta\phi = \left(\frac{8d}{c}\right)\left(\frac{v/c}{1-4v^2/c^2}\right)\left(N_{rev} \times \omega \times 100\right)$$

we obtain, for the anomalous precession of Venus' orbit, this numerical value:

$$\Delta\phi = 5.077 \quad \text{arc-seconds per century}$$

as calculated in accordance with the assumption of ballistic velocity of light.

The numerical value of the anomalous precession of Venus' orbit, as computed above on the assumption of ballistic speed of light in vacuum, is much higher than the numerical value of (*-0.05 ± 0.25 arc-seconds per century*) initially obtained by A. Einstein **[*Ref.* #*3*]**; but it's still less than the currently recalculated value of (*8.6 arc-seconds per century*) within the framework of Einstein's general theory of relativity as well as the observed value of (*8.4 ± 4.8 arc-seconds per century*) **[*Ref.* #*5.a*]**. However, the observations of Venus, in this regard, are more difficult and lower-grade than those of the planet Mercury. And therefore, the result of (5*.077 arc-seconds per century*), as calculated in accordance with the ballistic assumption, should be deemed acceptable and theoretically reasonable.

And furthermore, according to the ballistic assumption, the value of the anomalous precession of Venus' orbit varies linearly in direct proportion to the distance from the point of maximum velocity of approach and the point of maximum velocity of recession:

At a distance of *5.203 AU*, from the planet Jupiter, for example, the orbital shift of Venus is predicted, on the basis of ballistic speed of light, to have this numerical value:

$$\Delta\phi = 26.42 \quad \text{arc-seconds per century}$$

and at a distance of *19.2 AU*, on the planet Uranus:

$$\Delta\phi = 97.48 \quad \text{arc-seconds per century}$$

and at a distance of *30.05 AU*, on the planet Neptune:

$$\Delta\phi = 152.56 \quad \text{arc-seconds per century}$$

where $\Delta\phi$ is the anomalous precession of Venus' orbit.

6. The Ballistic Prediction of the Orbital Anomaly of Earth:

In accordance with the assumption of ballistic speed of light, no anomalous shift of Earth's orbit can be observed or deduced from transit data or any other observational data by observers located anywhere in the reference frame of the earth; i.e., in this special case:

$$\Delta\phi = 0''$$

where $\Delta\phi$ is the anomalous precession of Earth's orbit.

Nonetheless, if the earth is seen, for example, from the reference frame of the planet Mars, then the earth's orbit, as predicted on the basis of the same ballistic assumption, must appear to advance, in an anomalous manner, by a certain amount in the same direction of its orbital revolution around the gravitational center of the solar system.

Just as in the case of the transit observations of Mercury in the reference frame of the earth, transit data gathered in the reference frame of Mars with regard to the earth's orbit, ought to fit in properly with the apparent orbital period of the near-side half of Earth's orbit with respect Mars:

$$P_{nr} = P + \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the planet Mars.

And since the apparent orbital period, into which the timing points of such observational data correctly fit, is longer than the actual orbital period of the planet Earth, its orbit must appear to rotate anomalously in the same direction as that of the orbital revolution of the planet Earth around the barycenter center of the solar system, by an amount of arc-seconds per orbital revolution directly proportional to a factor of ΔP:

$$\Delta P = \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where ΔP is the time difference between the apparent orbital period and the actual period of Earth.

And correspondingly, the numerical value of the anomalous precession of Earth's orbit per century $\Delta\phi$, as computed on the basis of the ballistic assumption, can be obtained by using this formula:

$$\Delta\phi = \Delta P \times N_{rev} \times \omega \times 100$$

where N_{rev} is the number of orbital revolutions per year; and ω is:

$$\omega = \frac{360}{P} \quad \text{arc-seconds per second}$$

and where P is actual orbital period of the planet Earth in hours; since, within the above numerical relation, the number of *3600* for converting degrees to seconds of arc and the number of *3600* for converting hours to seconds of time cancel each other out.

And it follows, therefore, that if we insert the following observational data:

$$\begin{aligned}
&v = 29780 \quad \mathrm{ms^{-1}} \\
&c = 299792458 \ \ \mathrm{ms^{-1}} \\
&p = 31558118.4 \ \ \mathrm{s} \\
&\mathrm{d} = 227987154946.8 \ \ \mathrm{m} \\
&\mathrm{N_{rev}} = 1 \quad \mathrm{rev.} \\
&\omega = 0.041 \quad \text{arc-s/s}
\end{aligned}$$

into the following equation:

$$\Delta\phi = \left(\frac{8d}{c}\right)\left(\frac{v/c}{1-4v^2/c^2}\right)\left(N_{rev} \times \omega \times 100\right)$$

we obtain, for the anomalous precession of Earth's orbit, this numerical value:

$$\Delta\phi = 2.48 \quad \text{arc-seconds per century}$$

as calculated in accordance with the ballistic assumption in the reference frame of Mars.

In addition, according to the ballistic assumption, the value of the anomalous precession of Earth's orbit varies linearly in direct proportion to the distance from the point of maximum velocity of approach and the point of maximum velocity of recession:

At a distance of 9*.582 AU*, on the planet Saturn, for instance, the orbital shift of Earth is predicted, on the basis of ballistic speed of light, to have this numerical value:

$$\Delta\phi = 15.59 \quad \text{arc-seconds per century}$$

and at a distance of *19.2 AU*, on the planet Uranus:

$$\Delta\phi = 31.24 \quad \text{arc-seconds per century}$$

and at a distance of *39.48 AU*, on Pluto:

$$\Delta\phi = 64.25 \quad \text{arc-seconds per century}$$

where $\Delta\phi$ is the anomalous precession of Earth's orbit.

7. The Ballistic Predictions of Orbital Shifts of Outer Planets:

The orbits of outer planets, as seen from Earth, can show no Urbain Le Verrier's anomaly, in accordance of the assumption of ballistic velocity of light in vacuum; because those outer planets do not have any kind of any transit data to begin with.

Nevertheless, as in the case of the inner planets, in the moving reference frame of the earth, the straight line between the point of maximum approach, at which an outer planet is approaching directly the earth, and the point of maximum recession, at which the same outer planet is receding directly from the earth, divides the orbit of that planet into two equals parts: The near-side part and the far-side part.

Now, if we assume that P is the actual orbital period of an outer planet, throughout its entire orbit around the barycenter of the solar system, then the actual orbital time of the near-side half of the outer planet's orbit is *½P*, and the actual orbital time of the far-side half of the same orbit is *½P*.

I. The Apparent Orbital Period of the Near-Side Half:

The near-side half of an outer planet's orbit starts from the point of maximum velocity of approach and ends at the point of maximum velocity of recession, as observed in the reference frame of the moving earth.

Let's assume that v denotes the orbital velocity of an outer planet.

Since sunlight is reflected from an outer planet, its incident velocity c, as computed in accordance with the ballistic assumption, is increased, upon reflection, by twice the orbital velocity of the outer planet, in question; i.e., $2v$, at the point of maximum velocity of approach

$$c' = c + 2v$$

where c' is the velocity resultant of sunlight, upon reflection from an outer planet .

Conversely, at the point of maximum velocity of recession, the incident velocity of sunlight is decreased, upon reflection, by twice the orbital velocity of an outer planet:

$$c' = c - 2v$$

where c' is the velocity resultant of sunlight, upon reflection from an outer planet, at the point of maximum velocity of recession..

And as a result, if d is the distance between an planet outer and the observer on the earth, then the total travel time of sunlight, reflected from that outer planet at the point of maximum velocity of approach, can be calculated by this equation:

$$t_A = \frac{d}{c'} = \frac{d}{c+2v}$$

where t_A is the total travel time of reflected sunlight, from an outer planet, at the point of maximum velocity of approach, to observers on the earth.

And in a similar manner, the total travel time of sunlight, reflected from an outer planet at the point of maximum velocity recession, can be obtained by using this equation:

$$t_R = \frac{d}{c'} = \frac{d}{c-2v}$$

where t_R is the total travel time of reflected sunlight, from an outer planet, at the point of maximum velocity of recession, to the planet earth.

Subsequently, the apparent orbital time of the near-side half of an outer planet's orbit, T_{nr}, is always given by the following equation:

$$T_{nr} = \frac{1}{2}P + (t_R - t_A) = \frac{1}{2}P + \frac{4d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where P is the actual orbital period of an outer planet.

It follows, therefore, that the apparent orbital period of the near-side half of an outer planet's orbit, P_{nr}, can be calculated in accordance with this formula:

$$P_{nr} = 2T_{nr} = P + \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and the outer planet under consideration.

II. The Apparent Orbital Period of the Far-Side Half:

The far-side half of an outer planet's orbit starts from the point of maximum velocity of recession and ends at the point of maximum velocity of approach, as seen from the moving reference frame of the earth.

Let, once again, v stand for the orbital velocity of an outer planet.

Because sunlight is reflected from an outer planet, its incident velocity c, as calculated in on the basis of the ballistic assumption, is increased, upon reflection, by twice the orbital velocity of that outer planet; i.e., *2v*, at the point of maximum velocity of approach

$$c' = c + 2v$$

where c' is the velocity resultant of sunlight, upon reflection from an outer planet.

By contrast,, at the point of maximum velocity of recession, the incident velocity of sunlight is decreased, upon reflection, by twice the orbital velocity of an outer planet:

$$c' = c - 2v$$

where c' is the velocity resultant of sunlight, upon reflection from an outer planet, at the point of maximum velocity of recession..

And therefore, if it's assumed that d is the distance between an outer planet and Earth, then the total travel time of sunlight, reflected from that outer planet at the point of maximum velocity of approach, can be computed through the use of this equation:

$$t_A = \frac{d}{c'} = \frac{d}{c + 2v}$$

where t_A is the total travel time of reflected sunlight, from an outer planet, at the point of maximum velocity of approach, to observers on the earth.

Similarly, the total travel time of sunlight, reflected from an outer planet at the point of maximum velocity of recession, can be obtained by using this equation:

$$t_R = \frac{d}{c'} = \frac{d}{c-2v}$$

where t_R is the total travel time of reflected sunlight, from an outer planet, at the point of maximum velocity of recession, to the planet earth.

And accordingly, the apparent orbital time of the far-side half of an outer planet's orbit, T_{fr}, is given by the following equation:

$$T_{fr} = \frac{1}{2}P + \left(t_A - t_R\right) = \frac{1}{2}P - \frac{4d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where P is the actual orbital period of an outer planet.

And it follows, therefore, that the apparent orbital period of the far-side half of an outer planet's orbit, P_{nr}, can be calculated in accordance with this formula:

$$P_{fr} = 2T_{fr} = P - \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between the earth and an outer planet.

III. The Ballistic Precession of Outer Planets' Orbits:

As in the case of inner planets, observations of any outer planet, in the near-side half of its orbit, are consistent with a longer apparent orbital period; while observations of the same outer planet, in the far-side half of its orbit, are consistent with a shorter apparent period than the actual orbital period of that outer planet.

And as a result, the true planetary orbit appears to rotate, by a certain amount, in the direction of its orbital revolution, based on observations gathered from the near-side half of the orbit; and in the opposite direction, as deduced from observations gathered from the far-side half of the same orbit.

However, the calculated amounts of anomalous orbital shifts, in the case of outer planets, are considerably minute for two main reasons:

- The values of orbital velocity v, for outer planets, decreases with increasing distance from the barycenter of the solar system.

- The angle θ between the orbital velocity vector and the observer's line of sight approaches 90^o with increasing distance from the barycenter of the solar system.

And consequently, the values of maximum velocity of approach and maximum velocity of recession approach 0 as the orbital velocity gets smaller, and the angle θ approaches 90^o with increasing distance from the barycenter of the solar system.

Nevertheless, it is always possible, on the ballistic assumption, to obtain the time difference between the apparent orbital period and the actual period of any outer planet, through the use of this equation:

$$\Delta P = \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d the distance between the earth and an outer planet, as defined by this relation:

$$d = \sqrt{d_e^2 + d_p^2}$$

in which d_e and d_p denote the distance of the earth and the distance of an outer planet from the barycenter of the solar system; and both form the two sides of a right triangle with the barycenter of the solar system at the time maximum orbital velocity of approach and maximum orbital velocity of recession, respectively.

Also the velocity variable v, in the above equation, is defined in terms of the angle θ within the same right triangle, in accordance with the following formula:

$$v = v_p \cos\theta$$

in which v_p is the mean orbital velocity of an outer planet; and θ is the angle between the orbital velocity vector of an outer planet and the line of sight of an observer on Earth.

And since maximum velocities of approach and recession occur only when the earth and an outer planet form a right triangle with the barycenter of the solar system, the value of $cos\theta$, in the above equation, is readily computed by using this trigonometric relation:

$$\cos\theta = \frac{d_e}{\sqrt{d_e^2 + d_p^2}}$$

where d_e is the distance of the earth from the gravitational center of the solar system; and d_p is the distance of an outer planet from the same gravitational center; at the time of maximum velocity of approach and the time of maximum velocity of recession, respectively.

And, therefore, the above time-difference equation can be rewritten in this more convenient form:

$$\Delta P = \frac{8d_e}{c}\left(\frac{v/c}{1-\left(\frac{4v^2/c^2}{1+d_p^2/d_e^2}\right)}\right)$$

in which d_e has always this numerical value:

$$d_e = 1 \text{ AU} = 149597870700 \text{ m}$$

where AU stands for the astronomical unit.

And consequently, the numerical value of the anomalous precession of an outer planet's orbit in seconds of arc per century $\Delta\phi$, as computed on the basis of the ballistic velocity of light in vacuum, can be obtained by using the following equation:

$$\Delta\phi = \Delta P \times N_{rev} \times \omega \times 100$$

where N_{rev} is the number of an outer planet's orbital revolutions per year; and ω is the angular velocity of an outer planet in arc-seconds per second.

And it follows, correspondingly, that, in the case of Mars (the closest outer planet to Earth), if we insert the following observational data:

$$v = 13205.01 \ \ \mathrm{ms^{-1}}$$
$$p = 59355072 \ \ \mathrm{s}$$
$$\mathrm{d} = 272686020431.4 \ \ \mathrm{m}$$
$$\mathrm{N_{rev}} = 0.5317 \ \ \mathrm{rev./year}$$
$$\omega = 0.02183 \ \ \mathrm{arc\text{-}s/s}$$

into the following formula:

$$\Delta\phi = \left(\frac{8d_e}{c}\right)\left(\frac{v/c}{1-\left(\dfrac{4v^2/c^2}{1+d_p^2/d_e^2}\right)}\right)\left(N_{rev}\times\omega\times 100\right)$$

we obtain, for the anomalous precession of Mars' orbit $\Delta\phi$, this numerical value:

$$\Delta\phi = 0.372 \quad \text{arc-seconds per century}$$

as calculated in accordance with the ballistic assumption.

The calculated values of anomalous precession, for the rest of outer planets, $\Delta\phi$, on the basis of ballistic velocity of light in vacuum, are listed in the last column of the following table:

Planet	*Period p (s)*	*$v_p cos\theta$ (ms^{-1})*	*Distance d (m)*	*N_{rev} (rev/yr)*	*ω (arc-s/s)*	*Δϕ ("/cen.)*
Jupiter	*374335689.6*	2464.98	*792603472835.4*	0.0843	0.00346	0.005072
Saturn	*929596608*	994.71	*1441231849108.1*	0.0339	0.00139	*6.013×10^{-4}*
Uranus	*2651218560*	353.69	*2876172256906.7*	0.0119	*4.89×10^{-4}*	*5.269×10^{-5}*
Neptune	*5200329600*	180.6	*4497904475047.9*	0.0061	*2.49×10^{-4}*	*1.098×10^{-5}*
Pluto	*7824384000*	118.26	*5908018234677.8*	0.0040	*1.66×10^{-4}*	*4.129×10^{-6}*

Table: ***Ballistic Precession of Outer Planets***

With regard to the calculated predictions of planetary precession, on the basis of ballistic velocity of light in vacuum, the following points should be pointed out explicitly and made clear:

- The numerical values of ballistic precession, as computed in the reference of the earth, are significant only in the case of the two inner planets: Mercury and Venus.

- Although it's possible that the inclusion of orbital eccentricity and axial velocity, in more precise calculations, may, well, increase its numerical values slightly, the computed ballistic precession of outer planets is generally very tiny and insignificant.

- One of the two most striking aspects of ballistic precession, as demonstrated in this discussion, is the linear increase of its numerical values with increasing distance between the reference frame of the observer and the barycenter of the solar system or the barycenter of any other system in general.

- The other remarkable aspect of ballistic precession, as in the case of the solar system under discussion, is the counterclockwise precession of the near-side half of a planetary orbit by a certain amount, and the clockwise precession of the far-side half of the same planetary orbit by exactly the same amount.

- An integral part of ballistic precession, which was the main topic of debate between W. de Sitter and M. la Rosa during the early decades of the *20th* Century, is the increasing apparent orbital time of the near-side half of an orbit and the decreasing apparent orbital time of the far-side half of the same orbit with increasing distance between the observer and the binary system in question ***[P. #A]***. However, it seems clear, from their published papers on binary stars, that neither W. de Sitter nor M. la Rosa noticed that the sum of these two apparent orbital times is always equal to the actual orbital period of the celestial body under consideration.

- What is expected to happen after the apparent orbital time of the near-side half of an orbit becomes equal to the whole actual orbital period, and the apparent orbital time of the far-side half of the same orbit becomes equal to nil? As far as measuring observers are concerned, the the ballistic precession of the orbit, in question, should appear to them to start numerically from an absolute value of zero, and then to increase directly with distance as before. But this time around, the ballistic precession is in the opposite direction; and the apparent orbital time of the far-side half of the orbit increases, and the apparent orbital time of the near-side half of the same orbit decreases with increasing distance between the observer and the observed system. But the sum of these two apparent orbital times remains always equal to the actual orbital period of the celestial body under observation.

8. *Concluding Remarks:*

Is it possible, in accordance with the assumption of ballistic speed of light in vacuum, to carry out calculations of anomalous precession, with regard to planetary perihelia, in the reference frame, in which the barycenter of the solar system is at rest?

In principle, it is possible to perform such calculations; but their numerical values ought to be nil or very close to nil; since those planets, by their very orbital arrangements, have little or no amount of radial velocity component towards or away from their collective gravitational center.

And the same should apply, of course, to the orbital motion of the Moon around the barycenter of the Earth-Moon system, which also exhibits little or no velocity of approach or recession with respect to the common gravitational center of the earth and the Moon.

There is one more relevant question related to the anomalous precession of a celestial body that belongs entirely to a different category of its own:

Does the orbit of the Sun, around the barycenter of the solar system, show any measurable amount of ballistic precession, as observed in the moving reference frame of the earth?

Theoretically, the orbit of the Sun around the gravitational center of the solar system must have, in the ballistic calculations, at least, a non-zero value of anomalous orbital advance; but it's highly unlikely to be measured under any conceivable circumstances, because its computed value ought to be extremely tiny for two main reasons:

1. The mean value of the Sun's orbital velocity, around the barycenter of the solar system, is very small compared to the orbital speeds of the planets:

$$v_s \approx 12.55 \quad \mathrm{ms^{-1}}$$

   where v_s is the mean orbital velocity of the Sun around the barycenter of the solar system.

2. Because the Sun is an actual emitter of sunlight, and not a reflector of sunlight as in the case of every planet, the ballistic velocity of light is increased only by *1v*, not by *2v* as in the case of

an approaching planet, during maximum velocity of approach; and it's decreased by *1v*, and not by *2v* as in the case of a receding planet during maximum velocity of recession.

And therefore, all things being equal, the anomalous precession of the emitting Sun's orbit is always less by a factor of two than that of a reflecting planet's orbit; i.e.,

$$\Delta\phi = \left(\frac{4d_e}{c}\right)\left(\frac{v_s/c}{1-\left(\frac{v_s^2/c^2}{1+d_s^2/d_e^2}\right)}\right)\left(N_{rev}\times\omega\times 100\right)$$

where $\Delta\phi$ is the anomalous precession of the Sun's orbit; and d_s is the distance of the Sun's center of mass from the gravitational center of the solar system:

$$d_s = 0.005 \text{ AU} = 747989353.5 \quad \text{m}$$

For the above two major reasons therefore, the anomalous precession of the Sun's orbit, around the barycenter of the solar system, does not exceed, under any conceivable circumstances, by a significant amount the following numerical value:

$$\Delta\phi = 2.4388\times10^{-6} \qquad \text{arc-seconds per century}$$

where $\Delta\phi$ is the anomalous precession of the Sun's orbit around the barycenter of the solar system.

Now, once again, how exactly did the ballistic precession of Mercury's perihelion make a renowned and skilled specialist in celestial mechanics, like Urbain Le Verrier, think that there must be an inner planet named '*Vulcan*' between Mercury and the Sun?

Well, at first glance, the answer, to the above question, seems, even without taking the trouble to check for any historical evidence at all, straightforward and easy:

- Ballistic precession makes the interval of time spent by the planet Mercury, in the near-side half of its orbit, appear a bit longer than actually is; i.e., longer than its actual orbital time by an amount ΔT that varies directly with distance as determined by this formula:

$$\Delta T = \frac{4d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where d is the distance between Earth and Mercury.

And that, by logical necessity, means that the duration of every physical event related to Mercury, in this part of its orbit, appears longer than what actually is. And hence, the timing points of transit events such as ingress and egress, as done in the earth's frame of reference, imply an apparent orbital period, which is longer than the actual orbital period by an amount of time:

$$\Delta P = \frac{8d}{c}\left(\frac{v/c}{1-4v^2/c^2}\right)$$

where ΔP is the time difference between the apparent orbital period and the actual period of the planet Mercury.

- And accordingly, Urbain Le Verrier was bound to uncover a major discrepancy:

  If the timing points of Mercury's transit are projected mathematically one hundred years into the future, they converge collectively, as expected, on a specific orientation of Mercury's orbit in space. But, at the same time, if a sufficient sample of recorded Mercury's transits are statistically analyzed, they collectively converge on a different orientation of Mercury's orbit in space, which is, as a rule of thumb, always a head of the mathematically predicted orientation by an exact amount of seconds of arc in accordance with this relation:

$$\Delta\phi = \left(\frac{8d}{c}\right)\left(\frac{v/c}{1-4v^2/c^2}\right)\left(N_{rev}\times\omega\times 100\right)$$

where $\Delta\phi$ is the anomalous precession of Mercury's orbit.

Why is that? Because the former is based upon transit data measured during the longer apparent time of the near-side half of Mercury's orbit. While, by contrast, the representative sample of Mercury's transits corresponds always with the shorter actual time of the near-side half of the same orbit.

But exactly how? The planet Mercury, as observed in the reference of the earth, takes one actual orbital period to return to the same point of its orbit, because the ballistic apparent time of the near-side half and the apparent time of the far-side half of Mercury's orbit balance each other; i.e., the ballistic precession of planetary orbits is not secular. And therefore, the representative sample of Mercury's transits must follow the actual orbital period of Mercury, and not the apparent orbital time of the near-side half of its orbit.

- And subsequently, we can easily conclude that Urbain Le Verrier had no other choice but to assume the existence of the hypothetical planet Vulcan, in order to get rid of the aforementioned discrepancy between the two orientations of the planet Mercury in space.

  And that is it.

Finally, it should be pointed out, at the end of this discussion, that the treatment of the anomalous precession of planetary perihelia, on the basis of ballistic velocity of light, is much easier to carry out; and certainly it has a wider scope and range of applicability than those of all of the proposed theoretical schemes that have been proposed so far, in the published literature, for dealing with this particular phenomenon on the basis of different assumptions.

Obviously, the proposition of ballistic velocity of light in vacuum is a seemingly fundamental and somewhat drastic solution for the *43-arc-second-per-century* problem of Urbain Le Verrier's *200-year-plus* anomaly.

But, at the same time, it should be acknowledged, in this regard, that the ballistic-velocity-of-light proposition is, by no means, more visibly radical and extravagant than the modification of the square law of gravity by S. Newcomb, or curving abstract space and abstract time, altogether at once, by A. Einstein, to mention just a few drastic examples on this particularly highly inspiring topic.

Besides, the proposition of ballistic velocity of light seems to have a distinctly Copernican aura attached to it. In other words, the modern situation of a universally constant speed of light versus a regularly changing ballistic speed appears, on the face of it, to be eerily reminiscent of the old situation of a stationary Earth versus a moving Earth, during the middle ages and antiquity.

And furthermore, the notion of ballistic velocity of light in vacuum is not merely theoretical; and as a matter of fact, there is, actually, a substantial number of supporting experiments in its favor, the most important of which is the well-known Michelson-Morley experiment.

Certainly, one may, as well, explain away the null result of the famous Michelson-Morley experiment by using other alternatives. But as soon as one realizes that the notion of ballistic velocity of light explains, in a satisfactory manner, the reported result of that experiment, Occam's razor, naturally, will do away with many of those alternative interpretations in no uncertain terms and in no time at all.

In any case, it is not unrecommended, from any given scientific perspective, to seriously consider and thoroughly investigate all theoretically viable possibilities, within contexts like the current one. And surely, it wouldn't make Urbain Le Verrier's famous anomaly of Mercury's perihelion obsolete or less important to have one more new explanation added to the long list of its published interpretations.

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