MAXWELL'S EQUATIONS AND GALILEAN RELATIVITY

by: FREDERICK DAVID TOMBE,
21 Waterloo Park,
Belfast, Northern Ireland,
BT15 5HU, U.K.

We often hear that Maxwell's equations are not Galilean invariant. This, however, is based solely on the fact that they contain only partial time derivatives. If they were generalized to contain total time derivatives, then they would be Galilean invariant. Who has yet carried out experiments in a frame of reference travelling fast enough relative to the geocentric rest frame that the validity of such generalization can be questioned? It has not been done. One merely accepts the equations with partial time derivatives because that is the way in which they are presented in textbooks. But have all the factors been taken into consideration?

Consider the three equations:

$$\mathbf{F}/Q = -\nabla\phi = \mathbf{E} \qquad \text{(Electrostatic)} \qquad (1)$$

$$\mathbf{F}/Q = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = \mathbf{E} \qquad \text{(Magnetic)} \qquad (2)$$

$$\mathbf{F}/Q = \frac{1}{c}\,\mathbf{v} \times \mathbf{B} \qquad \text{(Magnetic)} \qquad (3)$$

where ϕ is the electrostatic potential and c is the speed of light. The vector **A** is the magnetic vector potential and is related to the magnetic induction vector **B** by the defining equation:

$$\nabla \times \mathbf{A} = \mathbf{B} \tag{4}$$

Equation (1) for the force per unit charge, **F**/Q, is an electrostatic equation. Equation (2) is a magnetic equation which applies when the particle being magnetically accelerated is initially stationary, but the magnetic source is either moving or contains an unsteady current, or both. Equation (3) is also a magnetic equation which applies when the particle being magnetically accelerated is moving with velocity **v** but the magnetic source is stationary and steady.

However, unlike both its magnetic counterpart, equation (2), and the electrostatic equation (1), equation (3) does not receive the title *electric field* **E** in modern day textbooks. But why not? There is no reason why the definition of field as a point function should not be generalized to relate to points in motion. Lorentz, the architect of equation (3) first presented it in **E** form.

Consider, for example the two magnetic equations together in **E** form:

$$\mathbf{E} = \frac{1}{c}\left[\mathbf{v} \times \mathbf{B} - \frac{\partial \mathbf{A}}{\partial t}\right] \tag{5}$$

Taking the curl of equation (5) leads to:

$$\nabla \times \mathbf{E} = \frac{1}{c}\left[\nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{\partial(\nabla \times \mathbf{A})}{\partial t}\right] \tag{6}$$

Using a vector identity and equation (4) we get,

$$\nabla \times \mathbf{E} = \frac{1}{c}\left[\mathbf{v}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{v}) + (\mathbf{B}\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{B} - \frac{\partial \mathbf{B}}{\partial t}\right] \tag{7}$$

The first term on the right hand side of (7) vanishes because of the standard result that $\nabla\cdot\mathbf{B} = 0$. The second and third terms both concern partial spatial derivatives of a velocity term, which in this case represents the velocity of a single particle. This represents a discontinuity and hence the second and third terms also vanish. We are now left with

$$\nabla \times \mathbf{E} = -\frac{1}{c}\left[(\mathbf{v}\cdot\nabla)\mathbf{B} + \frac{\partial \mathbf{B}}{\partial t}\right] \tag{8}$$

The convective and local terms on the right hand side of equation (8) sum to a total derivative term; i.e.,

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{d\mathbf{B}}{dt} \tag{9}$$

This is a total time derivative generalisation of Faraday's law of magnetic induction. As such, when the electromagnetic wave equation is derived, the wave speed becomes both relative to the moving particle and velocity dependent. Hence there is no infringement of Galilean relativity.

An observer may ask which frame of reference the v is relative to. At this stage the author will say no more other than that he believes in an elastic aether composed of a dense distribution of electrons and positrons which, like the atmosphere, is dragged along close to the Earth by gravity. It is the geocentric rest frame relative to which we measure v in Maxwell's equations.

The author remarks that he is not the first to observe that the correction of Maxwell's equations to include total time derivatives renders them Galilean invariant. In 1890, Heinrich Hertz [1] postulated total time derivatives to account for motion of the aether. To the author's awareness, only two other men in this century have independently arrived at a similar modification of Maxwell's relationships. They are T. E. Phipps Jr. [2] and Stanislaw Kosowski [3]. The interpretations of the physical significance of the total time derivative which these other investigators have made are different from the author's and also they differ from one another.

BIBLIOGRAPHY

[1] **Electric Waves,** by Heinrich Hertz, English translation by D. E. Jones, Macmillan, London, 1893; Dover, N. Y., 1962.
[2] Phipps, T. E. Jr., **First-Order Modification of Maxwell's Equations,** J. of Classical Physics, V. 2, #1, 1983, pp. 1 - 22.
[3] Kosowski, S., **Transformational Properties of the Maxwell Equations; Newtonian Dynamics Correct Lorentz Transformation as a Method.** Preprint of an unpublished(?) manuscript. Copies are probably available from the author: Ul. Pereca, Zm. 1214, 00-849 Warszawa, Poland.