and the surface-integral over the surface of the ocean, in each case north of $\theta=\alpha$ only.

Now

$$\int_0^{m\pi/s} h\zeta'\sigma a \cos s\chi \,.\, a \sin\alpha\, d\chi = \frac{m\sigma a^2 h \sin\alpha}{s}\int_0^{\pi} \zeta' \cos m\xi\, d\xi,$$

and we thus have the coefficient of a Fourier's cosine-series for ζ' along the parallel of co-latitude α. As m is at our disposal we can thus evaluate ζ' along any parallel of latitude when we know its values round the coast, and those of $\bar{\zeta}$ over the ocean.

Again, when

$$\frac{Z}{h} = \tfrac{1}{2}e^{-is\chi}\,\frac{\sin^s\theta Z_s^+/P_s - \sin^{-s}\theta Z_{-s}^-/Q_{-s}}{p_s/P_s - q_{-s}/Q_{-s}}$$

$$+\tfrac{1}{2}e^{is\chi}\,\frac{\sin^{-s}\theta Z_{-s}^+/P_{-s} - \sin^s\theta Z_s^-/Q_s}{p_{-s}/P_{-s} - q_s/Q_s}, \quad . \quad (6.31)$$

we have

$$Z = h\cos s\chi, \quad hU=0,$$

on $\theta=\alpha$, so that (3.4) will yield

$$\int h\zeta' N\,ds - \int_0^{m\pi/s} h\nu \,.\, h\cos s\chi \,.\, a\sin\alpha\, d\chi = i\sigma\iint \bar{\zeta} Z\, dS, \quad (6.32)$$

the integrals being taken as in (6.22). Again, we see that we can evaluate ν through its Fourier's series along any parallel of latitude.

LXII. *On Kinematics.*

By Charles L. R. E. Menges *.

THE following concerns the modern views related to electromagnetism. This subject being treated so clearly and concisely in Prof. J. H. Jeans' well-known excellent book †, a quotation therefrom will be the best beginning to attach my explanations.

Page 607 reads :—"In Fizeau's water-tube experiment, a stream of water was made to flow through a tube, its velocity of flow being u relative to the earth, and a ray of

* Communicated by the Author.

† J. H. Jeans, 'The Mathematical Theory of Electricity and Magnetism' 4th edition (Cambridge, at the University Press, 1923).

light was passed through the water in the direction of its motion. To an observer moving with the stream, the water would appear to be at rest, so that the light would be propagated relatively to this observer with a velocity u' connected with the refractive-index ν of the water by the relation (C=velocity of light in vacuo)

$$u'=\frac{C}{\nu}. \quad \dots \dots \dots \quad (1)$$

"According to classical laws of kinematics, the light ought to travel, relative to an observer at rest on the earth, with a velocity

$$u'+u \quad \dots \dots \dots \quad (2)$$

or

$$\frac{C}{\nu}+u. \quad \dots \dots \dots \quad (3)$$

"Fizeau found it possible to measure the actual velocity by an interference method and formula (3) was not confirmed. The formula

$$\frac{C}{\nu}+u\left(1-\frac{1}{\nu^2}\right) \quad \dots \dots \dots \quad (4)$$

was found to represent the velocity accurately both for water and other transparent media."

The generally admitted inference is :—

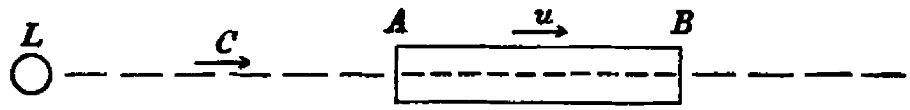
By applying the laws of kinematics to Fizeau's water-tube experiment, formula (3) is obtained, and this formula was not confirmed by experiments. Therefore those laws of kinematics are not true.

This conclusion is derived from results obtained by means of a moving *liquid*, water, which has obvious practical advantages for the experiment. But the theory, the kinematical insight of the question, is more intricate when the light-waves propagate in a *motionless* tube, wherein water flows, than when they traverse a moving *solid* transparent substance. It is therefore of importance, that formula (4) for flowing water has been very accurately confirmed indirectly by Prof. P. Zeeman's experiments on Fizeau's effect in moving *solid* transparent substances *.

The following concerns in particular Fizeau's effect in moving transparent *solids*.

* P. Zeeman, *Verslagen Akadem. Amsterdam*, xxvii. p. 1453 (1919).

In Zeeman's experiment the source of light L sends a ray through a glass cylinder AB moving with the velocity u relative to L. When the glass cylinder is still at rest, the frequency n^0 of the light-waves in it is the same as in the



source L. Then the light-waves propagate with velocity u^0 in the glass, its refractive index being ν^0 according to the relation

$$u^0 = \frac{C}{\nu^0}. \qquad (5)$$

We know that the refractive index of glass, when at rest, varies with the frequency of the light-waves. The frequency in the glass being n instead of n^0, the refractive index is by Lorentz's formula * :

$$\nu = \nu^0 + (n - n^0)\frac{d\nu^0}{dn^0}. \qquad (6)$$

When the frequency of the source L remains n^0, but the glass is moving with velocity u in the direction of the propagating light-waves, then the frequency in the glass is no longer n^0. By Doppler's effect it is then

$$n = n^0 \frac{C - u}{C}. \qquad (7)$$

Lorentz improved Fresnel's original formula with ν^0, by replacing ν^0 by ν from equation (6). The velocity u may be introduced in equation (6) by replacing $n - n^0$ by its value from equation (7). But it must be well understood that $\frac{d\nu^0}{dn^0}$ in equation (6) is not in the least related with movement; its value is taken from experiments *without any material movement*. Therefore, with ν according to equation (6) instead of ν^0, only a difference in frequency *apart from motion* is taken into account, but not that part of the observed total effect, which in particular results from motion.

To discuss the question clearly, three exactly defined cases will be considered.

CASE 1: The source of light L with frequency n^0 and the glass cylinder *at rest*; the frequency in the glass is also n^0.

CASE 2: *Without material movement* as in case 1, but the

* H. A. Lorentz, 'The Theory of Electrons,' p. 191; and note 69, p. 316 (B. G. Teubner, Leipzig, 1909).

frequency of the source L and in the glass AB both being now n instead of n^0.

Case 3 : *As in case* 2 the frequency of the light-waves is n *in the glass* AB, which, however, is now *moving* away from L with velocity u.

In this last case with the *moving* glass cylinder (Zeeman's experiment), the frequency in the source of light L must differ from n because of Doppler's effect. But immediate action at a distance is excluded, therefore the different frequency in L cannot *by itself* produce in the *distant* glass cylinder AB an effect differing from that in case 2. As experiment proves beyond any doubt that the optical effect in case 3 is really different from that in case 2, there must exist in the field, immediately in contact with the glass face A, in case 3 something which differs from that in case 2. Now, in case 2 *without material motion*, the velocity of the light-waves which fall on the face A is C. But in case 3 the face A *is moving* with velocity u relative to its position in case 2, where it receives the waves with velocity C. Therefore in case 3 the speed wherewith the *moving face* A *receives* the waves is *by kinematics* equal to $C-u$.

This being taken into account, I showed * by pure mathematics and kinematics, without introducing a "dragging coefficient" or any hypothesis of that kind, that ν in formula (1)—to make it kinematically correct for a moving substance be it water or glass—must be replaced by

$$\nu' = \nu - \frac{u}{C}(\nu^0 - 1). \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The true formula for u' in moving glass is then

$$u' = \frac{C-u}{\nu'} = \frac{C-u}{\nu - \frac{u}{C}(\nu^0-1)} \quad . \quad . \quad . \quad . \quad (1a)$$

It is now easy to explain the above given quotation: "Fizeau found it possible to measure the actual velocity and formula (3) was not confirmed."

For formula (3, namely

$$\frac{C}{\nu} + u, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

is obtained by inserting in formula

$$u' + u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the value of u' from

$$u' = \frac{C}{\nu}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* *Comptes Rendus*, clxxv. p. 574 (1922).

Formula (2) being taken in accordance with perfectly true laws of kinematics, then it will certainly become false, if the true value of u' be replaced by a value which is contradictory with said laws. This is the case with formula (3), because of the false value of u' from equation (1), which ought to be true for the *moving* glass. But in this sense C in it is certainly inconsistent with the laws of kinematics, as I above explained ; C must be replaced by $C-u$, giving the kinematically correct formula (1*a*).

Instead of the hitherto generally admitted inference that the laws of kinematics, as expressed in formula (2), are not correct, the true inference now is quite inverse. *For according to the laws of kinematics formula* (1) *must be replaced by formula* (1a), *and this is exactly confirmed by Zeeman's very accurate experiments.* It follows from the formula confirmed by Zeeman's experiments, his formula being obtainable from my new formulæ (1*a*) and (8) by pure mathematical transformations, as follows.

The glass cylinder of length l being at rest, the speed of the light-waves therein is $u^0=\frac{C}{\nu^0}$, and it contains

$$l:\frac{u^0}{n^0}=\frac{ln^0}{u^0}$$

waves. While the glass cylinder moves it contains

$$\frac{ln}{u'}$$

waves. The optical effect of motion is therefore

$$l\left(\frac{n^0}{u^0}-\frac{n}{u'}\right),$$

and with u' from formula (1*a*) it is

$$l\left(\frac{n^0}{\frac{C}{\nu^0}}-\frac{n^0\frac{C-u}{C}}{\frac{C-u}{\nu'}}\right)=\frac{ln^0}{C}(\nu^0-\nu').$$

With formulæ (6), (7), (8) and the wave-length $\lambda=\frac{C}{n^0}$ this optical effect is

$$\frac{lu}{C}\left(\nu^0-1-\lambda\frac{d\nu^0}{d\lambda}\right),$$

which is Zeeman's formula.