PHYSICS. — *On the theory of relativity and the experiment of Mr. Sagnac.* Note by Mr. **P. Langevin**.

Interesting remarks on the theory of relativity were presented recently per Messrs Painlevé and Picard. I will show later on how the difficulties raised by Mr. Painlevé are only apparent and how we can build on his criticisms.

I wish here to place myself from the purely experimental point of view and to recall that this theory is the *only* way which makes it possible currently to represent the whole of the known experimental facts and which has moreover the remarkable power of prediction confirmed so vividly by the deviation of the light rays and displacement of the spectral lines in the gravitational field of the Sun.

To show how much this synthesis is complete and to answer at the same time the desire expressed by Mr. Picard, I will show how the theory of generalized relativity explains, in a quantitative way, the result of the experiment of Mr. Sagnac and of gives at the same time the simplest interpretation and in conformity with the nature of the things.

One knows that Mr. Sagnac makes the interference of two light rays resulting from the same source after their having made to traverse, thanks to mirrors suitably placed, the same closed circuit in opposite directions. He notes that the setting in rotation with an angular velocity ω of the platform which carries the whole of the optical system produces a displacement of the fringes which corresponds to a difference $\frac{4\omega A}{c^2}$ between the time to course the same circuit in two directions, A representing the interior surface with the circuit projected on a normal level with the axis of rotation and c the speed of light.

Let us notice first of all that it is about a first order experiment (in $\frac{\omega R}{c}$, R being the radius of the platform), on which all the theories of optics, mechanics, electromagnetic or relativistic are in agreement, qualitatively and quantitatively, and cannot testify for or against any of them.

One must know, from any point of view, we cannot compare this experiment with that of Mr. Michelson. That one is of the second order according to the speed of traverse, and its importance holds so that it came to highlight in an acute way the need for introducing a new kinematics, imposed besides by the remarkable agreement between the equations of the theory of Lorentz and the whole of the electromagnetic and optical phenomena.

Although all the theories predict the result of Mr. Sagnac, one obtains it in the simplest way and most natural while placing oneself from the point of view of generalized relativity and by seeing there the influence on the light propagation of the field of gravitation particular to the observers related to the platform in rotation, the same field which appears mechanically by the effects of centrifugal force or gyroscopic.

This experiment, far from constituting a difficulty for the theory of relativity, thus provides an example of the more immediate applications.

The symmetrical character of the phenomenon of rotation, and in particular the fact that rotation changes direction with the orientation of the observer following the axis, requires that the running of clocks carried by the platform as well as the dimensions thereof for rulers associated with it could not be changed until the second order in $\frac{\omega \mathrm{R}}{c}$ compared to the running of clocks or the dimensions of the same rulers related to observers without rotation, these modifications should not change with the sign of ω. It is known for example that the elastic deformation of the platform and the apparatuses, deformations of which it would have to be held account if the precision could be thorough with the second order, do not depend on the direction of rotation.

It results from it that if we represent by (x, y, z, t) and (x', y', z', t') the coordinates space-time of the same event compared to rectangular axes dependant on the platform and axes without rotation respectively, the usual relations of old kinematics remain with the first order between these two frames of reference. The z and z' axes being both parallel to the axis of rotation, and the events origins in coincidence, one has

$$x' = x\cos\omega t - y\sin\omega t, \qquad y' = x\sin\omega t + y\cos\omega t, \qquad z' = z, \qquad t' = t.$$

The fundamental invariant, which for the observers without rotation has the usual Euclidean form

$$ds^2 = c^2 dt'^2 - dl'^2, \quad dl'^2 = dx'^2 + dy'^2 + dz'^2,$$

takes in the frame of reference linked to the platform the value obtained by substitution, and in which I neglected the terms in ω higher than the first order:

$$ds^2 = c^2dt^2 - 2\omega\left(xdy - ydx\right)dt + dt^2, \quad dl^2 = dx^2 + dy^2 + dz^2.$$

The propagation of a light ray, characterized by the condition $ds^2 = 0$, corresponds to the relationship

$$c^2dt^2 - 4\omega d\mathrm{A}\, dt - dl^2 = 0,$$

Or, $d\mathrm{A} = \frac{1}{2}\left(xdy - ydx\right)$ represents the surface of the triangle having the top origin of the coordinates and base projection on the pattern of x, y of the element of the light ray seen by the observers related to the platform.

This relation can be written, with the same order of approximation:

$$dt = \frac{dl}{c} + \frac{2\omega}{c^2} d\mathrm{A};$$

from where, by integration along a closed contour:

$$t_1 = \frac{l}{c} + \frac{2\omega\mathrm{A}}{c^2},$$

A representative area of the contour projected onto a plane normal to the axis of rotation.

For the ray which follows the same contour in opposite direction, the surface changes sign and one has

$$t_2 = \frac{l}{c} - \frac{2\omega\mathrm{A}}{c^2},$$

thus the difference $\frac{4\omega\mathrm{A}}{c^2}$ conforms to the experimental result of Mr. Sagnac.

With the general sense introduced by Mr. Einstein where the gravitational field is represented by all ten potentials g_{ik} the experiment of Mr. Sagnac measures the influence on the light propagation of the potentials g_{14} and g_{24} respectively equal to $2\omega y$ and to $-2\omega x$, and which only are modified *with the first order* by rotation.

It is the same for the effects of centrifugal force or gyroscopic composed in contrast to the effects of centrifugal force which is the static second order which corresponds to the potential g_{44} whose exact value is the second order $c^2 - \omega^2\left(x^2 + y^2\right)$.

This first order optical experiment is akin with the experiment of the pendulum of the Foucault pendulum or gyroscope and proclamation once more; since Newton, there is the opportunity to highlight the movement of rotation of a material system by experiments at the inertial system.