DISCUSSION

THE TIME-RETARDING JOURNEY AGAIN*

IN this rejoinder I must remind Professor Lovejoy again that we are dealing with *Einstein's* relativity. Einstein deduced his Special Theory from two postulates or "principles": the principle of the constancy of the velocity of light and the restricted principle of relativity. The latter, as stated by Einstein,[1] requires only that rates of *change* shall be describable by the same laws in any two unaccelerated systems. It says nothing as to inter-systemic interchangeability of *instantaneous configurations,* which being instantaneous do not "run their course".

The "classical principle of the relativity of motion", as understood by Lovejoy (p. 566), requires the interchangeability or reversibility of "*all* * relativity estimates". This means, for instance, that if, read from one system, a clock of this system registers $t = a$ and the coincident clock of another system registers $t' = b$, then, read from this other system, the first clock registers $t = b$ and the second $t' = a$. There is no such principle recognized either in classical mechanics or in relativity. For instance, when two meter rods, R and R', are moved relatively to each other in the direction of their length, the statement that at any moment the reading "22" of R, read from R, is opposite the reading

*Editor's Note: See "The Paradox of the Time-Retarding Journey" by Professor A. O. Lovejoy, this REVIEW, Vol. XL (1931), pp. 48 ff. and 152 ff.; discussion of the same by Professor E. B. McGilvary, pp. 358 ff., and "A Reply" by Professor Lovejoy, pp. 549 ff.

1 "The laws by which the states of physical systems *undergo change* * are not affected, whether these *changes* * of state be referred to the one or the other of two systems of coördinates in uniform translatory motion" (*Electrodynamics,* p. 41). "If relative to K, K' is a uniformly moving coördinate system devoid of rotation, then natural phenomena *run their course* * with respect to K' according to exactly the same general laws as with respect to K" (*Relativity,* p. 15).

Here as elsewhere throughout this paper starred italics are not found in the original texts quoted. Also I refer to cited works by abbreviated titles if, in the previous papers of Lovejoy or in my own previous paper, full titles have been given with identifying details. Reference to any of the three papers just mentioned will be by page only, with a prefixed "L." or "McG." when this is necessary. Since all these papers appeared in Vol. XL of this REVIEW, no confusion will result. Born's *Einstein's Theory of Relativity* is the work referred to when Born is mentioned. When I speak of "system" I shall mean an *unaccelerated system of rectangular Cartesian coördinates,* unless some further qualification is mentioned.

"35" of R' is not reversible, in the sense that at that moment the reading "35" of R, read from R', is opposite the reading "*22*" of R'. Now a clock is no more and no less a metrical instrument than a meter rod. If "relativity estimates" of the coincident readings of *meter rods* in relative motion are not "reciprocal", *i.e.*, reversible, why should "relativity estimates" of the coincident readings of *clocks* in relative motion be reciprocal? Coincident readings are *not* "rates of change", nor does any one pair of them "run their course". Any such pair is an *instantaneous configuration.*

If Eddington or any other relativist has said: "All relativity estimates are reciprocal" (L. p. 156), and *meant* thereby that estimates from the reading of an instrument on either of two systems, as to the reading of any similar instantaneously opposite instrument on the other system, are reciprocal, he was making an obviously untrue statement. It is therefore unlikely that he meant what Lovejoy thinks he meant. Be that as it may, the place to get any principle to which Einstein's relativity is subject is Einstein's own writings. Like any other mathematical theory, the Special Theory must be judged by its own postulates and its own equations. In relativity *no* "estimates are reciprocal" except those which the postulates and the equations of relativity say are reciprocal.

But Lovejoy does not judge relativity by its postulates and its equations. He judges it by what he is pleased to call "the classical principle of the relativity of motion". Every appeal he makes to this principle as requiring reversibility of coincident clock-readings is absolutely irrelevant. Delete from his two papers every such appeal and everything he deduces therefrom as bearing on the Special Theory, and you will have something that looks like a pacifist newspaper that has passed through the hands of a censor in time of war. On the other hand, my omission of any reference to that principle finds its parallel, not in "*Hamlet* with Hamlet left out" (L. p. 566), but in *Hamlet* with Falstaff left out. I have always preferred my *Hamlet* in this expurgated form.

II. My previous paper, with the exception of less than four pages, was devoted entirely to the relation of clocks and physiological processes in unaccelerated relatively moving systems. Whatever retardations such clocks underwent took place only during periods of unaccelerated motion. I therefore find it difficult to understand why Lovejoy should say (p. 557) that I failed "to meet, or even to mention, the consideration" which he had "presented as the decisive proof that . . . any . . . way of escape" from the contradiction involved in

the clock-paradox "is impossible". The gist of that "proof" by Lovejoy is contained in two assertions: "The whole paradox, in its symmetrical form, presents itself *during the outward journey,* before any such acceleration occurs" as is involved in a round trip (p. 164). "The mischief has already been done *before* * the acceleration occurs" (p. 557). On the contrary, by the use of Einstein's own equations I showed in my paper that *no* mischief was done "during the outward journey", and thus I met his contention squarely. Lovejoy does not directly attack my equational treatment, except in one place. In other places he dismisses my conclusions on the ground that they are "completely at variance with the orthodox relativistic doctrine" or that they are "self-contradictory". I shall examine these contentions before I am done.

III. His direct attack (pp. 564–566) upon the way I used my equations is made in the following passage:

> *They are made from the point of view of only one of the travellers . . .* [*viz.,*] *only from the point of view of Paul or his cousin.* But Peter is equally entitled, in a world of relativity, to make measurements on his own platform. . . . The computation of Paul's cousin's age from Peter's point of view McGilvary fails to make. This astonishing omission is apparently due to a confusion between computing times *for* both platforms and computing them *from* both platforms. With all his equations . . . McGilvary has too few by half, since in none of the specific cases with which he deals does he apply this formula [*i.e.,* the Lorentz transformation] bilaterally.

If this means anything, *it means that a mathematical transformation does not truly transform.* I had thought that *correct* transformation is just what a correct *transformation* actually accomplishes. The Lorentz transformation does consistently transform, in the sense that, starting from measurements made in *either* system, the transformation enables one by calculation to ascertain *truly* the corresponding measurements as they are (or would be) *actually* made in the *other* system.[2]

[2] Let us consider a single example. On Paul's platform the point P is, by hypothesis of actual measurement on that platform, $7c\sqrt{91}$ miles in the positive x-direction from A, the origin of that platform (*i.e.*, for P, $x = 7c\sqrt{91}$); and by hypothesis of actual observation on that platform, the coincidence of P with P' takes place at time $t = 0$ by the clock at P. I have shown that the Lorentz transformation gives the time of this coincidence by Peter's clock at P' as $t' = -212\ 1/3$ (p. 364). Now by use of the equation $x' = \beta(x - vt)$, substituting therein the above values of x and t, and the values $\beta = 10/3$ and $v = c\sqrt{91}/10$ used in my illustration, we get x' for P' as $x' = 70c\sqrt{91}/3$. Thus, calculating *from Paul's platform,* we get *for Peter's measurements* the distance of P' from A' as being $70c\sqrt{91}/3$, and the reading of Peter's clock there when P and P' coincide as being $t' = -212\ 1/3$.

IV. A familiarity with the use of the Lorentz transformation would have saved Lovejoy from another mistake here, and thus from using an argument to prove that I am wrong when the equations prove that I am right. He says: "But the distances M–A and A–P, from whichever platform determined, are *by hypothesis* * equal, as are the distances M'–A' and A'–P'" (p. 565). *This is not true.* On the contrary, the distance M–A, measured on either platform is 3 1/3 times the distance A–P, measured on that platform,[8] and the distance A'–P', measured on

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But of course there *is* some point on Peter's platform which is at the positive x-distance $70c\sqrt{91/3}$ from A', *i.e.*, some point whose $x' = 70c\sqrt{91/3}$, and is thus the point P' as that point is defined above; and there is *some* time when the clock at that point has the reading $t' = -212\ 1/3$. The question is what a computation made *from Peter's platform* says as to the point on Paul's platform coincident with P' ($x' = 70c\sqrt{91/3}$) at time $t' = -212\ 1/3$ by Peter's clocks; also what such a computation says as to the reading of Paul's clock at that coincident point. To answer this question we take the transformation equations used on Peter's platform, *viz.*, $x = \beta(x' + vt')$ and $t = \beta(t' + vx'/c^2)$, and make substitutions in them of the values of x' and t' just given, and of the values of β and v given in the preceding paragraph of this footnote. Thus we get $x = 7c\sqrt{91}$ and $t = 0$. But this point ($x = 7c\sqrt{91}$) is the point P, and this time ($t = 0$) is the time $t = 0$ at which Paul says that P' with its clock-reading $t' = -212\ 1/3$ is at P.

Thus whether we start *from* measurements made on *Paul's* platform and compute *for Peter's* or *from* measurements made on *Peter's* and compute *for Paul's*, we get the result that P ($x = 7c\sqrt{91}$) and P' ($x' = 70c\sqrt{91/3}$) coincide at time $t = 0$ by Paul's clocks and at time $t' = -212\ 1/3$ by Peter's clocks. To say this in another way: when my "omission is repaired and the transformation equation for time is applied from both sides, there manifestly results a proof of the conclusion which McGilvary has endeavored", *not* "with elaborate irrelevance to controvert" (L. p. 566), but with elaborate relevance to establish. It is "astonishing" that any one who has ever used the Lorentz transformation should make the mistake that Lovejoy here makes. No wonder that, not knowing what is meant by a "transformation", and not knowing how to use the Lorentz transformation, Lovejoy should usually shy at my equations and jump on my conclusions with both feet. No one who has not actually used that transformation and thus seen that it consistently transforms has a right to criticize a theory that is expressed in that transformation.

[8] The distance A–P, measured on *Paul's* platform is by hypothesis (McG. p. 364) $7c\sqrt{91}$, as is also the distance M'–A', measured on *Peter's* platform (p. 366). From the fact that by hypothesis (p. 366) M is coincident with M' ($x' = -7c\sqrt{91}$) at time $t' = 0$, the distance M–A, measured on *Paul's* platform, is obtained by use of the Lorentz transformation equation $x = \beta(x' + vt')$, from which, by substituting the above values for x' and t' and the values of β and v from footnote 2 above, we get for M $x = -70c\sqrt{91/3}$. Since the Lorentz transformation correctly transforms, we learn that the distance M–A, measured on Paul's platform is $70c\sqrt{91/3}$ and thus 3 1/3 times

either platform, is also 3 1/3 times the distance M'–A', measured on the same platform.

For the relativist's abashment Lovejoy concludes (pp. 565–566) from the false statement quoted above:

> Thus two persons . . . travelling on the same platform with the same velocity, cover *what for them are equal distances* * in the same direction in unequal times, one being 21 and the other 70 at the end of his journey, by the clocks on their own platform.

This conclusion would be entirely correct *if* the words I have italicized were replaced by *a true statement of the facts, viz.,* "what for them are distances unequal in the ratio of 3 to 10." But with this replacement there is nothing to shame the relativist. Lovejoy has here employed "the method of alternating options", the alleged use of which by relativists he finds so offensive. He opts the measure, as made on *Paul's* platform, of the distance A–P, and the measure, as made on *Peter's* platform, of the distance M'–A'. Finding these measures, thus made on *different* platforms, to be equal, he triumphantly reports that they are equal "from *whichever* * platform determined". What logic! No wonder that, not recognizing "the method of alternating options" when he himself actually employs it, he should characterize as "the method of alternating options" a *different* method used by the relativist. It is Lovejoy who "handles his equations in so original a manner", when he ventures to touch equations at all.

V. Lovejoy recognizes the necessity of defending his conclusion "that Peter and Paul's cousin will—if retardations actually occur as

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the distance A–P, measured on that same platform. I will not take the space to prove the incorrectness of the rest of Lovejoy's statement quoted above.

I had thought that "the first thing taught children in '*really* relativistic nurseries'" (L. p. 566) is that distances measured on either platform are found contracted when measured on the other platform. In our case the factor of proportionality is $1/\beta = 3/10$. Hence if the distance M–A *after* reduction on *Peter's* platform, is $7c\sqrt{91}$, it must have been, *before* such reduction, $70c\sqrt{91}/3$ on *Paul's* platform.

One cautionary remark is appropriate. Although M is 10/3 as far from A as M' is from A', each measured on its own platform, the measure of the distance M'–A' as obtained on *Paul's* platform is not 10/3 as great as that obtained on Peter's; on the contrary it is 3/10 as great. The reason for this is that in getting the former measure the end-points of M'–A' are located simultaneously by *Paul's* clocks and not by Peter's, just as on Peter's platform the end-points of M–A are located simultaneously by *Peter's* clocks and not by Paul's. A correct use of the Lorentz transformation requires the use of the simultaneity of the correct platform. The reader can easily get the correct results by using the correct method.

physical facts—each be younger than the other at the same place" (p. 559). For proving this he admits that it is "necessary to establish only two propositions (1) that Peter and Paul's cousin *could* be born simultaneously; (2) that, according to the Special Theory, the comparative retardation arising while they continuously approached one another would be bilateral and equal". I shall take up these two propositions in order. Of the first he says:

> It can be shown by two distinct arguments that the relativist must admit that the births of Peter and Paul's cousin could be simultaneous, whether the judgment upon the question were that of an observer on Peter's platform, or of one on Paul's.
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> *i.* In the first argument Paul serves, so to say, as *a logical intermediary* * between his cousin and Peter. For an observer at A', on Peter's platform, the births of Peter and Paul are simultaneous; he *sees* them both occurring together at the place where he is. He also *knows* that indefinitely many other events on Paul's platform must be occurring *simultaneously with Paul's birth* *; *i.e.*, that *the synchronized clocks on that platform all show the same reading when that event occurs* * . . . one of these events may conceivably be the birth of *a** cousin of Paul's at some point, P, to the eastward of A . . . McGilvary explicitly tells us that . . . "the difference between 'simultaneity at the same place' and 'simultaneity at a distance' is not a difference in the connotation of 'simultaneity'", but only in "denotation." If this is assumed, the first argument clearly follows . . . it is an application to our hypothetical case of the principle: If two events are in the same sense simultaneous with the same event they are simultaneous with each other.

V*a*. There are two fallacies in this argument. The first is that any proof that *a** cousin of Paul's may be born at P at the time mentioned is no proof that such a possible cousin is *the* cousin of Paul's who was by hypothesis born at P at time $t = 0$ by Paul's clocks. To avoid confusion hereafter, let us call this latter cousin "Zebedee". But this *ignoratio elenchi* is not the fallacy on which I wish to dwell. I refer to the *petitio* latent in Lovejoy's argument, which *assumes* a simultaneity-at-a-distance that is common to the two platforms, *in order to prove the possibility* of such simultaneity. This can easily be seen. If the observer at A' is a *relativist,* he knows that *simultaneously with Paul's birth,* as simultaneity is determined on *Paul's* platform by clocks synchronized by light-waves, many other events are occurring on that platform. He also knows that *simultaneously with Peter's birth,* as simultaneity is determined on *Peter's* platform by clocks synchronized by light waves,[4] many other events are occurring on Paul's platform.

[4] In any system distance-simultaneity is the time-relation between any two events such that at the places of these events clocks in that system, synchronized by light waves, have equal readings on the occurrence of these events. This is the invariable connotation of "simultaneity at a distance" in the Special Theory.

But he also knows that these two groups of "other events" are *not identical.* Paul's group includes all events that occur at time $t=0$ by Paul's clocks, and thus includes Zebedee's birth which occurred at time $t=0$. Peter's group includes all events that occur at time $t'=0$ by Peter's clocks, and thus *does not include* Zebedee's birth, which occurred at time $t'=-212\ 1/3$, as is shown by the fact that the Lorentz transformation gives this as the time of Zebedee's birth by *Peter's* clocks (McG. p. 364).[5] Only on the assumption that what occurred at time $t=0$ at P on Paul's platform also occurred at time $t'=0$ on *Peter's* platform at coincident P', does Lovejoy get his conclusion. The observer at A' also knows that synchronized clocks on Paul's platform all show the same reading $t=0$ at the time of Zebedee's birth, as 'time' is determined on *Paul's* platform, *not* as 'time' is determined on *Peter's* platform. He knows what is contained in the affirmative part of the immediately preceding sentence because he recognizes a tautology when he sees one. If, in concession to Lovejoy's preference, he is allowed to think, he knows what is contained in the negative part of that sentence, because he can correctly use the Lorentz transformation. To use Lovejoy's own words with his own italics: "There is no reason why an *intelligent* observer on Peter's platform should be supposed to be incapable of recognizing this" (p. 560). But when this is recognized the "logical intermediary" ceases to intermediate.

V*b*. Lovejoy's second and "more direct" argument for proposition (1) is that the relativist must admit "*simultaneity between events on two different systems*" (pp. 560–561); otherwise he would be "denying the possibility of relative motion. . . . There must be a 'common while' if there is to be a relative motion; and this common while must contain common moments." Of course the relativist admits all this. There *are* for the relativist "common moments" on the two platforms: the births of Peter and Paul or the coincidence of A and A' is one such common moment, and the arrival of Peter at P is another (although the latter has not the same *date* by the clocks on the two platforms). The "common while" here is the time-interval between these two "common moments." The relativist does not deny the community of this "while"; he merely denies that this "common while" has the *same measure* on the two platforms. As I said in my previous paper: "You and I, travelling together, may agree that we passed 10 milestones *while* your watch was ticking off 15 minutes and mine 14"

[5] I assume that Zebedee was not like William James's daughter, who was "born in rapid succession".

(p. 374). In this case we have a "common while with common moments"; but we lack a *common measure* for this "common while"; yet nobody thinks that this lack makes our motion relative to the milestones impossible. Why should a similar lack in relativity make a relative motion impossible?

Thus Lovejoy fails to establish one of the propositions that he declares it "necessary to establish" before it can be proved that the births of Peter and Paul can be simultaneous on both platforms with the birth of Zebedee. The first argument toward establishing it is an *ignoratio* and a *petitio*; the second is irrelevant.

VI. We proceed now to the examination of Lovejoy's proof of proposition (2), which he declares it is necessary for him to establish. He acknowledges that the establishment of this presupposes the establishment of proposition (1): "We have, *then*,* Peter and Paul's cousin [Zebedee] born 'at the same time' at A' and P respectively" (p. 562), which in the context means born at the same time, as 'time' is determined on *Peter's* platform as well as on Paul's. Since the proof of proposition (2) admittedly depends on the prior establishment of proposition (1), and since the latter proposition has not been established, the former proof has no relevance here. But it is so "curious" that I must ask the reader's indulgence while I examine it.

Proposition (2) is, as we recall, "that, according to the Special Theory, the comparative retardation arising while they [Peter and Zebedee] continuously approached one another would be bilateral and equal". Lovejoy does not have to establish this proposition against the relativist; it is one of the best-known articles of the relativist's creed. But Lovejoy does not understand the meaning of this article: he labors under a most insistent and most amazing confusion between the comparative retardation ("lagging") of a clock[6] and the comparative smallness ("slowness") of any of its instantaneous readings. A relativistic clock "lags", not when it is *momentarily more slow*, but when through a time-interval it *runs more slowly*. The fact that my clock is running fast (or slowly) as compared with another does not imply that, at any one time, it has a greater (or smaller) reading than the other. Yet Lovejoy argues *from* the fact that in relativity there is a reciprocal retardation or lag *to* the conclusion that there is a reciprocal

[6] "As a consequence of its motion the clock *goes more slowly** (*geht . . . langsamer*) than when at rest" (Einstein, *Relativity*, p. 44). Its going more slowly is a rate of change and thus falls under the restricted principle of relativity. Its being more slow at any time is not a rate of change and therefore does not fall under that principle.

slowness of coincident readings at any instant! For instance (pp. 562–563):

> But it is implied by the usual version . . . that the mutual lag . . . will *actually occur on the respective platforms* . . . of Peter and Paul's cousin. And *therefore**, when Peter and Paul's cousin presently meet, each traveller will *be* physiologically younger than the other and each clock will show a record of fewer elapsed hours than the other. And this is *equivalent to saying** that the situation when Peter and Paul's cousin meet will be precisely that which McGilvary says is not implied by the theory—*"a mutual inversion of unequal coincident clock-readings" and physiological ages.**

The argument here is from a mutual *lagging* to a *mutual* comparative slowness. Surely no logician except Lovejoy (or, for that matter, no common-sense man who does not claim to be a logician) would say that the following reasoning is correct: "My clock *runs* more slowly than yours; therefore it *is slower* now". Any familiarity with the ordinary run of every-day clocks would show that there is something wrong with such an argument, for experience often shows that slow-running clocks are sometimes fast (and fast-running clocks sometimes slow). Until Lovejoy can show *by a correct use of the Lorentz transformation* that the Special Theory asserts that there is "a mutual inversion of unequal clock-readings", his proposition (*2*), *interpreted as he interprets it, as a statement about coincident clock-readings,* is utterly ungrounded. In my previous paper, on the contrary, I showed in detail (pp. 369–372) by the use of the Lorentz transformation that "it is *because* the clocks at coincident" points "have unequal and non-interchangeable readings . . . that the clocks of either system *run slowly** [*i.e.*, "lag"] as measured by the synchronism of the other". I distinguished between non-reasoning observers and reasoning accountants just for the sake of bringing out the distinction between *observed* non-interchangeable clock-readings and *calculated* retardation. I trust that all my other readers saw the point. Lovejoy did not; but neither does he lay a finger on any specific error in my calculations. Instead of this, he blandly says: "If McGilvary thinks that any different consequence can be drawn from the Special Theory [*viz.*, thinks that clocks on relatively moving systems do not lag], he has Einstein and the other mathematicians against him, and his quarrel is with them" (p. 562), an assertion as true, as irrelevant, and as misleading, as an assertion that if Lovejoy thinks that color is a physical quality of physical objects, he has all the epistemological dualists against him and his quarrel is with them. Just as Lovejoy does not entertain such a thought, so I did not deny that the clocks mutually lag. On the con-

trary I asserted that they do mutually lag. I denied that their coincident readings are therefore interchangeable.

VII. Again Lovejoy says (p. 563):

> One of the most curious confusions in McGilvary's paper is to be seen in his remarks on the question of the "physical reality" of the retardations. This *question* * is, he declares, "ambiguous"; in fact, he suggests, it loses all meaning when we remember that "retardation, like largeness, is always a respective characteristic".

But I *didn't* say that the "question" is ambiguous. I said that *a quoted statement of Lovejoy's* on this question was ambiguous; and it *was* ambiguous, as will soon appear. Nor did I make any such suggestion as Lovejoy says I made. It is possible that my *remark* may have suggested to an epistemological dualist that, since largeness is a respective characteristic and therefore (?) is non-physical in epistemological dualism, and since retardation is like largeness, retardation is non-physical. But happily *I* am not responsible for any such "suggestion". On the same page, however, he seems later to have remembered what it was that I *did* call ambiguous; and then he claims that what I thus called ambiguous "is perfectly unambiguous".

> It means precisely what McGilvary expresses when he correctly says that . . . "it is agreed by all relativists that Peter's clock at *A'* ticked, and Peter's heart beat, *21* times from his birth till his arrival at *P*, and that the heart of Paul's cousin [Zebedee] beat, and Paul's clock at *P* ticked, *70* times from *this* cousin's birth till Peter's arrival at *P*." The (supposed) fact that Peter's clock ticked and his heart beat *21* times instead of *70* *is* the retardation.

But it *isn't*. In the case before us, that "(supposed) fact" is a *necessary* condition of the retardation, but not a *sufficient* condition. Another condition is necessary, *viz.*, that Peter and Paul's cousin Zebedee should have been *born at the same time*. Now in relativity, as we have seen, they were born at the same time by *Paul's* clocks, but *not* by *Peter's* clocks. This is the answer to the second question Lovejoy proceeds to ask: "Wherein, also, lies the respectivity of the assumed retardation?" The answer to the first question—"Where, then, is the ambiguity in the former proposition, or in [is?] the distinction between it and the latter?"—is that *both are equally ambiguous*, the latter more clearly so because stated more explicitly: it claims sufficiently to identify retardation by a characteristic that it shares with faster-running and with running at the same rate.

VIII. In my previous paper I began by charging Lovejoy with scrambling his eggs. He replies (in substance, but of course with more dignity): "Not guilty; it was the relativist fellow that did it".

Fortunately this reply furnishes us with an admitted *corpus delicti*: the eggs *were* scrambled. Let us see what in detail Lovejoy says the relativist did, and what in detail the relativist actually did do. Lovejoy says (p. 549) that the relativists presented a tale of a time-retarding journey "as a corollary of the Special Theory", and defended it "from the specific standpoint of that theory". In a footnote he refers to Born by page and adds:

> There are other examples, which I have not at hand where I write. It is true (as noted [in his previous paper] p. 159) that those who first deduce from the Special Theory *alone* * both the implication of a time-retarding journey, and also an ostensibly sufficient reply to the objection that the comparative retardation should be bilateral, usually add, when later setting forth the General Theory, that it is in this that the decisive reply to the objection is to be found.

I take it that Born, the only relativist here mentioned, is one of the relativists whom Lovejoy charges with actually doing this. The passage (Born, pp. 214–216) to which Lovejoy refers occurs in a context in which Born is discussing the Special Theory. It is too long for full quotation, but I must quote at some length from it.

> In the sequel we shall draw numerous *deductions* * from Einstein's theory . . . But here we wish to deal with an *argument* * which leads to particularly remarkable results. . . . We are referring to the so-called "clock-paradox".

The "argument" here mentioned may at first glance possibly be thought to be just one of the "deductions from Einstein's theory", *i.e.*, as Lovejoy puts it, "a corollary of the Special Theory". On the other hand, it is possible that Born did not intend it to be so regarded. We shall see. But meanwhile let me continue my quotation, intentionally garbling it, but also indicating by bracketed numerals the places where the garbling occurs.

> Let us consider an observer A at rest at the origin O of the [1] system S. A second observer B is at first to be at rest at the same point O, and is then to move off with uniform velocity along a straight line, say the x-axis, until he has reached a point C, when he is to turn round and return to O along a straight line with the same velocity. [2] this effect [*i.e.*, the effect of the acceleration involved] may be neglected. But then the clock of the observer B must have lost time compared with the clock of A after B's return to O. For . . . during the periods of B's uniform motion, which are the determining factors for the result, the proper time lags behind the time of any other [3] system.

VIII*a*. Taking this passage, even as it stands thus quoted, one might possibly object that "the Special Theory has *logically nothing to say* on the retardation involved in a journey *to and fro*. It makes no as-

sumptions whatever as to the effects of acceleration" (McG., p. 358). *Therefore,* no one, in discussing this theory, has a right to consider any case in which there *is* acceleration. Such a conclusion, however, would be a *non sequitur*. We must make a distinction between what a *theory* says, and what *some one discussing that theory* correctly says. My remark (pp. 358–359) about scrambling the eggs had reference to what Lovejoy said was "assumed (*in the Special Theory*)"*, not to what *Born* assumed in the passage under discussion. Take a parallel case: the Newtonian theory of gravitation has *logically nothing to say* about the effects of air-resistance on falling bodies. These effects have to be discovered by experiment; they cannot be deduced from the theory *alone*. But a physicist, in discussing that theory, may and often does consider a case in which a falling body encounters such resistance, and then say that, *if* that resistance is negligible, the gravitational equation $s = gt^2/2$ holds approximately for such a case. The theory of gravitation, having nothing logically to say about air-resistance, has *no denial logically to make* of the physicist's assertions, so long as he does not say something that contradicts what the theory *does* logically say. The same relation holds between what the Special Theory alone says and what a physicist says in discussing that theory. Let us see what *Born* says, when what is omitted at "[2]" in the foregoing quotation is restored.

> Let both observers carry with them ideal clocks which indicate their proper-time. The times lost in getting started, in turning round, and in slowing down on arrival at *B* [*O*] can be made as short as we please by making the times occupied in moving uniformly there and back sufficiently great. If, say, the rate of the clocks should be influenced by the acceleration, this effect will be comparatively small *if** the times of the journey are sufficiently great. . . . But *then** the clock of the observer *B* must have lost time compared with the clock of *A* after *B*'s return to *O*.

VIII*b*. It is clear, when attention is called to the words I have italicized, that Born does not deduce his results "from the Special Theory *alone*"* (L. p. 549) but from that theory *plus* the assumption that in *certain* cases the effects of acceleration are negligible; nor does he call his results a "deduction" (p. 216) *till this additional assumption has been made and utilized.* (This, I suggest, accounts for the change in the earlier passage from "deductions" to "argument". It also accounts—this is not merely a suggestion—for my saying (p. 359): ". . . or until we explicitly make some further assumption, we are *not* concerned" with a round-trip.) We thus see that Born did not confuse the Special Theory *alone* with the Special Theory *plus* an ad-

ditional assumption necessary for his "deduction". In any mathematical discussion no postulate that is employed may be ignored. But Lovejoy, in speaking of this "deduction" as "a *corollary* * of the Special Theory" deduced "from the Special Theory *alone*" *, has failed to distinguish, *i.e.*, he has "scrambled", what Born left unscrambled. This, however, is not his only scrambling.

VIII*c*. A much more serious garbling of the passage I have quoted from Born was my omission at "[1]" and "[3]" of just one word. That word was "inertial". Now, what is an "inertial system"? To answer this question we must first consider what the law of inertia is. Early in his book (pp. 27–28) Born says:

> In our experience we do not know of bodies that are really withdrawn from all influences from without. . . . For the present, then, we shall interpret the law of inertia in the restricted sense *in which Galilei meant it.**

He illustrates this law by a sphere resting at first on a smooth horizontal table and then given an impulse when

> it will continue to move in a straight line and will lose only very little of its speed. This retardation was called a secondary effect by Galilei, and it is to be ascribed to the friction of the table and the air. . . . The law of inertia is at any rate confirmed for motion on the table. It has been established that *in the absence of forces the velocity remains constant in direction and magnitude.** Consequently *the forces will be associated with the change of velocity, the acceleration.** In *what* * way they are associated can again be decided only by *experiment.**

This is "the law of inertia in its original [Galilean] form", by which later (p. 62) Born defines an "inertial system":

> We must now alter the terms used in our definition of the principle of relativity, for in it we still spoke of a coördinate system at rest in absolute space, and this is clearly without sense physically. To arrive at a definite formulation the conception of *inertial system* . . . has been introduced, and it is taken to signify a coördinate system in which *the law of inertia holds in its original form.** There is not only the one system at rest as in Newton's absolute space . . . but *an infinite number of others.** . . .

Thus in relativity as interpreted by Born, an inertial system is one whose velocity relative to an infinite number of systems remains constant in direction and magnitude *because of the absence of forces.* This definition still holds—it has not meanwhile been modified—when in the same chapter with (and less than twenty pages before) the passage Lovejoy criticizes, Born derives the Lorentz transformation. On beginning this derivation he says (p. 198):

We once more repeat the hypothesis of Einstein's *kinematics.** 1. *The Principle of Relativity.*—There are an infinite number of systems of reference (inertial systems) moving uniformly and rectilinearly with respect to each other, in which all physical laws assume their simplest form (originally derived for absolute space or the stationary ether).

But Lovejoy will have none of this (pp. 61–62); he says:

If the effect *is* unilateral, then . . . *either** the two systems are not in merely relative motion, *i.e.*, the so-called inertial system (the earth) is in a state of rest in an absolute space or with respect to some absolute reference body; *or else** the dislocations of objects on Paul's system are *not* consequences solely of the reversal of direction of relative motion with respect to the earth. The former alternative would take us back to the *Newtonian universe**: the latter might conceivably be brought under the well-known hypothesis of Mach about the nature of inertial forces. Paul's system would be affected differently from Peter's because its relations to the matter composing *the rest of the universe** would be different. It is unnecessary to ask here which alternative is preferable . . . it is assumed (in the Special Theory) that the *only** motion in question *is* the relative motion between the two systems. . . .

Could there be a clearer case of *petitio?* Lovejoy in criticizing Born uses a premise ("either . . . or") in direct contradiction to the principle under which Born was operating, which is *neither* Lovejoy's "either" nor Lovejoy's "or"[7]. Does Lovejoy do this because he has not read the whole book, or even the whole chapter, from which he takes a passage to criticize? or because in his eagerness to find fault he has forgotten what he has read and what if remembered would have given an entirely different meaning to the passage?

Immediately preceding the passage just quoted from Lovejoy, he mentions an argument urged in support of the unilateral character of the effects of acceleration: Paul's projectile, being shot from the earth, "*will* be jolted," and his clocks also.

The fundamental objection to this argument is . . . that the jolting could not conceivably be confined to Paul's system *if it were assumed that the acceleration (change of direction) is merely relative to Peter's system,* and therefore reciprocal. . . . To say that it is so confined would (upon these assumptions) be to say that precisely the same phenomenon . . . produced a given effect in one case and no effect in the other . . . when the other conditions remain invariant.

Of course. *If* the acceleration is merely a change of direction (or

[7] It is not "the matter composing the rest of the universe" that is under consideration in the motion of an inertial system. It is such matter as is in any of the infinite number of inertial systems with respect to which any inertial system under consideration moves uniformly. Whatever matter belongs to any other system can be dealt with in connection with the Special Theory only by *referring its changes to the systems that are by hypothesis inertial.*

of velocity or of both), then Paul cannot undergo any *such* change with respect to Peter without Peter's undergoing *ipso facto* a symmetrical change with respect to Paul. Thus *if* the jolting is due to *that kind of acceleration,* it must be reciprocal. But Einstein's kinematics, as interpreted by Born, whom Lovejoy is criticizing, presupposes, as we have seen, that its unaccelerated reference-systems are *inertial, i.e., not acted on by external forces,* and that *accelerated systems are acted on by such forces.* Lovejoy in the last-quoted passage presupposes that the Special Theory assumes that *purely spatial characteristics* (changes of direction, etc.) do the work that Born (with Einstein) attributes *to external forces.* Born had said, when treating of inertial systems (p. 62): "We here see clearly how intimately the problem of space is connected with mechanics. It is not space that is there and that impresses its *form* on things, but *the things and their physical laws determine space.*" * A force is thus some physical *thing* that acts on some other physical thing according to physical laws; and an "accelerated" system is not merely one in which there are *just* speeds increasing (or decreasing), or just directions changing with the lapse of time. It is one in which these increases and these changes are determined by *things acting as forces.* And an inertial system is one on which things are *not* acting as forces.

As Born says, these "forces will be associated with the change of velocity, the acceleration. In what way they will be associated can again be decided only by experiment." Now it does not require a pre-relativist with his absolute space nor an astronomer with his celestial lore, to discover "by experiment" whether, when a glass of water is spilt on the table, it was the glass or the table that was "associated" with the force at work. Neither does it require a Newton or a Mach to discover "by experiment" whether, when the observer B gets parted from "the origin O of the inertial system S", it was B whose muscles (or vehicle) produced the separation or whether the system S slipped from under B's feet in a gigantic landslide. It is only in the *General Theory* that, when there is acceleration, *either* of the systems involved may be considered as inertial and the other as non-inertial. To appeal to a "reciprocity" of accelerated systems when criticizing a relativist who is dealing with a problem under the *Special Theory* (considered in connection with an additional assumption), is to "scramble the eggs" as Born does not scramble them. In the Special Theory there is no "reciprocity" between an inertial system and an accelerated system. In fact, Born expressly warns against such scrambling (p. 216), and Lovejoy has read the warning (L. p. 57); but he heeds it

not. His insistence that the characteristic feature of his first paper was its scrupulous care in keeping the eggs intact only proves his happy inability to recognize what logical scrambling is when performed in dealing with relativity.

VIII*d*. Lovejoy thinks it "a topsy-turvy sort of logic" that permits Born to attribute the retardation to *B*'s clock, which has been accelerated, and to withhold it from *A*'s, which has not been accelerated, in spite of the fact that "the occurrence of a retardation is inferrible only from the Special Theory", which "applies exclusively to inertial, *i.e.*, unaccelerated, systems" (p. 57). Here Lovejoy confuses a *reference-system* with *what is referred* to that system. When the tracks of a railway together with the land on which they lie are the reference-system of a railway, a passenger does not say, when suddenly jolted out of his berth at night, that it is the tracks and the land that suffered the jolt. The topsy-turviness belongs to the logic of the critic who should maintain that the traveller's shaken nerves should be attributed to the tracks, since the tracks are the reference-system used in the time-tables.

VIII*e*. But suppose that the effects of an acceleration are *negligible*. Does this not carry with it the supposition that we must also ignore the fact that there is acceleration that is due to forces? Lovejoy seems to require an affirmative answer when he says (p. 62):

> we must still conclude . . . (b) that if there *were* any retardation—*i.e.*, if the acceleration in question were treated as theoretically negligible—the comparative retardation inferrible would necessarily be reciprocal.

Whatever Lovejoy may mean here, the sentence as it stands reminds me of what I once heard Tommy say. Tommy had grossly slandered Jimmy, who in consequence had lost caste with the gang. But Jimmy was a real Christian and, when he next saw Tommy, told him that he forgave him, saying that the mischief done didn't matter. [He would have said it was "negligible" if he had known or remembered this word.] Tommy met Jimmy more than half way. "Fine!" he shouted. "Then we'll say it was *you* who lied about *me*, and that makes us all even." Tommy had evidently been taken by his mother to hear a popular lecture on relativity.

VIII*f*. If Lovejoy really wants "reciprocity" and "reversibility" between the clocks of *A* and *B*, he can get it, provided he grants symmetry to the conditions under which they separate and reunite. (No one can get something for nothing in physics, not even in relativity-physics.) To see this, let us make more specific the way in which in

Born's set-up B moved from O. Let us say that in Born's set-up B moves from O in the *positive* x-direction relatively to the inertial system S, and returns in the negative x-direction; and let us call this "set-up (1)". Now (2) let us reverse this set-up, by having A move from O in the inertial system S, and by having B left stationary at O. A's motion from O is to be in the *negative* x-direction, till he reaches a point D, whose distance from O is equal to the distance of C from O in set-up (1). At D, A is to turn round and go back to O in the positive x-direction. In all other respects this second set-up is to be similar to the first. By Born's reasoning, it is A's clock that on his return to O has lost time compared with B's—of course mere difference in direction makes no difference in the results of motion.

Now let us combine the two set-ups. *Both A and B* are to set out from O, leaving a clock stationary at O. (There is no limit in relativity to the number of available clocks.) B is to go from O in the positive x-direction as in set-up 1, and A in the negative as in set-up 2, each returning in the respectively opposite direction. When they meet again at O, the clocks of *both* (again by Born's logic) will have lost time compared with—*what? With each other? No! Both clocks will have lost time equally, compared with the clock left stationary at O.* Thus Lovejoy will have his "reciprocity" and "reversibility" since the relation of equality is symmetrical, and can be reversed and reciprocated without a single bat of even a meticulously logical eye.[8]

In my former paper, instead of assuming that the results of acceleration are negligible, I assumed that the acceleration involved had "*no* * disturbing effects . . .[9] other than that after" the acceleration physical

[8] If Lovejoy, in his examination of Einstein and other "mathematical expositors of the theory", has found that they all "agree that *in any possible case* of such a journey", travellers, born at the same place and time, and returning to that place after wayfaring, "would *not* be 'of equal age physiologically'" (p. 553), this can only be because he has not read these mathematical expositors *mathematically*. The very equations (or Minkowski diagrams) these expositors use in discussing a to-and-fro journey in connection with the Special Theory are, by hypothesis, applicable only to journeys with respect to some inertial system. When two travellers move symmetrically with respect to such a system they will undergo similar changes with respect to clocks in that system. In reading mathematical writers, it is always profitable to pay attention to their definitions, their postulates, and their equations, and not merely to *some particular instance* in which their reasoning is illustrated.

[9] Lovejoy rightfully disowns the "violent supposition" I imputed to him. This imputation was a mistake, which I regret and for which I sincerely apologize. I must assume sole responsibility for the supposition. Fortunately for my argument, if not for my accuracy, the mistake does not impair the logic of my discussion.

processes should "carry on just at the same rate" as required by their new uniform velocity (p. 375). The reason I employed this assumption was that I thought that by it I could more definitely eliminate everything except what happens during periods of uniform motion. In the case where only one traveller reversed his direction I got exactly the same result Born got. In the case where both travellers reversed their directions I got the result Born would have got had he assumed that both A and B moved away from the origin O of the inertial system S, their motions relative to that system being completely symmetrical.[10] When Lovejoy says that my "equations yield a result precisely opposite to that to which the equations of Einstein and other mathematical physicists lead them, on this point" (p. 554), his assertion can be condoned only on the assumption that he does not know to what those equations actually do lead. Will Lovejoy by the use of equations show the correctness of his charge?

VIII*g*. But there is still another objection that Lovejoy makes to my conclusions: I get "opposite results for . . . two instances of precisely the same kinematic phenomenon" (p. 555). The reason he says that there is no "kinematically relevant difference" in the two cases is that the "relative speed" in the two cases is equal. I am willing to grant for the sake of argument (since the point is irrelevant) that in classical kinematics the two instances would be identical in character. But I was and still am "unaware of my self-contradiction" because I thought and still think that I was dealing with *relativistic* kinematics. Any kinematics is precisely stated only in terms of its definitions, its postulates, and its equations. It so happens that there are very great differences between relativistic and classical kinematics. Now whatever classical kinematics may say of our two instances, relativistic kinematics has to maintain that where only one traveller reverses his direction relatively to another system with respect to which he has previously been in unaccelerated motion, we have an entirely different kinematic situation from that in which two travellers thus reverse their direction. It must be remembered that the basic relative speeds that the Special Theory deals with are speeds relative to some inertial system or systems. When Peter takes John's platform to get back to Paul (McG. pp. 375–376), his speed relative to the inertial platform he

[10] By referring to Born's Minkowski diagram (p. 215) the reader can see that Born would have got the result I got. Draw in that diagram to the left of the t-axis another dotted line, symmetrical with the dotted line there shown, the symmetry being with respect to the t-axis. The two dotted lines are thus equal, and what is true of Born's own line is also true of the line now added.

has left is (by the relativistic Theorem of the Addition of Velocities) $2v/(1+v^2/c^2)$, whereas Paul's speed relative to his own platform remains zero. When both Peter and Paul reverse their directions by transfer each to the other's platform (pp. 274–275), the speed of each relative to the platform he has left is v. A mere glance at these numbers will suffice to convince any one that the two instances are not kinematically identical. Here again Lovejoy is misled by his belief that what he calls "the classical principle of the relativity of motion" controls every relativistic transaction. If Lovejoy could only bring himself to try to *understand* relativity before using his "critical faculty" on it American philosophy would be much 'forwarder'; for all of us recognize the keenness of that faculty when it has chosen to exercise itself on appropriate material, *i.e.*, on what it understands. When on the contrary it tries its teeth on what it does not understand, it degenerates into mere cavil.

IX. I must repeat what I have been constantly emphasizing. One cannot understand a mathematical theory without approaching it *as* a mathematical theory, *i.e.*, as a theory with its own definitions, its postulates, and its equations. Such a theory says just what and only what its equations say, not what a critic says who approaches it with other postulates and other definitions and without equations. The reason I have been interested in calling attention to Lovejoy's critical methods in his dealing with the mathematical theory of relativity is that his criticism of this theory is motivated by a desire to erect his epistemological dualism on its ruins. The last paragraph of his "reply" shows this clearly to any one who may not have already discovered it from the reading of *The Revolt against Dualism*. Now criticism of a philosophical view is often criticism that cannot be brought to book for the reason that what is criticized cannot be so precisely formulated in all its details as can be done with a mathematical theory. But criticism of a mathematical theory is different. A critic who cannot handle the logic of a mathematical theory he attacks can hardly be trusted as competent to deal with a philosophical view whose logic is in many respects that of the mathematical theory. Not being an epistemological dualist in Lovejoy's unrestricted meaning of that term, I consider it fortunate that he has ventured so unhappily into the mathematical field where the limitations of his logic become manifest. In dealing with the present-day variety of philosophical views, a critic needs, not the ability to make two hairs grow (by splitting) where only one grew before, but a sympathetic discernment, which is only etymologically suggestive of splitting.

The reader will have guessed why I have declined to follow Lovejoy into the General Theory. Either he does not, or I do not, understand the *Special Theory*. In either case the one who does not understand can hardly be expected to contribute to a profitable discussion of a more advanced and more difficult theory which presupposes the one misunderstood.

P.S. After having corrected the proofs of this article I received those of Lovejoy's reply which follows. I had not seen this reply before, and it is obviously impossible now to do justice to it. However I will select two or three points at random to comment on.

If my argument "divagates into several side-issues," that is because these issues were brought forward as critical in Lovejoy's previous papers.

Lovejoy lays down as Proposition IV: "The two clocks may legitimately be supposed to be set in motion, and the two persons born, at the same time." As applied by Lovejoy to Peter's birth at A' and Zebedee's at P, these points not being coincident, this proposition asserts what the Special Theory denies, *viz.*, a simultaneity at a distance common to two relatively moving systems. He justifies his assertion by saying: "there is no reason why a clock at P' should not be *set* going with the same reading as the clock at P, when these clocks are coincident." Of course one *can* set a clock going with any reading, but if one is to set it so as to be synchronous with some other clock at some other place, there is only one reading that one *may* give it. Lovejoy's arbitrary choice of setting, from which he gets his desired result, is at variance with relativity, and therefore his result does not show any inconsistency in relativity. Lovejoy's proof that there is a common simultaneity at a distance for the two platforms is given in just one sentence: "And $t = 0$ cannot be the same moment as $t' = 0$ at A (which $= A'$) but a different moment at P." This assertion when applied to both platforms is an assertion of what relativity denies, and therefore does not disprove the correctness of that denial. Lovejoy says: "McGilvary's argument for the non-simultaneity of their births [i.e., the births of *Peter* and *Paul*] has therefore no relevance." Neither has this statement any relevance, since I have never denied the simultaneity of *these* births. On the contrary my argument assumes that they are simultaneous on both platforms.

And so forth and so forth.

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