DISCUSSION

"THE PARADOX OF THE TIME-RETARDING JOURNEY"

PROFESSOR Lovejoy's article in this REVIEW (this volume, pp. 48–68, 152–167) calls for extended discussion, more extended indeed than is possible within the limits of a single paper.[1] I therefore shall confine myself mainly to what he says about the retardation of clocks and of senescence in the Special Theory of Relativity. This theory has to do only with systems in relative motion with constant velocity. With such motion there is of course no return journey. But I must touch here and there upon the problem of a return journey, since Lovejoy mixes this problem up with that involved in unaccelerated motion, as will be seen from the first quotation below. Some scrambled eggs have to be unscrambled—if we are to get live eggs out of the scramble.

In speaking of the return journey he says:

> it is assumed (in the Special Theory) that the only motion in question *is* the relative motion between the two systems, and also that any effects . . . of the acceleration (if there are any such effects) will be due solely to the change in this relative motion, *viz.*, to the change of the direction of the motion of each system with respect to each other. Any *such* effect, then, resulting from the acceleration should be the same on both systems (p. 62).

It cannot be too strongly emphasized that the Special Theory has *logically nothing to say* on retardation involved in a journey *to and fro.* It makes no assumptions whatever as to the effects of acceleration. It deals only with journeys to an undiscovered country from whose bourne no traveller returns. Einstein did, indeed, in 1905 draw a conclusion from the Special Theory as to what would happen if a traveller did get back therefrom. But there was nothing in his equations at that time to warrant this conclusion, and this is now generally recognized by those who are familiar with them. But let us continue our quotation:

> There is, it is true, a tendency among physicists, even when expounding the Special Theory, to drag in a third (and essentially an absolute) reference-body,

[1] Since in that article Lovejoy makes no reference to another which he recently published in the JOURNAL OF PHILOSOPHY (Vol. XXVII, Nos. 23 and 24), I assume that in the former he is not building upon conclusions he thinks he has established in the latter. The two articles seem to be logically independent. All page-references in my paper are to the article in this REVIEW.

> as an explanation of the postulated absoluteness of acceleration. [This shows the danger a critic of the Special Theory incurs when he relies on non-mathematical expositions of that theory, instead of going back to its equations.] This conception, however, has a place—if anywhere—only in the General Theory; its use in that theory will be considered later. Until such a conception is explicitly introduced and justified, we are concerned only with the motion which is a private affair between Peter and Paul;

the latter having made a stellar journey and returned (with physiological complications, so Lovejoy thinks) to his twin brother. Until the General Theory is introduced, I must repeat, or until we explicitly make some further assumption, we are *not* concerned (except perhaps emotionally) with this private affair between principals dealing with each other under conditions for which the Special Theory makes no provision. But if we do interfere, Lovejoy is more correct than he realizes when he goes on to say:

> and from this point of view, *i.e.*, that of the Special Theory, we must still conclude (a) that no comparative retardation whatever can be deduced for the case of a journey in which there is a reversal of direction.

He is correct because no conclusion whatever, either about comparative retardation, or about comparative *non*-retardation, on reversal of direction, can be deduced from the Special Theory alone. But he commits a fallacy and arrives at a false conclusion when he goes on to say:

> but (b) that if there *were* any retardation—*i.e.*, if the acceleration in question were treated as theoretically negligible—the comparative retardation inferrible would necessarily be reciprocal (p. 62).

Here he reaches a result that is comparable with that of a critic who should argue that, *if* Lovejoy were, along with all his present views, also an epistemological monist, he would be a most self-contradictory philosopher. Obviously this conclusion would be false, but no more so than Lovejoy's, as will be shown before this discussion is concluded.

But the contention that I wish especially to discuss is put forth in the following passage, which I must quote quite at length. "The paradox of the twins—in its symmetrical form—arises even though no reversal of Paul's motion, and no acceleration whatever, is supposed to occur." We are to

> imagine Peter to be on a flat platform extending as far as we please in either direction, and Paul to be on a similar platform immediately adjacent to Peter's and in uniform unaccelerated motion relative to it and parallel with it. If, *while the two were at rest, synchronized clocks* and automatic cameras were placed at intervals along the inner edges of both platforms, the event of any

reading of any one of Peter's clocks will be simultaneous with the reading of any clock of Paul's which may be passing, for both clocks will be in this case virtually in the same place.

Now, "in order to avoid any complication involved in getting the motion started", we are to make a further supposition:

> Peter and Paul, not now brothers, must be supposed to have been born simultaneously at points A and A′ when these points on the two platforms were passing each other, and each remains throughout at the place of his birth on his own platform. In both directions from A and A′, on both platforms, observation-posts are placed at wide intervals; at each of these assistant observers are stationed, duly provided with *clocks originally synchronized.* It is the law on each platform that no one can be appointed an assistant observer unless he was born *at the same time as Peter and Paul.* Assume that 70 years have elapsed on Peter's platform up to the moment when he passes observation-post P′ on Paul's platform. Given sufficient velocity, he, an old man of 70 gazing at his *coeval*, the assistant-observer at P′, will seem to that observer to be a young man of 21; and assuming, as is done in the customary story, that *a retardation observed from one system is a physical fact on the other*, Peter will *be* [Lovejoy's italics] twenty-one as well as seventy. At the same time his *coeval* at P′ will appear to Peter to be 21, and will therefore be of that age, as well as of the age of seventy. We thus eliminate any acceleration, and therewith all attempts to evade the symmetrical form of the paradox by invoking accelerations (pp. 63–64).

The reason why I have italicized certain words in this passage is that in one case they are ambiguous and in the others they signalize the points at which Lovejoy introduces into the problem features that do not appear in it as it is faced in relativity-physics. *They insinuate into the relativistic problem a Newtonian postulate.* Reasoning from this Newtonian postulate, innocently smuggled among the relativistic assumptions that are left intact, it is no wonder that Lovejoy gets a result that is neither Newtonian nor relativistic, but a jumble of contradictions.

The statement that "a retardation observed from one system is a physical fact on the other" is at best ambiguous. Any retardation of a clock on its platform is of course a physical fact, and this clock's retardation as measured on the other platform is a physical fact. But if the clock is a good Einsteinian one, its retardation is not a retardation with respect to the time-system of its own platform; its being a good clock is a guarantee against that. Its retardation is with respect to the time-system of the platform relatively to which it is in motion. The kind of retardation under consideration is analogous to largeness. A man of average size may be large. His largeness is a largeness where he happens to be; he could not well be large where he was not, any more than he could be of average size where he was not. But just as his being of average size is a characteristic he has with respect to the standard size of his own people, so his being large is a characteristic he has with respect, let us say, to the standard size of a race of pigmies. Retardation, like largeness, is always a relative (or, if Lovejoy prefers, a respective) characteristic. Anything that changes too slowly changes too slowly (*where it is*) in comparison with some *other* change (*where that is*) which is used as the measure of the rate. But this retardation is not a physical fact *where that other change is occurring.*

The first italicized words, and the word "originally" next italicized, show that Lovejoy wishes to have had the clocks on the two platforms synchronized while they were relatively at rest. If this is done and the clocks are afterwards left untouched, acceleration is introduced into the transaction, for when the platforms are eventually set in motion there is of course acceleration. *The clocks in the Special Theory can be properly synchronized only when the relatively moving systems are moving with unaccelerated velocity.*

The word "coeval", appearing twice, and the phrase "born at the same time as Peter and Paul", taken in the context in which they appear, seem to indicate that simultaneity of birth is tested by clocks synchronized while the platforms were relatively at rest. But leaving this aside, a relativist would call attention to the fact that in relativity-physics any babies born (at different places along the line of relative motion) at the same time (by Paul's clocks) as Peter and Paul, are not "coeval" with any babies born (at different places along the line of relative motion) at the same time (by Peter's clocks) as Peter and Paul. Here then the problem is set by Lovejoy in terms of a *universal simultaneity*, which is directly at variance with the relativistic application of the conception of simultaneity to events at a distance from each other. This is as if a critic, after surreptitiously introducing a Euclidean postulate among those of Riemann, were to show that from all the postulates now on hand contradictory conclusions followed.

The fact that a philosopher with Lovejoy's reputation for carefulness can fall into such fundamental mistakes about a theory he is criticizing makes it desirable to go somewhat into detail in stating just what the Special Theory maintains with regard to the retardations of clocks and of other physical processes. In order to give an accurate picture of what we are to consider, it will be necessary to introduce measure-numbers into our discussion. In doing this, I shall give the results of calculations in the text, putting the calculations into footnotes.

In this way a reader who is interested only in the picture need not bother about the calculations upon which the picture is based; and the more exacting reader can check up the calculations. Any one who has had a few weeks of high-school algebra can easily follow them. One who has never used the Lorentz transformation can, by reading the footnotes, get some idea of the technique of its employment.

Let us take Lovejoy's general set-up, consisting of two straight platforms of indefinite length, moving relatively to each other and always keeping a constant clearance between them sufficient to prevent disaster; for they are to have the uniform speed of 177,425 miles per second.[2] Peter's platform is to move relatively to Paul's in what we shall take to be the positive direction—let us say eastward. Hence Peter's velocity v is positive, *i.e.*, $v = 177{,}425$. All the clocks on the two platforms are to be such that, *if* they were relatively at rest, they would run at exactly the same rate, and of course that rate would be such that light *in vacuo* would have the velocity $c = 186{,}000$ miles per second.

Distributed as they are in our set-up, the clocks on each platform are to be synchronized by light-signals in the well-known way, while the platforms have the constant relative speed v. Any reading of 'Paul's clocks' (*i.e.* clocks on Paul's platform) we shall designate by t. Any reading of 'Peter's clocks' (*i.e.* clocks on Peter's platform) we shall designate by t'. On either platform every point with which we shall deal is to have, not only a clock, but also a milestone, marking its distance, measured on that platform, from the 'origin' of that platform, *i.e.* from the point on that platform from which all distances on that platform are to be measured. The origin of Paul's platform we shall designate as A, that of Peter's platform as A'. The reading of any of Paul's milestones we shall designate as x, that of any of Peter's milestones as x'. When one of Peter's milestones is directly opposite one of Paul's, we shall for brevity say that the milestones 'coincide', since the small distance between them at that instant is at right angles to the relative motion of the platforms and is therefore not relevant to our present problem. Thus the two milestones are

[2] The reason why this speed is selected is that we wish the β of the Lorentz transformation to have the value $\beta = 10/3$, since Lovejoy uses the ages 70 and 21 in his story. In that transformation β is defined as $\beta = c/\sqrt{(c^2 - v^2)}$, where c is the velocity of light and v the relative speed of the moving systems. When $\beta = 10/3$, we write $c/\sqrt{(c^2 - v^2)} = 10/3$; and by solving for v we get $v = c\sqrt{91}/10$. Now $\sqrt{91}$, to three decimal places, is 9.539. Hence $v = 177{,}425$ miles per second, if we take the velocity of light as $c = 186{,}000$ miles per second. Our units from now on, till further notice, will be miles and seconds.

for that instant considered as "virtually in the same place"; and any events that occur, one at one milestone and the other at the coincident milestone, we shall consider as occurring "at the same place". The importance of emphasizing this is that, in relativity-physics, any two events that occur "at the same place", if simultaneous in any system, are simultaneous in all systems moving perpendicularly to the direction between the two points at which they occur. Therefore events that occur simultaneously at coincident points on our two platforms are simultaneous on both platforms. The reader knows that in relativity there is an important difference between 'simultaneity at the same place' and 'simultaneity at a distance'. This difference, however, is not in the connotation of 'simultaneity', but in its denotation; just as 'father' has the same connotation for all English-speaking peoples, but different persons apply the word to different individuals, you to one man and I to another.

We shall lift the embargo Lovejoy places on observation by any one who is not "coeval" with both Peter and Paul. This is necessary because we shall need legitimate observers, and cannot get them under Lovejoy's law; that law, as we have seen and shall see more clearly as we go on, is unconstitutional within the domain of relativity. Besides, we shall need witnesses to the births of Peter, Paul, and sundry other babies: we do not wish to get so far away from plausibility as to allow babies to date their own births without assistance. Therefore we station two ideally competent observers, one at each of any momentarily coincident points upon which our attention may fall. This competence consists in the possession of vision so keen and alert that each observer can see at any time at least six things: (1) the coincidence of his milestone with a milestone on the other platform, (2) the reading of the latter milestone (he is supposed to have learned by heart the constant reading of his own), (3) the reading of the clock beside the milestone on the other platform, (4) the reading of his own clock, (5) any other fact of interest, such as the birth of a baby or the exact physiological age of any one, at any momentarily coincident station of the other platform, and (6) any such fact at his own station. Having seen, he is to make a true and complete record. It is to be especially emphasized that the reading of *each* of any two coincident clocks is to be witnessed and recorded by *two* observers, one on *each* platform.

A and A' are to coincide on January 1, 1900, exactly at noon, which is to be the zero hour for both systems; thus Paul's clock at A has then the reading $t = 0$, and Peter's clock at A' the reading $t' = 0$. At exactly this time Paul is to be born at A and Peter at A'. Here we have simultaneity at the same place (and therefore simultaneity shared by the two platforms) of five events, *viz.* the coincidence of A and A', the two clock-readings, and the two births.

Needing more babies we call for the birth of a cousin of Paul's at P on Paul's platform at time $t = 0$ by Paul's clock there. P is $7\sqrt{91}$ (= 66.773) light-seconds—a light-second being the distance light travels in a second—or $7c\sqrt{91}$ miles from A, as measured on Paul's platform, and its direction from A is that in which Peter's platform is moving relatively to Paul's. Peter will therefore in due course pass this baby. To even matters up, a cousin of Peter's is to be born at P' on Peter's platform, P' being the point coincident with P when Paul's cousin is born at P; and the birth of Peter's cousin is to occur simultaneously with the coincidence of P' with P, and consequently simultaneously with the birth of Paul's cousin at P. Here again we have simultaneity of five events at the same place, *viz.* the coincidence of P with P', two births, and two clock-readings. One of these readings, that of *Paul's* clock, is $t = 0$. But the reading of *Peter's* clock is *not* zero; it is[3] $t' = -212\frac{1}{3}$. Thus this clock of Peter's at P' is at this time slower by $212\frac{1}{3}$ seconds than Paul's coincident clock at P. This means that, since on Peter's platform the clocks at A' and P' are synchronous, the five events at P and P' that we have just been considering occur earlier by $212\frac{1}{3}$ seconds, by Peter's time-system, than the other five events we considered previously, *viz.* those at A and A' when these points were coincident; since the events at coincident P and P' occur at time $t' = -212\frac{1}{3}$ by Peter's clocks, whereas the events at coincident A and A' occur at time $t' = 0$ by Peter's clocks. (The reader will bear in mind that although *we* have had to calculate in order to get this result, the two

[3] This result is obtained in the following way. We have four known values: the reading of the milestone at P is $x = 7c\sqrt{91}$; the reading of Paul's clock at P is $t = 0$; and the constant values in our problem are $\beta = 10/3$ and $v = c\sqrt{91}/10$. We wish to ascertain the value of t' (the clock-reading at P') simultaneous with t and "at the same place" with x. We look among the Lorentz transformation-equations for one that has as its terms our five values, x, t, β, v, t'; and by substituting into it the four given values we obtain the fifth desired. The Lorentz equation that has these five terms is

$$t' = \beta(t - vx/c^2).$$

By making the indicated substitutions we get

$$t' = 10/3 \times (0 - [c\sqrt{91}/10 \times 7c\sqrt{91}]/c^2) = -(7 \times 91)/3 = -212\tfrac{1}{3}.$$

The minus sign tells us that the time t', when P and P' are coincident, is before twelve o'clock.

observers at P and P' *do not have to calculate.* Being competent observers, both of them *see* the readings of both clocks.)

There is nothing in this that the logical reader need boggle at. He is already familiar with the fact that two witnesses may agree that two events take place *at the same time*, and yet may disagree as to *what* that time is, and thus may not give *the same time* to the double occurrence. In other words, the expression 'the same time' is ambiguous. Your clock and mine at the same time may not give the same time. To say 'at the same time' may be to assert (1) undated simultaneity, *i.e.*, simultaneity without reference to any time-system or 'calendar'. (In relativity, this is possible only of events at the same place.) On the other hand, it may be to assert (2) an agreement (congruence) between two clocks or calendars as to the dates of events at the same place, or (3) an equal dating of events at different places by synchronous clocks, *i.e.* by the same calendar. In the case before us, to say that the events at coincident A and A', or (*not* and) those at coincident P and P', occur at the same time on *either* platform is to make assertion (1). To deny that the events at coincident P and P' occur at the same time on *both* platforms is to deny assertion (2). To deny that the events at coincident A and A', *and* those at coincident P and P', occur at the same time on Peter's platform is to deny assertion (3).

Lovejoy's initial mistake, *fons et origo malorum*, in his misconception of the retardation of clocks in the Special Theory, is his assumption, entirely at variance with the doctrine he is criticizing, that *because by Paul's clocks* all the four births we have been considering are simultaneous, *therefore* they are simultaneous *without qualification*, and *therefore* simultaneous *on Peter's platform.* (If I remember aright, this would be called in the Aristotelian logic a fallacy of accident. Or is it a *petitio?*) By *Paul's* calendar all four babies are "coeval" in the sense that they are 'born at the same time'. In this sense, the babies born at P and P' are *not* "coeval" *by Peter's calendar* with those born at A and A'. But to be "coeval" may mean to be 'continuously of the same age'; in this sense, only Paul and Paul's cousin at P are relativistically "coeval", and they are "coeval" only by Paul's clocks. In Newtonian time these two meanings of "coeval" are exactly convertible, but they are not exactly convertible in relativity. This difference will be made clear later on in this paper.

Meanwhile let us follow Peter as he speeds along Paul's platform until he comes directly opposite Paul's cousin at P. By *Paul's* clock there, this event occurs at time $t = 70$, *i.e.* at 70 seconds past twelve o'clock.[4] But this is *not* the time as given by *Peter's* clock, which on his arrival at P has [5] the reading $t' = 21$.

Now let us turn our attention to point M' on Peter's platform *west* of A' and distant from A' by $7c\sqrt{91}$, as measured on that platform. Let M be the point on Paul's platform coincident with M' at time $t' = 0$ by *Peter's* clock at M', *i.e.* at the same time by *Peter's* clocks as the coincidence of A with A' and the births of Peter and Paul. This coincidence does *not* occur at time $t = 0$ by *Paul's* clock at M; it occurs [6] at time $t = -212\frac{1}{3}$. Thus, whereas by *Peter's* clocks the coincidence of M with M' is simultaneous with the coincidence of A with A', and therefore simultaneous with the births of Peter and Paul, by *Paul's* clocks the former coincidence was earlier by $212\frac{1}{3}$ seconds than the latter coincidence and the births at A and A'. Please note again that *we* have had to calculate to find the reading of Paul's clock at M when it was coincident with M', but that the two *observers* at these points do not have to calculate; they *see* the reading.

Note also the chiastic symmetry between the readings, on the one hand, of Paul's clock at P and Peter's clock at P' when these points are coincident, and the readings, on the other hand, of Peter's clock at M' and of Paul's clock at M when these latter points are coincident. In the former case, where P and P' are in the positive direction from

[4] This result is obtained thus. Peter, who was born opposite A at time $t = 0$ by Paul's clock there, and who travels with velocity $v = c\sqrt{91}/10$, measured on Paul's platform, covers the distance $7c\sqrt{91}$, measured on Paul's platform, by time $t = 70$ by Paul's clock at P, since the time taken is equal to the distance covered divided by the velocity.

[5] This result is obtained thus. We have the four known values: $x = 7c\sqrt{91}$, $t = 70$, $\beta = 10/3$, and $v = c\sqrt{91}/10$; and we wish to obtain the value of t'. The Lorentz transformation-equation that contains these five terms, *viz.* x, t, β, v, and t', is the one we used in footnote 3, *viz.* $t' = \beta(t - vx/c^2)$. By substituting into this equation from the equations giving us the known values, we get the desired value $t' = 21$. What *we* here get by calculation, the two *observers* at A' and P get by looking and *seeing*.

[6] This result is obtained from a Lorentz transformation-equation, but not from the one we used in footnotes 3 and 5. That equation contained the five terms, t', t, x, β, and v. *Now* we have the following values as given: $x' = -7c\sqrt{91}$ (the minus sign because M' lies in the negative direction from A'), $t' = 0, \beta = 10/3$, and $v = c\sqrt{91}/10$; and we wish to obtain the value of t. Thus we have a term x' not found in that Lorentz equation, and that equation has a term x which we do not have as given in our present problem. But there is a Lorentz transformation-equation that does have our five terms, t', t, x', β and v. That equation is

$$t = \beta(t' + vx'/c^2).$$

We substitute into this transformation-equation the given values of t', x', β and v, and we get

$$t = 10/3 \times (0 - [c\sqrt{91}/10 \times 7c\sqrt{91}]/c^2) = -212\frac{1}{3}.$$

A and A', Paul's clock at P has the reading $t = 0$, and Peter's clock at P' has the reading $t' = -212\frac{1}{8}$. In the latter case, where M' and M lie in the *negative* direction from A' and A, and M' is as far from A' as P is from A, it is *Peter's* clock (not Paul's) that has the reading $t' = 0$, and it is *Paul's* clock (not Peter's) that has the reading $t = -212\frac{1}{8}$. The 'reciprocity' of clock-readings on the two platforms is *not* mutual reversibility of readings of clocks 'at the same place'; it is symmetrywith respect to similar *kinematic* conditions. The point M, going westward, is at a distance from A', measured on Peter's platform, equal to the distance, measured on Paul's platform, at which the point P', going eastward, is from A. Hence Paul's clock at M and Peter's clock at P' (M and P' *not* coincident) have *equal readings under similar kinematic conditions*, just as Paul's clock at P and Peter's clock at M' have equal readings under similar kinematic conditions. But this equality of readings is not simultaneity of readings, since the clocks with these equal readings are not on the same platform, and therefore are not synchronous.

Now, just as we followed Peter till he was passing Paul's cousin at P, let us follow Paul till he passes Peter's cousin at M'. Having passed A' at time $t' = 0$ by Peter's clock there, and having travelled with speed $c\sqrt{91}/10$ over a distance $7c\sqrt{91}$, as measured on Peter's platform, he arrives at M' at time $t' = 70$, by Peter's clock at M'. But *Paul's* clock at A, then coincident with M', does *not* have this reading; it has [7] the reading $t = 21$.

Here we have another example of the chiastic 'reciprocity' of clock-readings on the two platforms. Since Peter and Paul both started from kinematic scratch, *viz.* from coincident A ($x = 0$) and A' ($x' = 0$) respectively, at time $t = t' = 0$, the ratio between the readings of Peter's clock at A' and Paul's coincident clock at P, after Peter has traversed a scalar distance *eastward*, is equal to the ratio between the readings of Paul's (not Peter's) clock at A and Peter's (not Paul's) clock at M', after Paul has traversed an equal scalar distance *westward*. The symmetry does not consist in the mutual inversion of *coincident* clock-readings, as Lovejoy supposes when he makes Peter attribute to Paul's "coeval", whom he is passing, the age that the latter attributes to Peter. In relativity there is no such mutual inversion of unequal coincident clock-readings. *The relativistic reciprocity is symmetry with*

[7] Here we have the given values $t' = 70$, $x' = -7c\sqrt{91}$, $\beta = 10/3$, and $v = c\sqrt{91}/10$, and we wish to determine the value of t. The Lorentz transformation-equation that has these five terms is the one we used in footnote 6, *viz.* $t = \beta(t' + vx'/c^2)$. By substituting our known values into this equation we get $t = 21$.

respect to kinematic scratch. Here then we have another fundamental misunderstanding under which Lovejoy labors in seeking to criticize relativity.

So far we have been dealing chiefly with clock-readings 'simultaneous at the same place'. If relativity is true, observers, such as we have assumed, *consentiently* see, record, and report, all the facts we have recorded in the text. The computations we have made in the footnotes are *ours*, not theirs. We have had to make them because we were not bodily present to witness the occurrences. It is impossible to over-emphasize the fact that there is no disagreement as to what is directly observed by competent observers at any two coincident points. If there were any, observations under similar conditions would have to be repeated till agreement should be reached. If such agreement were unobtainable, we should have on our hands "a condition and not a theory". But there is no such disagreement; we have in each case, at the mouth of two witnesses, reliable data consisting of statements such as that the two coincident clocks at such and such points had severally such and such readings, and then such and such events occurred there. Where then does the 'disagreement' come in?

We are all familiar with the difference between the reading of a clock and the 'correct time' when the clock has this reading. In general practice we get 'correct time' second-hand from Washington by consulting local Western-Union clocks, intermittently synchronized by electro-magnetic impulses. But *if* every *identical* electro-magnetic impulse is to have constant velocity on each, and equal velocity on both, of our relatively moving platforms, the clocks synchronized on Peter's platform cannot have the same synchronism as those synchronized on Paul's.

Synchronism is a relation between clocks, and it consists in the fact that they (*a*) run at the same rate, and (*b*) are so set that any reading of any one of them is simultaneous with the equal reading of any other of them. If (*b*) is true (*a*) is also true, and therefore our definition is redundant. Still, it is important to distinguish (*a*) and (*b*), for the reason that (*a*) may be true and yet (*b*) may be false. But what for our present purpose is more important is that the asynchronism of Peter's clocks as tested by Paul's, and of Paul's as tested by Peter's, cannot be understood without this distinction. According to the current *Newtonian* conception, we may have two groups, each consisting of clocks running at the same rate and set together, the rates, however, of the two groups not being the same. In such a case, says the Newtonian, *any observer will see that the equal readings of the* *clocks of either group are simultaneous*, but that readings of the clocks of one group are not, in general, simultaneous with equal readings of the clocks of the *other*. The clocks of one group will lose time as compared with those of the other; and *all* observers will agree *which* group is the loser. Any one who thinks only in terms of *this* conception will find the relative asynchronism of Peter's and Paul's relativistic clocks contradictory—but the contradiction will be with the terms in which he thinks, not an internal contradiction in relativity.

In order to understand relativistic time, we must not *start* with the idea that the clocks of either platform run slowly with respect to those of the other; we must start with the fact that *at no time, registered by the clocks of either platform, does any one of the clocks of the other have a reading equal to the reading of any other clock of that other platform* (unless the two latter clocks lie in a plane perpendicular to the relative motion of the platforms). In other words, the italicized statement of the preceding paragraph, which is true of Newtonian time, is not, in general, true of Einsteinian times. This is the basic difference between relativity and Newtonianism. The fact that the clocks of either platform run slowly, as measured by the synchronism of the other, is *a logical consequence* of the italicized statement of this paragraph. The relative retardation of clocks is a corollary of the intersystemic asynchronism of intra-systemically synchronous clocks, *i.e.*, of the fact that *simultaneity* is not in general shared by the two platforms. The popular expositions of the Special Theory, being popular, fail to bring out this logical relationship between 'retardation' and the fact that two relatively moving systems have disparate simultaneities. This logical relationship can be made clear only by following the *reasoning* of relativists as they proceed from the Lorentz transformation which they have derived, to the 'paradoxical' conclusions which they deduce. Let us see how this reasoning runs. In this paper it is obviously impossible to consider the *deduction* of the Lorentz transformation from the relativistic postulates. Those postulates, *if* accepted, compel the acceptance of the transformation.

To distinguish between the data involved in the acceptance of the Lorentz transformation, and the logical conclusions that follow, we shall now impose a hard law upon our 'observers'. *They* shall not *think* at all, in the sense of comparing any observation they may make at any time with those they may make at any other time, or with those which other observers may make 'at other places'. They are to do nothing but attend strictly to the six things their 'competence' enables them to do; they are merely to observe and record and report—

like private soliders, theirs is not to reason why. All the records thus made on Paul's platform are to be sent to the Office of the Register of Physical Deeds at A, and all the records made on Peter's platform are to be sent to a similar office at A'. At each office is to be an intelligent accountant, whose task it is to scan all reports and to make what he can out of them. He is an ideal statistician with no axe to grind; he is impartial between the clocks of the two systems, knowing that on each platform the clocks have been properly synchronized. Of course he must begin somewhere, and we let each accountant begin with the known fact that the clocks *on his own platform* are synchronous. We are now to look over the shoulder of the accountant on Paul's platform as he works. He writes the following memoranda.

"(1) *Paul's clocks are synchronous.* At time $t = 0$ *by Paul's clock* at A, A' was at A, and at that time *Peter's* clock at A' had the reading $t' = 0$. At time $t = 70$ *by Paul's clock* at P, A', having meanwhile travelled eastward, was at P, and at that time that identical clock of *Peter's* at A' had the reading $t' = 21$. Thus during a time-interval of 70 seconds (*i.e.*, the interval between $t = 0$ and $t = 70$), *as measured by Paul's synchronous clocks at A* and P, that identical clock of *Peter's* at A' advanced its reading by only 21 seconds. Therefore that clock of Peter's lost 49 seconds during a time-interval of 70 seconds, as measured by Paul's synchronous clocks at A and P; *i.e.* it lost time at the rate of 7 seconds in every 10."

Now the same accountant, knowing that *Peter's* clocks also had been properly synchronized, goes on to write as follows.

"(2) *Peter's clocks are synchronous.* At time $t' = -212\frac{1}{3}$ *by Peter's clock* at P', P was at P', and at that time *Paul's* clock at P had the reading $t = 0$. At time $t' = 21$ by Peter's clock at A', P, having meanwhile travelled westward, was at A', and that identical clock of Paul's at P had the reading $t = 70$. Thus during a time-interval of $233\frac{1}{3}$ seconds (*i.e.*, the interval between $t' = -212\frac{1}{3}$ and $t' = 21$), *as measured by Peter's synchronous clocks at P'* and A', that identical clock of *Paul's* at P advanced its reading by only 70 seconds. Therefore that clock of Paul's lost $163\frac{1}{3}$ seconds during a time-interval of $233\frac{1}{3}$ seconds, as measured by Peter's synchronous clocks at P' and A'; *i.e.* it lost time at the rate of 7 seconds in every 10."

Resorting to algebraic instead of arithmetical numbers,[8] and being

[8] Let us call Paul's platform S, and Peter's S'. At *any* point P ($x = n$) on S at *any* time $t = a$, the S' clock at P', then coincident with P, has the reading $t' = \beta(t - vx/c^2) = \beta(a - vn/c^2)$.

P', moving with constant velocity v in the positive x-direction of S, arrives at an impartial accountant, who desires to avoid the appearance of making statistics lie by failing to tell the whole truth, he writes:

"*Measured by the synchronism of Paul's clocks,*[9] any one of *Peter's* clocks loses time, in going from any point to any other point on Paul's platform, at the rate of 7 seconds in every 10; *i.e.* it runs only 3/10 as fast as Paul's clocks". Then he adds as a footnote: "*Measured by the synchronism of Peter's clocks,* any one of *Paul's* clocks loses time, in going from any point to any other point of Peter's platform, at the rate of 7 seconds in every 10, *i.e.* it runs only 3/10 as fast as Peter's clocks."

If we now look over the shoulder of Peter's accountant, we shall find that the only difference between what he writes, and what Paul's wrote, is that the memorandum which Paul's accountant numbered (2) Peter's accountant numbers (1), and *vice versa;* and that what Paul's accountant put into a footnote Peter's accountant puts into the text, and *vice versa.* Such unanimity among accountants is surprising —at any rate it is refreshing—but what are we to expect from impartial and competent accountants who have exactly the same data to interpret?

Let us review carefully what each accountant has done. He compares (*a*) the reading of a clock on one platform with the reading of a coincident clock on the other platform, and (*b*) a later reading of the former clock with the reading of the then coincident clock on the other platform; and he treats the clocks on that other platform as synchronous, since they have been synchronized on their platform. Each of these comparisons gives the same result for both accountants. The only difference that can arise is a difference of *preference;* each

any point $Q(x = n + m)$ at time $t = a + m/v$. At that time the clock at P' has the reading $t' = \beta(t - vx/c^2) = \beta(a + m/v - v[n + m]/c^2)$. Thus during the time-interval which has the measure m/v by the S clocks at P and Q, synchronous on S, the clock at P' on S' advances its reading by $\beta(m/v - vm/c^2) = m/\beta v$. Thus whatever clock on S' is coincident with *any* clock on S at *any* time, will, after traversing *any* distance, have lost time, as measured by synchronous clocks in S, at the rate of $(\beta - 1)$ seconds in every second.

A symmetrical result is obtained for any clock in S, as measured by synchronous clocks in S', by using the transformation-equation $t = \beta(t' + vx'/c^2)$.

[9] It is to be observed that our accountant uses the word 'measured' instead of the word 'judged', frequently employed by other accountants. In this connexion they mean the same thing; but the latter is often misinterpreted as if the 'judgment' were not necessary in view of the numerical data furnished and of the fact that the clocks in each system have been physically synchronized. There is nothing 'subjective' in our accountant's statement, any more than if he were to say, "Measured by this meter-stick, this bar of iron is 2 meters long".

naturally prefers to use the synchronism of the clocks on his own platform—it is more convenient, and in fact the convenience amounts to a physical necessity for *observers*, since an observing physicist has at his disposal only his *own* instruments, including chronometers. Our set-up with 'competent' observers is an ideal one. A physicist would be decidedly embarrassed if, with chronometers flying by him at every conceivable speed short of the velocity of light, and in every direction, *he had to read every one*. He uses his own synchronism for the same reason that the terrestrial astronomer uses as *the* astronomical year the time-interval that is required for the *earth* to complete a revolution around the sun, although he is aware that *each planet has its own year*. Which is *the* true, *the* absolute, *the* cosmic year? The answer to *this* question is in principle the relativistic answer to the question: Which is *the* correct time-system? It happens that the transformation from the terrestrial year into the Martian year is not the same as that from Paul's synchronism into Peter's, but the latter is not necessarily 'subjective' for that reason.

If we have followed the reasoning of the accountants, we have seen that the reciprocal retardation of the clocks on the two platforms is not reciprocal inversion of *coincident* clock-readings. It is *because* there is *no such* reciprocity, *e.g.*, it is *because* the clocks at coincident P and P', and at coincident A' and P, have unequal and non-interchangeable readings when these points are respectively coincident, that the relativist concludes that the clocks of either system run slowly as measured by the synchronism of the other. If the readings of coincident clocks *were* interchangeable, as Lovejoy supposes, there would be no reciprocal retardation—there would be temporal chaos. Just try it out.

Now let us suppose that Peter's heart and the hearts of all his cousins beat once every second *by his clocks*, and that Paul's heart and the hearts of all his cousins beat once every second *by his clocks*. Let us further suppose that all the other physiological processes of each of these babies run at a normal rate as measured by his heart-beats. In accordance with this assumption, Peter is 21 seconds old *physiologically* when he arrives at P, since his clocks have ticked off 21 seconds since his birth. On the other hand Paul's cousin at P is 70 seconds old *physiologically* when Peter arrives at P, since Paul's clocks have ticked off 70 seconds since this cousin's birth. The two competent observers at A' and P agree in recording these respective physiological ages. Neither observer attributes to Paul's cousin the physiological age that the other attributes to Peter, nor *vice versa*. Our observers have to do only with clock-readings and with events and facts simultaneous with these readings at the same place with these readings. The automatic relative motion of the platforms and the automatic movements of the clocks and of physiological processes furnish them with events and facts to be observed.

But the *accountants*, when they receive and examine the reports sent them, *compute the time-intervals involved*, and in doing this they discover that they must *distinguish between physiological and calendar ages*. Before they became relativists they had been familiar with this distinction. For instance, a child born in Moscow on January 1, 1800, O.S., was one year old, by the Russian calendar, on January 1, 1801, O.S. But by the calendar used elsewhere in Europe he was one year and one day old. The fact that the child had two calendar ages did not give him two physiological ages. Where two different calendars are employed, the same physiological age is compatible with two calendar ages. In relativity each of two relatively moving systems has its own distinctive calendar. By the supposition made in the preceding paragraph, the calendar age of any person, by the calendar of his *own* system, is equal to his physiological age. But his calendar age by the calendar of another system, which we shall for short call his 'heterochronic age', is not equal to his physiological age. We now let our accountants work out the problem.

"*Paul's clocks are synchronous.* Peter, having been born at time $t = 0$ by Paul's clocks, arrived (21 seconds old physiologically) at P at time $t = 70$ by Paul's clocks. Hence on Peter's arrival at P his heterochronic age by Paul's calendar (*i.e.*, the *measure* of the interval between Peter's birth and his arrival at P, as given by Paul's clocks) was 70 seconds, and this heterochronic age was $3\frac{1}{3}$ times his physiological age.

"*Peter's clocks are synchronous.* Paul's cousin at P, having been born at time $t' = -212\frac{1}{3}$ by Peter's clocks, arrived (70 seconds old physiologically) at A' at time $t' = 21$ by Peter's clocks. Hence on Paul's cousin's arrival at A' his heterochronic age by Peter's calendar (*i.e.*, the *measure* of the interval between Paul's cousin's birth and his arrival at A', as given by Peter's clocks) was $233\frac{1}{3}$ seconds, and this heterochronic age was $3\frac{1}{3}$ times his physiological age."

Note that neither the heterochronic nor the physiological ages of Peter and of Paul's cousin are respectively interchangeable. It is true that Peter's heterochronic age is equal to Paul's cousin's physiological age; but the reverse is not true. The 'reciprocity' involved here is in the *ratio* of heterochronic to physiological age. Each has a heterochronic age 10/3 his physiological age. We may not like the calendars the relativist employs on his platforms, and therefore may not choose to run with him on them. But this does not justify us in misrepresenting the facts on his platform and then accusing him of contradiction—unless we choose to adopt the kind of logic that Lovejoy (or for that matter any of us at times, unless we are very careful) employs in dealing with those he criticizes.

Lovejoy thinks (pp. 155–156) that since "number (of discrete individuals) is an absolute quantity", the Special Theory is committed to an absurdity in treating durational lengths as respective. But the absurdity is in his imagination, not in relativity. All the *numerical* data involved, in the case of Peter and Paul, are agreed upon by both our accountants; just as you and I, travelling together, may agree that we passed 10 milestones while your watch was ticking off 15 minutes and mine 14. We agree on the count of everything we count, when we count the *same* things: milestones, your watch-ticks and my watch-ticks. We might under these circumstances dispute as to *how long* it took us to pass those milestones, unless we agreed that it took 15 minutes by your watch and 14 by mine. So far as the analogy holds, this is just what happens in relativity. The physiological ages of Peter and Paul, counted by heart-beats, are analogous to the milestones, and their heterochronic ages to your 15 minutes and my 14. All observers agree on the count of everything they count. It is agreed by all relativists that Peter's clock at A' ticked, and Peter's heart beat, 21 times from his birth till his arrival at P; and that the heart of Paul's cousin beat, and Paul's clock at P ticked, 70 times from *this* cousin's birth till Peter's arrival at P. The "discrete individuals" here are these clock-ticks and these heart-beats. Their number is "absolute" in the sense that *all* agree on the count. But when the question arises *how long* it took for these two sets of heart-beats and clock-ticks to run off, there is a double answer, depending upon the synchronism used. By Paul's synchronism, Peter and Paul's cousin were born at the same time; therefore, since Peter's heart, 70 seconds later by Paul's clock, had beat only 21 times, and that of Paul's cousin had beat 70 times, Peter was growing old physiologically only 3/10 as fast as Paul's cousin. By Peter's synchronism, Paul's cousin was born 212⅓ seconds before Peter; therefore, since Paul's cousin's heart, 233⅓ seconds later by Peter's clock, had beat only 70 times, and that of Peter 233⅓ times, Paul's cousin was growing old only 3/10 as fast as Peter. There is no difference as to any count; there is difference only as to the *dates* at which the counting begins and ends; and on the two platforms the dates are different because different calendars are used. Again, the idea that Paul should have eaten only 365 and also 25,550 breakfasts during a certain trip is of course absurd, but it is Lovejoy's idea (p. 155) attributed to the relativist. There is surely no absurdity in the idea that the number of hours taken to eat 365 breakfasts varies with the clocks used to measure the time-factor of the breakfast-eating process. The number of breakfasts any one on a relativist platform eats in making a trip is "absolute". The *time* it takes to make the trip, *when measured*, gives different measure-numbers according to the different clocks used in measuring. Lovejoy himself admits this when he says that "even this [lack of "accord" in "time-measurements"] would not amount to a contradiction" (p. 51). The correctness of his article lies mainly in his verbal insistence on thinking straight.

We are now ready to answer the question, What would happen if Peter were to *reverse his direction* and return to Paul, and "if the acceleration in the reversal were treated as theoretically negligible"? We shall treat "theoretically negligible" as synonymous with 'having no physical effects'. Let us add another platform to our set-up and call it 'John's platform'. It is to be parallel to the other two and in unaccelerated motion relatively to them. It moves relatively to Paul's platform in the direction opposite to that in which Peter's moves; and its speed relative to Paul's is 177,425 miles per second. It is equipped as the others, *i.e.* with clocks properly synchronized, with milestones, and with observers. We designate as A'' the point on it that is coincident with A and A' when these points are coincident with each other; at that time John's clock at A'' is to have the reading $t'' = 0$. We designate as K'' the point on John's platform coincident with P and A', when these points are coincident with each other, *i.e.* at time $t = 70$ and $t' = 21$. Thus on Peter's arrival at P, K'' is passing P going the other way.

This set-up is in accord with the Special Theory of Relativity. We now introduce a new assumption, not contemplated in that theory: we imagine both Peter and his clock at A' to be instantaneously transferred to K'' when directly opposite K''. This transfer thus takes place when Peter is 21 seconds old physiologically, and when his clock has the reading $t' = 21$. The transfer is to have no disturbing effects on Peter's physiological processes or upon Peter's clock other than that after it both Peter and his clock are to carry on just at the same rate as if they had been indigenous to John's platform. (This is of course a most violent supposition, as the transfer is a most violent transfer; but we are not responsible for either. We are following Lovejoy's supposition, and merely wish to see whether his inference from it is correct.) Peter's heart continues to beat with his clock-ticks, and his clock now runs at the same rate as the clock originally at K''; the difference between its reading and that of the original clock at K'' is to remain constant. Now that original clock at K'' has the reading $t'' = 445\frac{2}{3}$ when the transfer is made, and Peter's transferred clock has the reading $t' = 21$. Hence after the transfer Peter's clock is to be constantly slower than John's at K'' by $424\frac{2}{3}$ seconds. Peter, now retracing his movement with the same speed back as he had on his out-bound journey, reaches A at time $t = 140$ by Paul's clock at A, at which time John's clock at K'' has the reading $t'' = 466\frac{2}{3}$. Hence Peter's transferred clock has then the reading 42, and Peter's physiological age, which has been advancing in the rhythm of his clock, is 42 seconds.[10] There is no 'reciprocal' retardation of clocks after the return journey has been completed. *It is the clock that reversed its direction that is slow.* Lovejoy desires that, since relative motion is relative and reciprocal, reversal of relative motion shall be reciprocal in its effects upon clock-readings and physiological processes. But we are dealing with *Einstein's relativity;* and when dealing with Einstein's relativity, we must use his relativity-*equations*. The physical theory of relativity is not just the classical theory of the relativity of motion dressed up in mathematical clothes. The only *logical* consistency that any theory is required to have is consistency *with itself*, not consistency with some one's preconceived notion, or even with some one else's internally consistent notions.

Now let us see what the comparative ages of Peter and Paul would be when they met, if *each* were to be transferred instantaneously to the other's platform, on the assumption that the transfers are to have no disturbing effects, and that they are made under similar kinematic conditions. Peter is to be transferred after he has travelled a given distance along Paul's platform, and Paul after he has travelled an

[10] This result is obtained by using for John's clock at K'' the Lorentz transformation-equation $t'' = \beta(t + vx/c^2)$. Into this we first substitute $x = 7c\sqrt{91}$, and $t = 70$, and the constant values for β and v, getting $t'' = 1337/3 = 445\frac{2}{3}$ for the reading of John's clock at K'' at the time of the transfer; and then we substitute $x = 0$, $t = 140$, and the constant values for β and v, into the same transformation-equation and get $t'' = 1400/3 = 466\frac{2}{3}$ for the reading of John's clock at K'' when it arrives at A. Thus John's clock has advanced its reading by 21 seconds in going from P to A. Peter's clock, having the reading 21 on the transfer, and advancing its reading *pari passu* thereafter with John's, has the reading 42 on Peter's return to A.

equal distance along Peter's. Let Peter be transferred when he has arrived at P, and Paul when he has arrived at M'. Each is thus transferred to the other's platform when he is 21 seconds old physiologically. When they meet, each is 42 seconds old physiologically.[11] Thus the equal accelerations undergone by the two babies under similar kinematic conditions, and under the assumption that these accelerations have no disturbing effects, bring them back to each other with equal physiological ages.

* * * * * *

Let us now in terms of years instead of seconds tell the story of Peter and Paul moving relatively at the furious speed of 177,425 miles a second on their unaccelerated platforms.

'Peter at the physiological age of 21 years, on January 1, 1921 (by his calendar), was passing one of Paul's "coeval" cousins at Q (born simultaneously with Paul by Paul's calendar). This man was at that time 70 years old physiologically; for he was born on January 1, 1900, by Paul's calendar, and his heart had been beating once every second by Paul's clocks; and now it was January 1, 1970, by Paul's calendar. Peter had been told some ten years and nine months ago[12] that news had been received of the birth of this cousin of Paul's as having occurred on September 1, 1687, by Peter's calendar. This man therefore was 233⅓ years old by Peter's calendar when Peter was

[11] Let us work out the answer algebraically, and then apply the answer to our arithmetical set-up. Let v be any positive constant less than c, and let n be any positive constant. Peter, having been at $A(x = 0)$ at time $t = 0$, reaches point $D(x = n)$ at time $t = n/v$ by Paul's clock there, and Peter's clock has then the reading $t' = \beta(t - vx/c^2) = \beta(n/v - vn/c^2) = n/\beta v$. His clock is therefore at that time slower than Paul's clock at D by $n/v - n/\beta v = n(\beta - 1)/\beta v$.

Paul, having been at A' at time $t' = 0$, reaches $E'(x' = -n)$ at time $t' = n/v$ by Peter's clock there, and Paul's clock then reads $t = \beta(t' + vx'/c^2) = \beta(n/v - vn/c^2) = n/\beta v$. His clock is therefore slower than Peter's clock at that time by $n(\beta - 1)/\beta v$.

The points D and E' pass each other at time $t = t' = n(\beta + 1)/\beta v$, this being the reading of the *indigenous* clocks at D and E'. We get this result by substituting $x = n$ and $x' = -n$ into the Lorentz equation $x' = \beta(x - vt)$, and solving for t; and then into the Lorentz equation $x = \beta(x' + vt')$, and solving for t'.

Since the transferred clocks are slower, each than the indigenous clock beside it, by $n(\beta - 1)/\beta v$, both, on passing each other, have the reading $n(\beta + 1)/\beta v - n(\beta - 1)/\beta v = 2n/\beta v$. Since the heart-beats of Peter and Paul have been running off in the rhythm of their clocks, they are both $2n/\beta v$ seconds old in passing each other.

If now we substitute for n the value $7c\sqrt{91}$ and for v and β their respective numerical values in our set-up, we get the readings of the transferred clocks, on their passing each other, as being 42.

[12] The reader is now probably able to work this out for himself.

passing him. Peter was astonished to see this man still alive, in spite of his nearly twelve-score years—he surely looked like a mighty fine specimen of a super-bicentenarian. Peter could not understand it, for in the life-time of this prospective Methuselah the fountains of youth had run completely dry. There was something devilishly queer about it all; for now that he came to think of it, he had passed several other men who were said to be very old but looked quite otherwise. He began to wish that he knew the secret of living two and one-third centuries without showing the ravages of labor and sorrow; he even had the passing wish that he had been born on the other platform, where age sits so lightly on the sons of men. However, being 21 and in love, he gave these matters only a moment's thought; there were other things more important.

'Paul's cousin, on his side of the anti-friction clearance, was likewise puzzled. He had learned, some three years and three months ago,[12] of the birth of Peter as having occurred when he himself was born, and he had lately been kept informed by the *Daily Platform* of Peter's rapid approach. When Peter did whirl by, he could not believe his eyes. Instead of being a septuagenarian like himself, Peter proved to be a remarkably handsome lad, not more than 21; he must have resorted to goat-glands or hormones or whatever it was that in 1970 had long been noised abroad as effective antidotes for old age. Whatever it was, he himself had not found it effective in his own case; the practitioners had taken his money and left him his years. Perhaps he had been born on the wrong platform; on the other, youth-preservers must be more potent! Having heard of relativity but not knowing more about it than that it proposed rapid travel as the way to keep from growing old, he now suspected that it must be true. Finally in a mood, mixed of high resolve and desperation, he jumped over to the other platform, where under the ministration of relativists he might get the benefit of speed for the rest of his days.—And that was the last seen or directly heard of that cousin of Paul's. Neither hide nor hair of him could be found.

'Peter, however, soon thereafter got word by wireless that there had been a terrific platform-quake some way down the line. Coming so soon after the passing of Paul's cousin, so old and yet so spry, this news set him a-wondering whether there might not be some connexion between the strange events that were taking place in his neighborhood. The miracle that had transformed a man over 233 years old into a sprightly though gray-haired buck might account for anything else. He bethought him of the accountant he had heard of as a very learned man, and betook himself to him and introduced himself as Paul's "coeval". He now remembered that word, and one thing leading to another he asked for news of Paul. He was told that Paul was probably getting along splendidly; he must now be getting his first molars, for he must be over six years and three and a half months old.[12] This competely bowled him over; he was done with this accountant and all his ilk. He set out hot-foot for Hannah, who soothed him by promising to keep her world-line always as close to his as possible in this new Space-Time which people were beginning to talk so much about.'

This is the authentic story of the Paradox of the Time-Retarding Journey in the Special Theory of Relativity, as told by relativists in the privacy of their nurseries. All my relativistic friends tell me that at bedtime their children cry for it, and when they get it they "just lap it up". Like all children, they know by heart the stories repeatedly told them; and when a father, just to tease his little ones, mixes up the ages of Peter and Paul's cousin, they shout the correction in unison of protest. I am afraid that in any of these nurseries Lovejoy's story would provoke a storm of derision. Just where he picked it up I do not know. He is certainly correct in speaking of it as "pleasing"; but why did he call it a "pleasing *relativistic Märchen*"?

EVANDER BRADLEY MCGILVARY

UNIVERSITY OF WISCONSIN

ON NEGATIVE FACTS

I shall argue in this paper that logic may dispense with negative facts. By a negative fact I mean whatever would make a negative proposition true and yet could not be reduced to any positive fact, *i.e.* to a fact expressible in terms of an affirmative proposition. In denying the necessity of negative facts in logic I do not deny that negative propositions are significant and essentially different from affirmative ones.[1] Nevertheless it may be shown that these negative propositions are true in virtue of some positive fact or other.

Though I cannot prove that there are no negative facts I may mention the following general consideration which prompts my disbelief in their existence. Their existence would imply that there is

[1] That there are essentially different propositions which are true in virtue of the same fact may be seen from the following example. The propositions 'this is green' and 'this is colored' may be based on the same fact of the existence of a certain green patch. But they are significantly different, since the understanding of 'this is colored' does not entail the knowledge of the other proposition.