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THE RELATIVITY DEFLECTION OF LIGHT[1]

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The claim of the relativists, which has attracted the greatest popular interest, is that of "bent light"; the claim that light has weight and falls towards the earth in a manner entirely similar to that of the famed apple of Newton. And this interest has been intensified by the widely heralded eclipse expeditions to Africa, to South America, and to Australia to test and to verify the predictions of Einstein, and by the repeated assertions that these expeditions have fully confirmed all the wonders of the relativity theory by obtaining results which "are in exact accord with the requirements of the Einstein Theory".

But just what these requirements of the theory really are, and how they result from the theory, neither Einstein, nor any of his followers, has explained in simple, understandable language. Einstein, himself, has given two very definite predictions as to the amount by which the light of a star should be bent, or deflected in its passage by the sun. In 1911 he fixed this amount as 0".83; in 1916 he doubled this and made the deflection, according to his theories, 1".70. But the way in which Einstein derived these two different values is not given in any general works on relativity. Such works of the relativists are replete with philosophical contemplations, with vague speculations and generalizations as to the structure of the universe, with references to the principle of

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equivalence, to warps and twists in space; but they one and all fail to give a direct explanation of the basis of Einstein's claim as to the deflection of light rays, and of the ways in which he arrived at the two different and conflicting values. The statement of Einstein, contained in his general work on relativity, is probably as clear and definite as any that can be found, and that statement is:[2]

> "According to the theory half of this deflection is produced by the Newtonian field of attraction of the sun, and the other half by the geometrical modification ('curvature') of space caused by the sun."

If this be taken literally then it would appear that Einstein, in 1911, evolved the theory that light has weight and is acted upon by gravitation in exactly the same way as is a particle of matter; that he afterwards and prior to 1916 found that the sun warps and twists space in its neighborhood, and that light is further deflected by its passage through such warps and twists. Thus it would seem that his 1911 prediction of a deflection of only 0″.83 was based upon some direct effect of Newtonian gravitation upon light; that his revised prediction of 1″.70 in 1916 was based upon some additional and newly discovered effect of gravitation upon space. The summation of these two supposed effects of gravitation, the one directly upon a body, the other indirectly through an intermediary action upon space, has been termed a "new", or the "Einstein" law of gravitation. And the deflections of light, observed at solar eclipses, have been cited as tests between these two theories, or laws of gravitation:—the Newtonian and the Einsteinian.

The importance of finding out just how Einstein made these calculations is thus evident; for such calculations must indicate clearly the basis upon which depends his prediction of "'bent light", and the exact change in that basis, or additional assumption, which turned the computed 0″.83 of 1911 into the final 1″.70 of 1916. Fortunately these calculations are set forth in considerable detail in his original mathematical papers which form the basis of the relativity theory, and which were published in the "Annalen der Physik". This present paper is based upon the formulas and

[2]EINSTEIN: "Relativity: the Special and General Theory." Translated by R. W. Lawson, 1920. Appendix III, page 153.

computations contained in these basic documents of relativity, and is an attempt to explain just what Einstein's theory as applied to light really is, and how and why his predictions differ.

Einstein's researches as to gravitational fields and the possible effects of gravitation upon the physical conditions of bodies and upon light, were purely philosophical and mathematical in character, and did not directly involve any experiments whatsoever. Begun in 1907, they culminated in 1911 with the enunciation of the celebrated "principle of equivalence", which has been stated and restated in many different ways. As actually used by Einstein in his mathematical calculations, however, this principle may be stated as:

> The effect of gravitation upon ideal "clocks" and "measuring-rods" at rest at a given point in a gravitational field is identically the same as that caused by a motion of the "clock" and "rod" through free space with a velocity equal to that which they would have acquired had they fallen, under the action of gravitation, from infinity to that point.

And these effects can, under the theory of relativity, be calculated by the Lorentz formulas. Thus, an ideal clock, at rest in a gravitational field, will run more slowly than a similar clock at rest in free space, for it is a basic tenet of relativity that a clock in motion in free space "goes more slowly than when at rest". Einstein, himself, states this new assumption, or principle of relativity as meaning that:[3]

> "'The clock goes more slowly if set up in the neighborhood of ponderable masses."

And it is from this assumed behavior of "clocks" in a gravitational field that Einstein deduced his formula for the shift in spectral lines and the 1911 value for the deflection of light.

The cardinal principle of relativity is that the measured velocity of light is a fundamental constant of nature. This measured velocity depends not only upon the actual speed of light through space, but also upon the units of space and time in which the measurement is made. If the unit of time be lengthened, then, if

[3]EINSTEIN: "Die Grundlage der allgemeinen Relativitätstheorie," Annalen der Physik, 49: 1916, page 820.

the measured velocity is to remain constant, the actual speed of the ray in and through space must be decreased in like proportion. The velocity of light is 186,000 miles per second, and if this same velocity be measured by a clock "running slow", then in each of the longer seconds of the new clock light would travel more than 186,000 miles:—the measured velocity per second would be different and the cardinal principle of relativity would be violated. This can be adjusted, and the velocity, as measured in the new unit, be maintained at 186,000 miles only by an equal slowing down in the actual speed of the ray itself. Thus Einstein's new principle of equivalence, combined with the cardinal precept of relativity, necessitates the abandonment of the hitherto accepted principle that the actual velocity of light in space is constant, and forces the adoption of the assumption that the actual velocity of the ray through space decreases as it approaches the sun, or other gravitational body. Einstein alludes to this complete change in the fundamental concept regarding the propagation of light through space as:[4]

> "The principle of the constancy of the velocity of light holds good according to this theory in a different form from that which usually underlies the ordinary theory of relativity."

In accordance with this assumption, or new tenet of relativity, all rays of light, which pass through a given point in space, and regardless of the direction of propagation, will have the same velocity, and that velocity will become less and less as the point nears the sun, or other gravitational body. Thus a ray, proceeding directly towards the sun, will be retarded, not accelerated, as would be a falling body; and Einstein's new assumption, therefore, is that light is *repulsed,* or *repelled* by the sun.

The amount of this retardation must be exactly proportional to the slowing down of the ideal clock, as given by the principle of equivalence. At any point in space the square of the "velocity from infinity", under the Newtonian law of gravitation, is $2m/r$, where m is the gravitational mass of the sun. And in accordance with the Lorentz formula a "clock" moving with any velocity v

[4]EINSTEIN: "Uber den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes." Annalen der Physik, 35: 1911, page 906.

is slowed down by the proportionate amount $1/2\ v^2/c^2$, where c is the velocity of light; so that a clock, moving with the "velocity from infinity", would be slowed down by m/rc^2; and the light ray must be slowed down in like proportion. The factor m/r is the well-known Newtonian potential; and making use of this symbol, Einstein writes his formula for the speed of light at any point in the gravitational field of the sun, as:[5]

$$c = c_0\left(1 + \frac{\Phi}{c^2}\right)$$

where c is the velocity of light at an infinite distance from the sun, and Φ is "the (negative) gravitational potential". As the value of the potential function increases, the speed of the ray decreases; and thus this basic formula of relativity indicates clearly that light is retarded by gravitational action.

From this formula for an assumed, or hypothetical, change in speed, Einstein deduced his value for the bending of a ray of light as it passes near the edge of the sun. In doing this, he made use of the ordinary differential formula of physical optics, which expresses the rotation of the wave-front in terms of the change in velocity along the front. The entire shift in the direction of the wave as it passes any fixed point, or origin of coordinates may be obtained by integrating this fundamental formula between the limits of minus and plus infinity. Thus one has for the mathematical expression for the entire bending of the ray:

$$B = \frac{1}{c}\int_{-\infty}^{+\infty} \frac{\delta c}{\delta x}\, dy$$

The meaning of this formula is readily understood by reference to the diagram (Fig. 1), which is similar to one used by Einstein. The beam of light starts at A and travels towards B in a direction parallel to the axis of Y, and passes the origin of coordinates at the distance Δ. If the speed of different portions of the beam remain the same, then the factor $\delta c/\delta x$ is zero, and the path of the beam

[5]EINSTEIN: Loc. cit. page 906.

will be a straight line. But, if through any cause whatsoever the relative velocities of the inner and outer portions of the beam change, then the ray will be deflected from this straight and narrow path; and the above formula gives the entire deflection from the instant the ray started at *A* until it vanishes into space in the direction of *B*. The amount of this deflection depends upon the factor $\delta c/\delta x$, which represents the variation in the speed of various portions of the wave-front; it does not depend upon the cause of

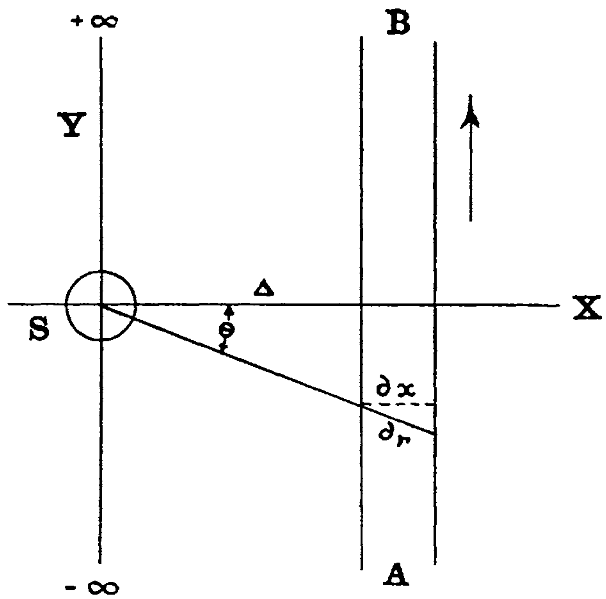

Fig. 1. The Deflection of Light.

such differences in speed. Such differences might be caused by the inner portion of the beam passing through a denser medium than the outer. This is the case of ordinary astronomical refraction: and this formula is used daily to compute the apparent change in the direction of a star, due to the passage of light through the earth's atmosphere.

Einstein used this simple formula of wave motion, and inserted in it his theoretical change in the speed of the wave as it passes through the gravitational field. This hypothetical change, as found from his speed formula, depends solely upon the varying value of the gravitational potential. And as this potential is a simple function of the distance of the wave at any instant from the sun, the formula may be greatly simplified and the integration readily effected by replacing the rectangular coordinates, x and y, by their equivalents in terms of the polar coordinates, r and θ. When this is done and the proper numerical values of the factors are introduced, the total deflection of a ray, passing near the edge of the sun, is found to be:

$$0''.83$$

Now it must be borne in mind that this computation is one of simple mathematics, that the fundamental formula is one of ordinary physical optics and does not involve any curvature of space, or non-Euclidean geometry. The predicted relativity "bending of star light" thus depends upon the truth of the wave theory of light, and upon an assumption that gravitation decreases, or retards, the actual speed of the wave in and through space. The particular value, 0″.83, of the 1911 prediction depends solely upon the special numerical assumption as to the rate of decrease in the speed of the wave as it approaches the sun.

In 1916 Einstein collected his various independent researches and wove them into the "General Theory of Relativity"; and, in so doing, was obliged to change his formula for the propagation of light in and through a gravitational field. The principle of equivalence, which had theretofore been applied solely to changes in clock rates, was now seen to be equally valid for changes in the lengths of measuring-rods. As such rods when in motion shrink, in accordance with the fundamental concepts of relativity, solely in the direction of motion, so this new development assumed that a measuring-rod, when placed radially in a gravitational field, would be shortened by a definite amount, but would retain its unchanged length, when in the tangential position. And this new development, in order to maintain the cardinal precept of relativity as to the measured velocity of light, involved and necessitated the corresponding assumption that a ray of light, proceeding radially, will be slowed down. But a ray of light passing tangentially through the gravitational field will have the velocity prescribed by the 1911 formula. Thus in the General Theory, as announced in 1916, the actual velocity of a ray of light, passing through a given point in a gravitational field, depends upon the direction in which it is moving:—the ray will travel faster when moving transversely, than when travelling directly towards or away from the sun.

The amount of this new radial retardation in the speed of light must be exactly proportional to the shrinkage in the length of the ideal measuring-rod, as given by the principle of equivalence. This shrinkage at any point in the gravitational field is determined by the "velocity from infinity" used in connection with the Lorentz formula. And these formulas readily give this radial shrinkage as m/rc^2, or in terms of the Newtonian potential, Φ/c^2. For a ray proceeding diagonally through the gravitational field, this new retardation will be similarly found from the component of the "velocity from infinity" in that direction. And this retardation becomes less and less as the ray becomes more nearly tangential, vanishing entirely for transverse ways.

The total amount by which a ray will be slowed down at a given point in a gravitational field thus depends upon two factors:—one a general slowing down of the ray, due solely to the gravitational effect upon "clocks"; the other, varying with direction, due to the effect upon "measuring-rods". Einstein united these two effects in a single formula, and gave the velocity of a ray of light, proceeding in any direction in a gravitational field, as:[6]

$$\gamma = \frac{c}{c_0} = 1 + \frac{\Phi}{c^2}\left[1 + sin^2\theta\right]$$

where θ is the angle between the radius-vector at any point on the path of the ray and the normal, drawn from the sun to the path. And this formula differs from that of 1911 solely by the addition of the term depending upon the direction in which the ray is propagated.

Now in revising his prediction as to the deflection of light by the sun to accord with this new development, Einstein used the same simple formula of physical optics, which he had used in 1911. In this formula he merely inserted the new value for the theoretical

[6]EINSTEIN: "Die Grundlage der allgemeinen Relativitätstheorie," Annalen der Physik, 49: 1916, page 822.

Einstein's notation has here been changed to conform to that used in his earlier papers; so that the 1911 and 1916 formulas can be directly compared.

variation in the velocity along the wave-front; so that the whole computation of 1916 involves nothing but a simple formula of physical optics and an assumed, or hypothetical, change in the speed

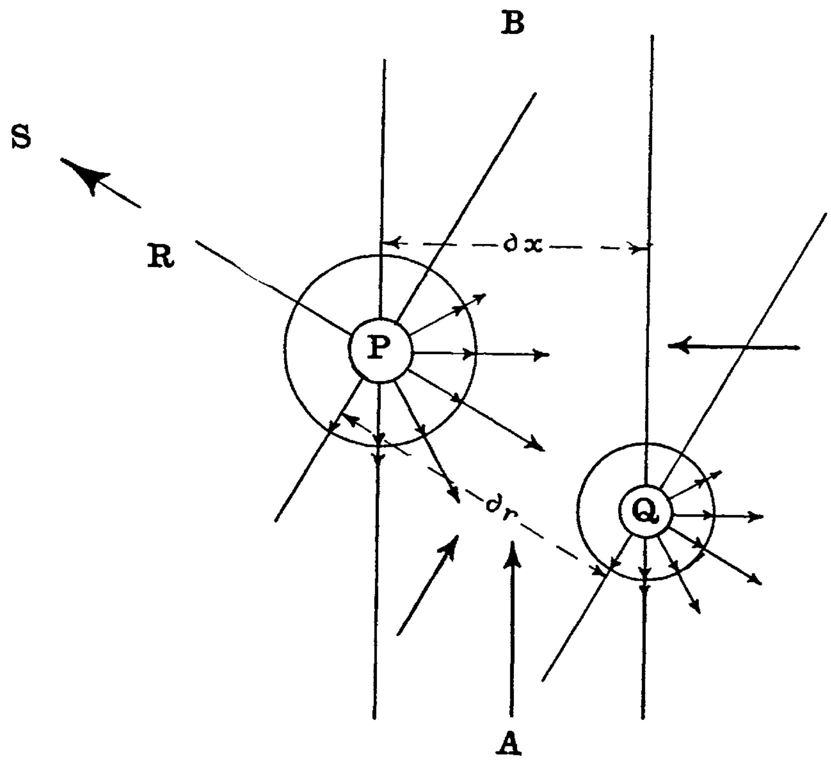

Fig. 2. EINSTEIN: Retardation of Light.

The 1911 assumption is represented by the arrows within the circles at P and Q, drawn contrary to the direction of the ray. The retardation depends solely upon the distance from the sun, being independent of direction, and is the same for rays not shown. The retardation at P is greater than that at Q.

The 1916 assumption is represented by the longer arrows extending beyond the circles. The additional retardation depends upon the direction in which the ray is travelling, being equal to the 1911 general retardation for radial rays and zero for transverse rays.

Einstein's Error. The essential factor for calculating the deflection of any ray is the difference in the lengths of corresponding arrows at P and Q. In his calculation Einstein used the difference in the lengths of radial arrows at P and Q instead of the difference corresponding to the path of the ray, which travels from A towards B.

of light as it passes through a gravitational field. In these formulas, and in Einstein's computation, there is not the slightest trace of non-Euclidean geometry, nor of "curvature of space". Unfortunately Einstein does not give the numerical calculation in detail: he gives merely the formulas and the result, stating that by carrying out the calculation:[7]

> "According to this, a ray of light going past the sun undergoes a deflection of 1″.70."

This is exactly double the value given in 1911, and this doubling of values has given rise to many speculations, and to many and varied explanations, on the part of the relativists.

An inspection of the formulas, which Einstein used, shows exactly what he did and how he derived this result. The essential factor in the formula is that for the rate of change of velocity along the wave-front $\delta c/\delta x$; and this is the only factor in which any change can be made. All the other terms and factors of the formula are always identically the same. But, according to specific statements and to the speed formula, the value of this essential factor for a transverse ray, at its nearest approach to the sun, is identically the same in the 1916 theory as in that of 1911, but its value for a ray passing through the same point and proceeding directly towards the sun is, in the 1916 theory, just double that amount. Einstein used in his calculation the value of this factor for a radial ray, not that of a transverse ray; and thus introduced the mysterious factor two (2). This, of course, was a plain, straight mathematical error:—the rate of variation in the speed of a radial ray has not the slightest thing to do with the "bending" of a transverse ray.

Nor would it be mathematically correct to use, in this computation, the special value of this factor for a transverse ray at its closest approach to the sun, or the value of a transverse ray at any point. The use of such values would give identically the same result as did the 1911 theory, namely 0″.83 for the total bending. For as the ray approaches the sun from a distant star, it is neither radial, nor transverse:—at a great distance from the sun it is very nearly radial; at its closest approach it is exactly transverse. Hence the value of this essential factor $\delta c/\delta x$ in the calculation will never be equal to that used by Einstein in 1916, and will always (except at the moment of closest approach) be greater than that used in 1911. But, as the greater portion of the "bending" occurs in the central portion of the ray's path, the total deflection will be much nearer the 1911 value than to the 1916 erroneous calculation. If

[7] EINSTEIN: Loc. cit. page 822.

the correct value of this essential factor for this varying ray be used in the formula, and the result be integrated in the usual way, one will then obtain for the full deflection according to Einstein's assumptions and basic formulas:

$$1''.10$$

and not the 1″.70 of Einstein's paper.

All of this may be checked and confirmed by the work of Eddington as given in his famous "Report" to "The Physical Society of London". He used a different notation, but his formula for the velocity of a light-ray in a gravitational field is identical with that of Einstein. That this basic formula of relativity indicates a difference in the velocities of radial and transverse rays is made specific by the statement[8]

> "The velocity thus depends upon direction."

Before computing the deflection of a ray of light by the gravitational action of the sun, however, Eddington makes a change in this speed formula. Upon the plea that it is[8]

> "inconvenient to have the velocity of light varying with direction"

he shifts his origin of coordinates, or point of measurement for *r,* from the fixed centre of the sun to a moving point, which travels on the circumference of a small circle about the sun's centre. That is, he finds that by using an approximate and variable value of *r* for measuring the velocity of the ray at various points in the gravitational field, it is possible to so transform Einstein's formula as to make the velocity appear the same in all directions; to make the radial and transverse velocities the same function of the different and variable *r*'s. But this is merely a mathematical trick of using a shifting origin, of measuring velocities of different rays from different points and in terms of different coordinates.

With this new formula the retardation of the ray, due to the gravitational action of the sun, appears as $2m/rc^2$, or exactly twice that of the 1911 theory. Neglecting the fact that he had "slightly" altered the coordinates, Eddington proceeded to use this expression for the retardation in identically the same way that the 1911 expression was used, and, of course, obtained a result just twice as

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[8]EDDINGTON: "Report on the Relativity Theory of Gravitation," 1918, page 53.

great, namely 1″.70. But this result, so obtained, does not represent the curvature of the actual path of the ray by the sun; it represents a fictitious curvature as measured from a moving origin of coordinates. Eddington's result is neither mathematically correct,

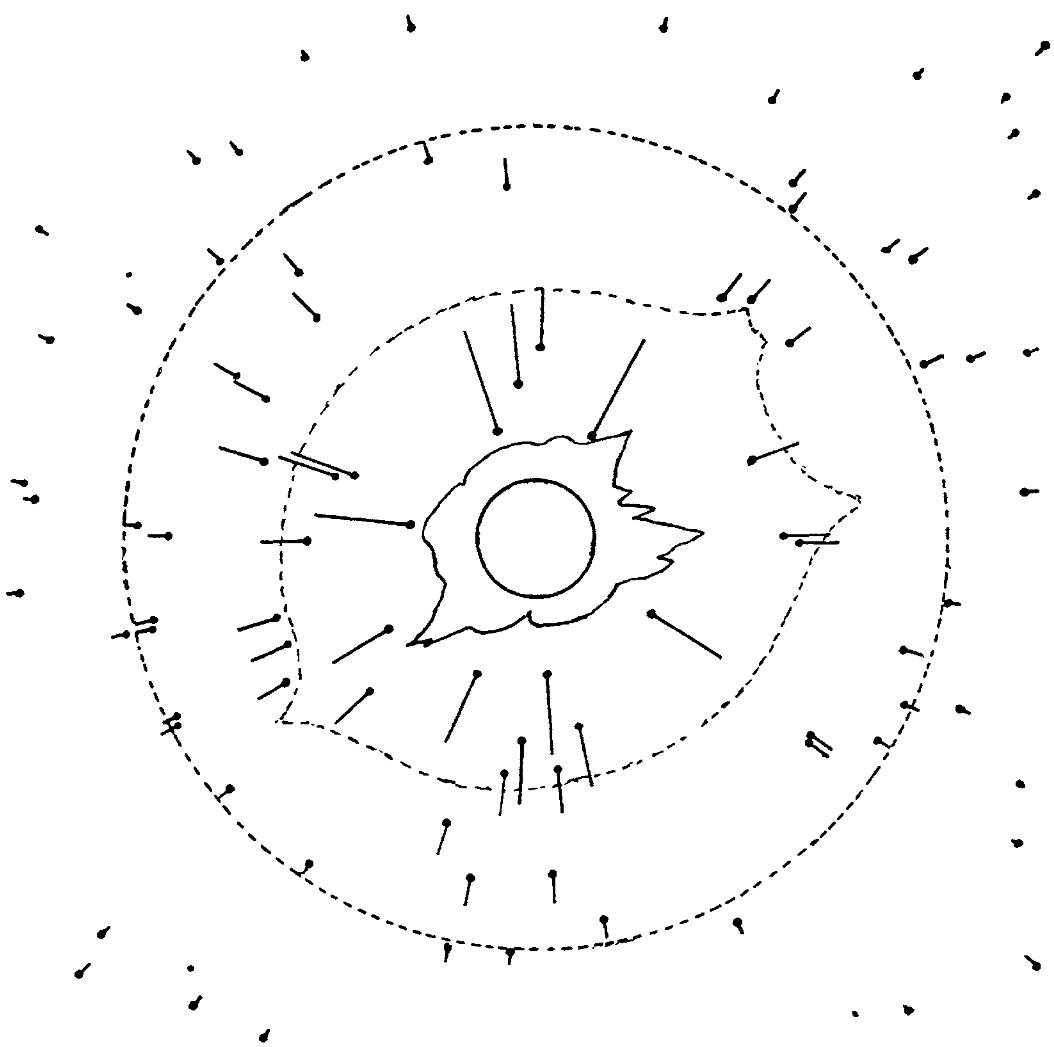

Fig. 3. The Einstein Prediction.

This figure shows the displacements, according to the prediction of Einstein, of the 92 stars observed by Campbell and Trumpler. These theoretical displacements are not the full predicted star displacements, but are the relative displacements derived from them, by Campbell and Trumpler, by applying the scale correction used in their Star Chart.

nor in accord with the basic assumptions of relativity in regard to the propagation of light. If Einstein's assumptions be correct, that the actual velocities of rays passing through a given point in space are different, if it be true, as both Einstein and Eddington state, that the actual velocity of a radial ray at any point is less than that of a transverse ray, passing through that point, then by no trick of changing origins of coordinates, by no possible mathe-

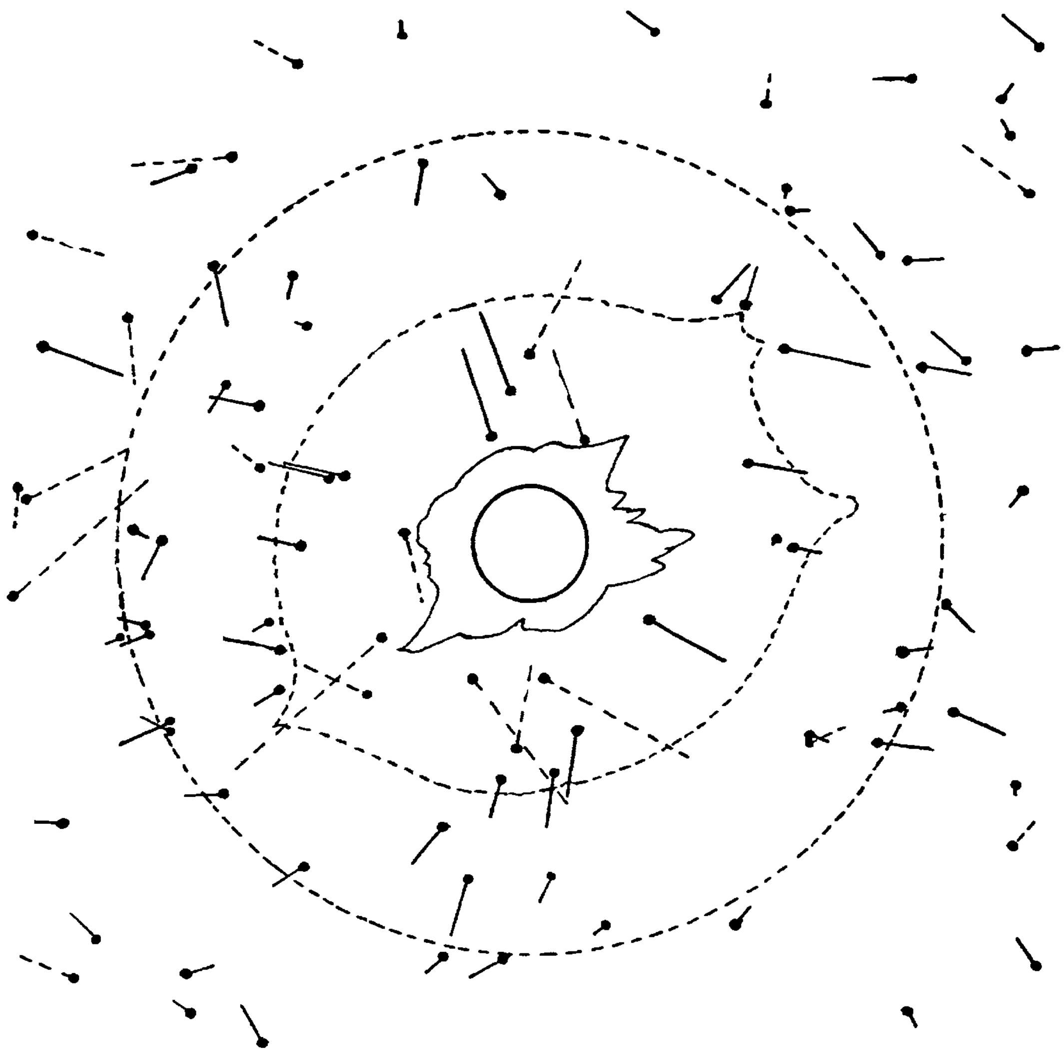

Fig. 4. The Eclipse Observations.

This figure shows the relative displacements of 92 stars as observed by Campbell and Trumpler at the total solar eclipse of September 21, 1922, and is from a direct tracing of the Star Chart in Lick Observatory Bulletin, No. 346. In that chart, however, the displacements of 21 stars were omitted: these have been added to make the above diagram complete and are shown by dotted lines.

matical means, can these two different velocities be made the same.

From all this it would appear to be clear that Einstein made an error in mathematical computation, when in his "Foundation of the General Theory of Relativity" he gave the figure 1″.70 for the theoretical relativity bending of light-rays by the sun. In accordance with his own assumptions and fundamental formulas this figure cannot be over 1″.10. The discordant and often elusive photographs of the various eclipse expeditions have shown, according to the published reports, a light deflection, varying between 1″.61 and 1″.98 for the 1919 eclipse, and between 1″.41 and 2″.16 for the 1922 eclipse; Campbell's result being first reported as 1″.72, but afterwards modified by the use of corrected data and made 2″.05[9]. If these observations represent actual deflections of light, then some cause other than the Einstein theory of relativity must be called upon to account for them.

In considering the results of these various expeditions it must also be remembered that no checks or controls were applied to determine either the cause of the observed stellar displacements, or even the point in space where the apparent bending occurred. No evidence has been given to show that the rays were bent in the vicinity of the sun: the observed effects may have been entirely local and due to abnormal conditions in the earth's atmosphere caused by the passage of the eclipse shadow. Simple and effective checks for the elimination of the effects, due to such possible causes, were available, but were not used.

It would thus seem that the general conception regarding Einstein's theory of light deflection is not in accord with the facts. In his actual formulas there is no trace of non-Euclidean geometry, or "curvature of space", nor do these formulas involve, or depend upon any "new" law of gravitation. The truth or falsity of the Newtonian law is not involved in his concepts, formulas, or predictions. The changed prediction of 1916 was due to the adoption of a new or revised formula for the hypothetical "retardation" of light in its passage through a gravitational field.

It would seem further that Einstein made a mathematical error in the practical application of his formulas to the eclipse problem: an error so large as to destroy the "confirmation" by the various eclipse expeditions.

New York City, December 15, 1926.

[9]CAMPBELL AND TRUMPLER: "Lick Observatory Bulletin, No. 346," page 54.