THE TRAVELS OF PETER, PAUL, AND ZEBEDEE

THE foregoing reply of Professor McGilvary is somewhat random in its arrangement and divagates into several side-issues, some of them chiefly of a personal interest. It is therefore likely to obscure rather than make clearer for the reader the nature of the main philosophical question under discussion and the general logical outline of the reasoning against which the reply is directed. It becomes needful, then, if the discussion is to reach any definite outcome, to restate the main question and to recapitulate in a connected way my argument with respect to it. I shall then proceed to examine whether McGilvary has successfully controverted any of my essential premises or shown that the conclusion which I have propounded does not follow from them.[1]

A. The gist of that argument consists of the following propositions:

I. Retardations of clocks and of physiological processes are, in the special theory of relativity, said to occur in any system S which is in unaccelerated motion relatively to another system, S'. II. These retardations are comparative; *i.e.*, the rates of standard clocks and of physiological processes in S are said to be slower than those in S'. III. But since both S and S' can equally legitimately be said to be in equal unaccelerated motion relatively to one another, equal retardations must be ascribed to the clocks, *etc.*, in *both* systems. IV. The two clocks may legitimately be supposed to be set in motion, and the two persons to be born, at the same time. V. If (as is implied by the story of the twins-of-unequal-age) the comparative lags, or retardations, are supposed to occur as physical facts in the systems to which they are referred, the foregoing propositions lead to the following contradictions: 1. The clock in each system goes more slowly than that in the other, and the person in each system ages less rapidly than the other person. (This follows from I–III alone.) 2. The reading of each clock, on S and S' respectively, will be slow in comparison with the other at the same moment, and each of two persons will be physiologically younger than the other at the same time. 3. Each clock will thus have two readings at once, and each person will be of two ages at once. VI. If, however, the retardations and resultant inversions of ages and readings are regarded, not as physical facts in the systems to which they are referred, but merely as a "mutual disparity of appearances" —*e.g.*, if the retardation of the clock or of the physiological age in S

[1] The previously published papers in this discussion will be hereafter referred to as L. I, McG. I, and L. II.

is held to exist only for, or from the point of view of, observers in S' —*these* contradictions do not arise. (I do not discuss the question whether no others remain.) The contradictions in V, 2 and 3, were, further, said by me to arise (upon the supposition mentioned) in either of the two possible cases: *i.e.*, V, *2a*. if the two clocks are, at the moment under consideration, *at the same place*, there will be a simultaneous reciprocal inversion of *coincident* clock-readings, and of the ages of persons at that place; V, *2b*. if the two clocks, or persons, in their respective systems, are, after a period of relative motion, at points distant from one another, there will be a similar mutual inversion of readings or of physiological ages at the same time but not at the same place. *2a* is the case in which the unaccelerated relative motion is a motion of approach, *i.e.*, in which the two clocks or persons start at a distance from one another and eventually meet; *2b* is the case of a motion of separation, in which they start from the same place and (since the motion is by hypothesis unaccelerated) do not return to one another. *Mutatis mutandis*, *3a* and *3b* correspond to *2a* and *2b*, and are corollaries of them.

How many of these propositions does McGilvary accept? I and II are undisputed. It might at first reading be inferred from some of his remarks about "reversibility" that he disputes III; but in fact, of course, he does not do so. If Peter and Zebedee, on their respective moving platforms, are supposed to start from distant points, A and P', then, says McGilvary, it is true (as I had observed) "that, according to the special theory, the comparative retardations arising while they continuously approached one another would be bilateral and equal"; this is "one of the best-known articles of the relativist's creed".[2] There *is*, in short, a "*mutual* lag". McGilvary's objections relate to

[2] *Cf.* also McG. I, p. 371: Postulating certain values for the variables in the equation, "*measured by the synchronism of Paul's clocks*, any one of *Peter's clocks* loses time, in going from any point to any other point of Paul's platform, at the rate of 7 seconds in every 10; *i.e.*, it runs only 3/10 as fast as Paul's clocks; *measured by the synchronism of Peter's clocks*, any one of *Paul's* clocks loses time, in going from any point to any other point of Peter's platform, at the rate of 7 seconds in every 10; *i.e.*, it runs only 3/10 as fast as Peter's clocks." What is, in McGilvary's paper above, called "Zebedee's clock" is "one of Peter's clocks". The reader will observe that this mutual lag results from the application of one of the transformation-equations, and that since McGilvary has himself made the computation, it is unnecessary that I should repeat it. But *this* "result of the equations" is fundamental in my argument; the rest consists in an analysis of the meaning of certain concepts employed in the theory and in pointing out that McGilvary's own computation asserts (as well as denies) the simultaneity of the birth of Peter and Zebedee.

the propositions under IV and V; and the principal part of his argument has to do with IV, *2a*. It is with these issues, then, that I shall deal. It will be observed that if it is shown that *any one* of the contradictions under V can be shown to follow from the theory—together with the assumption of the physical reality of the retardations—the general conclusion of my previous papers is established.

B. My Proposition V, 1 relates, not to the comparative readings of the two clocks or the ages of the two travellers at a single moment, but to the rates at which clocks go and physiological processes take place during the *period* in which the clocks or bodies are in unaccelerated relative motion. And this proposition follows directly from, or rather is equivalent to, Proposition III, which is admitted. Now if this mutual lag is conceived to be a physical fact, if each clock *really* goes only 3/10 as fast as the other during the same period, Proposition V, 1 is, I submit, a contradiction; though it is not a contradiction if each clock is merely judged by observers on the other platform to go at this comparative rate. To say that throughout a common duration the S-clock actually goes more slowly than the S'-clock and the S'-clock goes more slowly than the S-clock, and therefore that each also goes faster than the other, is as contradictory as to say that, at a given moment, each clock *is* slow in comparison with the other. And if it be suggested that there is no common period during which both clocks are going and moving, I reply that this is equivalent to saying that they are *not* in motion relatively to one another. To predicate relative motion of them is to assert that *while* one of them is becoming nearer to the other, the other is becoming nearer to it.[3] This "article of the relativist's creed", then, can be saved from contradiction only if the mutual lag is construed as merely apparent.

C. I pass to Proposition V, *2a*. In the concrete hypothetical case assumed by both of us, Peter is born at A' on the S'-platform, at a time when A' is coincident with A on the S-platform; and Zebedee is born at P on the S-platform (opposite P' on the S'-platform). The platforms are already moving; and each traveller from the moment of his birth is in an unaccelerated motion of approach relatively to the other. Also, at the moment of the birth of each, a standard clock is set going beside him, and accompanies him throughout his journey. Both travellers and clocks, on their own platforms, remain throughout at A' and P respectively. We may also assume an observer, Batholomew, who remains at point P'. When A' and P become (virtually) coincident, the two clocks are said to be "in the same place". Supposing

[3] With this last point I shall deal further under *D*.

that the mutual retardations are construed as physical facts, does it follow that *at this place* Peter's clock will be slow in comparison with Zebedee's and Zebedee's slow in comparison with Peter's by the same amount, and that Peter will be physiologically younger than Zebedee and Zebedee younger than Peter by the same amount?

McGilvary denies that (according to the special theory) there will or can be such a mutual inversion of coincident clock-readings, and of ages at the same place. His reason for denying it (repeated in many passages and in varying phrase) is that (*i*) this consequence does not follow from Propositions I, II, and III *alone;* (*ii*) that it would follow only if Peter and Zebedee could be born, and their clocks set going, *at the same time;* (*iii*) that, upon the principles and equations of the special theory, it is inconceivable that the two infants should be born, or the two clocks set going, at the same time. He also represents me as maintaining that the consequence *does* follow from Propositions I, II, and III alone. He asserts that I ignore the distinction between "the comparative retardation ('lagging') of a clock, and the comparative smallness ('slowness') of its instantaneous reading". Though the distinction, he rightly intimates, is obvious to the meanest intelligence, he assures the reader that I "labor under a most amazing and insistent confusion" on the point, since I reason directly "from the fact that in relativity there is a reciprocal lag to the conclusion that there is a reciprocal slowness of coincident clock-readings".

I reply that I have never "confused" the two ideas in question and do not argue in the way in which McGilvary says that I do. What I have said, and am now obliged to repeat, is the following: if two clocks on two platforms in relative motion are *set going with the same reading,* say 12 o'clock, *at the same moment,* and if either of them thereafter continuously lags in comparison with the other, this will *result* in a difference between their readings when, at any given subsequent moment, they are compared: the reading of the clock that has lagged will necessarily be "slow" or "smaller"; and if each lags equally in comparison with the other, the reading of each at any such moment will be "slow" in comparison with the other's. The simultaneity of the starting of the clocks (and of the births of the infants) and the initial identity of the readings of these clocks were explicitly and repeatedly specified as premises of the argument to which McGilvary is here referring.[4] I offered a proof of this premise, *i.e.,* of Proposition IV; and it was solely from Propositions I, II, III, *and* IV that I deduced the consequence which he denies. Whether that proof

[4] See, *e.g.,* L. II, pp. 560, 561.

is valid I shall consider in the next section; but it is a gross perversion of my argument on this point to give the reader to understand that Proposition IV was not an express part of it, and therefore to represent me as confusing the two conceptions mentioned. McGilvary, moreover, is aware that Proposition IV was set down by me as an essential premise of the argument for V, 2, and therefore that I distinguished the two questions; for in another passage above he writes: "Lovejoy . . . acknowledges that the establishment of proposition (2) presupposes the establishment of proposition (1)." In the numeration of my preceding paper, to which he is here referring, "proposition (1)" is the same as that which I have designated as "Proposition IV" in the above résumé of my argument as a whole, and Proposition (2) is the same as V, 2.

D. The issue concerning Proposition V, *2a* thus resolves itself into the question of the validity of Proposition IV; in other words, McGilvary's thesis that there is no "reversibility of coincident clock-readings" (or physiological ages) depends wholly upon the proposition that Peter and Zebedee—being on different moving platforms and at distant points—cannot, on any relativistic principles, conceivably be "born at the same time".[5] I have shown, and shall again show, that this proposition is false—that the relativist must, and constantly does, make assumptions which imply the possibility of what McGilvary, to justify his case, is compelled to declare to be impossible. The *particular* supposition that the two events of Peter's and Zebedee's births can occur simultaneously at A' and P on their respective platforms, can, obviously, be excluded only on the general ground that, in a relativistic universe, *no* two distant events on different moving bodies can be simultaneous. Now it is true that Einstein has sought to prove—

[5] This can most plainly be seen at McG. I, p. 370. McGilvary there computes that, in spite of the fact that each clock during their relative motion goes only 3/10 as fast as the other, *Zebedee's* clock will not, at the moment of their meeting, be slow in comparison with Peter's—the reason being that the clock at P' has a reading 212⅓ seconds less than Zebedee's, at the moment of the coincidence of P with P', and that Peter's synchronized clock at A' therefore will, at that same moment on Peter's platform, be correspondingly behind Zebedee's. This slowness of Peter's clock at the start more than offsets Zebedee's comparative lag. It is to be observed, however, that there is no reason why a clock at P' should not be *set* going with the same reading as the clock at P, when these points are coincident, and no reason why, at what McGilvary admits is the same moment on Peter's platform, Peter's own personal clock at A' should not be set going with the same reading as the clock at P'. This consideration is equally fatal to McGilvary's argument; but for the sake of brevity I do not develop it here.

though by arguments which I have elsewhere, I think, shown to be unconvincing—that no *pair* of events on *one* platform which are found by observers on that platform to be simultaneous with one another can conceivably be regarded as simultaneous with one another by observers on the other platform. That this (whether true or not) is a "genuinely Einsteinian premise" I have already recognized.[6] But it is not equivalent to a proof that no two distant events on *different* bodies, or in different systems, can be simultaneous; and it is the latter proposition that I assert to be inconsistent with the theory of relativity. The reason why it is lies in the fact that the two bodies in question are assumed to be in relative motion; and that to say "two bodies are in relative motion" is to say (among other things) that there is a series of moments identical for both, and therefore the possibility of a corresponding series of simultaneous events occurring on the two bodies.

For—as I have previously pointed out—the two bodies can conceivably be in relative motion only during a "common while". Distance, or spatial separation along a common axis, is a symmetrical relation; and a change in this relation between the two cannot occur in the case of one of them unless it at the same time occurs in the case of the other. One body cannot start to become nearer to the other unless the other at the same instant starts to become nearer to it; if the latter is denied, it is *thereby* denied that any relative motion between them has begun. McGilvary now grants that for the relativist there is a common while. Such a common while is, *e.g.*, "the time-interval between the births of Peter and Paul at A and A' and the arrival of Peter at P", where he meets Zebedee. But McGilvary adds that in relativity theory this time-interval does not have "the same measure on the two platforms". It does not, of course, have the same measure for observers on the opposite platforms; if it did there would be no retardation in any sense. But he apparently supposes that there are "common *moments* on the two platforms" only for events which occur at coincident points. But if the "common while" is admitted, the restriction of common moments to events occurring at coincident points is inadmissible. "Whiles" contain "whens"; and if the time-interval between Peter's birth at A, and his arrival at P, is common to both systems (as we have seen that it must be) there must also be throughout this interval a sequence of common moments. It must be possible to say—and plainly *is* said by the relativist, in the very act of asserting relative motion of Peter and Zebedee—"*Now*, at the common moment m_1, the mutual approach of Peter and Zebedee

[6] *E.g.*, L. II, p. 559.

is beginning"; and again: "*Now,* at the common moment m_2, Peter is nearer to Zebedee, and Zebedee is reciprocally nearer to Peter, than at m_1"; and so on, until he can say: "*Now,* at moment m_n, Peter and Zebedee have reached the same place, *i.e.*, A' and P have become coincident." This community of moments is similarly implicit in the notion of comparative retardation. Peter's clock and his physiological age are not supposed to get their retardation all at once, at the instant of their arrival at P. Unless there are before that instant times at which it can be meaningfully said: "*Now* the ratio of Peter's age to Zebedee's is no longer the same as at the moment of his birth", the assertion of the actual retardation of Peter's physiological processes in comparison with Zebedee's in the course of his motion can have no meaning whatever.

The major premise necessarily presupposed by the proposition that Peter and Zebedee cannot be born at the same time is, then, plainly inconsistent with the theory of relativity; for it is tantamount to the proposition that both relative motion and continuous comparative retardation are inconceivable. This I had, as I supposed, already made clear; but as McGilvary seems to have hitherto been unable to grasp the point, I have been compelled to repeat it here somewhat more fully. McGilvary has—with some relativistic physicists—failed to ask himself the question: *What precisely is meant by relative motion?* This concept is obviously fundamental in the theory; and it is peculiarly in the province of the philosopher to insist upon a definition of it, or to make fully explicit the definition which is plainly presupposed by those who employ it. To neglect to do this is a singular piece of *philosophical* naïveté. When the question is asked and answered, it becomes evident that the relativist is completely committed to the admission that events at distant points on different plaforms can really occur simultaneously and that therefore there is no reason why the births of Peter at A' and of Zebedee at P should not be supposed to occur simultaneously—that is, at one of the "common moments" during the time-interval in which the two platforms are in relative motion. And when this is recognized, it becomes evident that the foundation of McGilvary's entire argument—on the present issue—collapses.

The identity of the times of birth of Peter and Zebedee can also be shown in another way, from McGilvary's own admissions. Peter is admittedly born at A' simultaneously with the birth of Paul at A; and Paul's birth is admittedly simultaneous with Zebedee's birth at P. That the "connotation" of simultaneity is the same in both cases

McGilvary again agrees.[7] To the proposition which, as I had pointed out, directly follows from these admissions—*viz.*, that two events which are in the same sense simultaneous with the same event are simultaneous with each other—McGilvary offers no real reply and no specific denial. He does, indeed, reiterate his previous argument that Zebedee's birth cannot be admitted by a relativist to occur at the same time by Peter's clocks as by Paul's. But he neglects to meet *my* previous argument.[8] If (as he grants) Zebedee's birth occurs at $t = 0$, and Paul's birth occurs at $t = 0$, and Peter's birth occurs at $t' = 0$, and t' and t are the same moment—then Peter's birth occurs at $t = 0$, *i.e.*, his birth and Zebedee's occur at the same moment. And that this t' and this t are the same moment is expressly declared: "A and A' are to coincide on January 1, 1900, exactly at noon, which is to be *the zero hour for both systems;* thus Paul's clock at A has *then* the reading $t = 0$, and Peter's clock at A' the reading $t' = 0$".[9] Obviously two points cannot be coincident at separate moments. And $t = 0$ cannot be the same moment as $t' = 0$ at A (which $= A'$) but a different moment at P; nor does McGilvary assert that it is. If an "*intelligent* observer" at P knows that the time of Zebedee's birth there "exactly at noon" is simultaneous with Paul's birth "exactly at noon" at A, and that Paul's birth at A and Peter's at A' are happening at that identical place at the same time by both of these coincident clocks, he also knows that all three births are happening at once, though at two different places. There is thus no way in which the simultaneity of Zebedee's birth with Peter's can be denied without denying McGilvary's initially assumed simultaneity of Paul's birth with Peter's—which in turn cannot be denied to be a legitimate assumption unless it is also denied that it is possible for the points A and A' ever to be coincident. Now this argument must be met *directly,* not by pointing out *other* relativistic theses with which its conclusion is inconsistent; "for such latent inconsistencies in the theory", as I have already remarked, "a critic of it is not responsible". But no attempt to meet it directly has been made by McGilvary—no doubt for the excellent reason that none is possible.

[7] McG. I, p. 363, and again above.

[8] L. II, p. 558.

[9] McG. I, p. 363: my italics. In his present paper McGilvary remarks that I "get my conclusion only on the assumption that what occurred at time $t = 0$ on Paul's platform also occurred at time $t' = 0$ on Peter's platform". But he has himself repeatedly assured us that this assumption is true. The reader should perhaps be reminded that he is not to understand from the use of the term "zero-hour" that the platforms are not already in motion at the time in question.

But though he fails to join issue with the crucial proposition that events simultaneous with the same event are simultaneous with each other, he does attack in other ways the argument that Peter's and Zebedee's birth may be and, given the initially accepted hypothetical situation, must be simultaneous. His objections are four in number.

i. One of them is put thus: "A proof that *a* cousin of Paul's may be born at P at the time mentioned, is no proof that this possible cousin is '*the* cousin' who was by hypothesis born at time $t = 0$ by Paul's clocks", *i.e.*, at the time when A was passing A' and Peter and Paul were being born at A' and A respectively. The exquisite pointlessness of this remark must, I suppose, be plain to every reader. We are dealing with a hypothetical situation. Any *kind* of event may, so far as the theory of relativity is concerned, conceivably occur at the point P on Paul's platform, at the time $t = 0$ by the synchronized clocks on that platform. There is consequently nothing to forbid us to suppose that an infant is born there at that time—any more than there is anything to forbid us to call that infant, for convenience of reference, "Paul's cousin Zebedee". That Zebedee, then, is born at P at that moment, is a permissible part of the hypothetical set-up.

ii. My second fallacy in this matter is, it seems, a *petitio*: I "*assume* a simultaneity-at-a-distance which is common to the two platforms". I do not "assume" this; I have proved, in the previous paper and again above, that it is and must be assumed by the relativist, and is assumed by him for two distinct reasons. There is therefore no *petitio*.

iii. McGilvary's third objection here is based upon the observation previously mentioned, that the "measure" (in terms of clock-readings) of the time-interval between two events on Zebedee's (or Paul's) platform will not be the same for observers on Peter's platform. To this I have already replied. McGilvary now, however, adds to his previous irrelevance on this matter the remark that "the lack of common measure" does not "make relative motion impossible". This is equally irrelevant, since I have never suggested that it does make relative motion impossible. What I have asserted is that the denial of common moments of occurrence of events on the two platforms would make relative motion inconceivable.

iv. McGilvary's principal indirect argument on the point runs as follows: If we suppose an observer at P', this observer will, according to the transformation-equation for time, find that Zebedee's birth occurs, not at time $t' = 0$, but at time $t' = -212\ 1/3$, *i.e.*, that number of seconds (or hours, or years, according to the units assumed) *earlier*

than the time of Peter's birth, *viz.*, $t' = 0$, or "12 o'clock". Let us assume that, given suitable values for the variables, the transformation-equation would yield this result. Does this invalidate my foregoing argument and therefore show that Zebedee and Paul could not be born at the same time? It does not, for the following reason. By McGilvary's own statement, already cited, $t' = 0$ and $t = 0$ are the same moment; and Peter is born at $t' = 0$ and Zebedee is born at $t = 0$, in short, both are born at the same moment. If the equation shows that Zebedee is *not* born at $t = 0$, or that $t' = 0$ is *not* simultaneous with $t = 0$, we are simply confronted with a contradiction in the theory—and in McGilvary's argument.

McGilvary's fundamental error in this argument arises from the fact that he has not really reflected upon the consequences—on relativistic principles—of the admittedly legitimate hypothesis of the two adjacent parallel platforms in unaccelerated relative motion. He recognizes two separate groups of events, simultaneous on the two platforms respectively, *i.e.*, by Peter's and Bartholomew's synchronized clocks, and by Paul's and Zebedee's synchronized clocks. And he tells us that this simultaneity of clocks synchronized by light-waves on the same platform is "the invariable connotation of 'simultaneity' in the special theory". But it notoriously isn't. The special theory also recognizes that two events *occurring together at the same place* are simultaneous, apart from the procedure of distant-synchronization by light-waves.[10] And two events may thus occur together at the same place *on different moving platforms,* if the platforms are immediately adjacent; this is recognized by McGilvary himself when he grants that Peter and Paul may be born at the same place at the same time, *viz.*, at the time when their birth-places, A' and A, are momentarily coincident. There is, then, a *cross-simultaneity between the two platforms*; $t = 0$ and $t' = 0$ are temporally identical. The attempt at a rigorous segregation of the groups of events on the one platform and on the other thus breaks down, when the platforms are in immediate juxtaposition. The synchronism on the one becomes entangled with the synchronism on the other. For it is not asserted by McGilvary—or by Einstein—that the one word 'simultaneity' stands for two opposite kinds of temporal relation.

It will perhaps be suggested that the relativist ought to assert this

[10] This is elsewhere admitted by McGilvary; *cf.* McG. I, p. 363: "Events that occur simultaneously at coincident points on our two platforms are simultaneous on both platforms." There is, indeed, only one *connotation* of simultaneity, but there are two classes of cases in which it is exemplified and verifiable.

and thus maintain that Peter and Paul, and Paul and Zebedee, were not born at the same time in the same sense. But this could hold good only if the clocks on Paul's platform were not truly synchronized. When A and A' are coincident, clocks or stop-watches at each point may obviously be set going with the same reading; McGilvary himself assumes that both clocks there have then the reading of 12 o'clock. This is the moment of Paul's birth; it is admittedly also the moment, on Paul's platform, of Zebedee's birth at P. This means that, if P is 100 light-seconds distant from A, and if at the instant of Paul's birth a light-ray is emitted from A, it will reach P 100 seconds after Zebedee's birth, by any clock situated at P, and, if reflected back, will return to A 200 seconds after Paul's birth. A relativistic observer at A, knowing the distance from A to P and learning that his light-ray reached P 100 seconds after Zebedee's birth there, must necessarily say to himself: "Since the light-ray really travels along my platform (or any other) with a constant velocity, and since by the accepted test of synchronism it is shown that Zebedee was born at the same time as Paul, it is clear that he was born at the same time as any other event occurring at A when Paul was born there. But Peter's birth occurred at A when Paul was born there; for A and A' were then coincident, *i.e.*, were separated only by a minimal distance which the theory treats as negligible. It is true that the light-signal returns to me here at A at a moment which is not that of the birth of either Peter or Paul, and that Peter is no longer here. But I am not asking whether Zebedee's birth is simultaneous with an event which occurs at A 200 seconds after Paul's birth; I am asking whether it *was* simultaneous with an event which occurred at the same place as Paul's birth at the moment of that birth; and Peter's birth *did* occur at that place at that moment. If the test by light-signals from A to P on my platform shows Zebedee's birth to have been simultaneous with Paul's, it also shows that, by the same test and therefore *in the same sense,* Zebedee's birth was simultaneous with Peter's."

It may, however, be objected that this demonstrated univocal simultaneity of the births of Peter and Zebedee holds good, so far as the reasoning in the preceding paragraph shows, only for observers on Paul's (and Zebedee's) platform. But it is to be borne in mind that the thesis which McGilvary is defending is that the readings which observers at A or P ascribe to Peter's clocks, and the ages which they impute to him, are the actual readings and ages on Peter's platform, *i.e.*, are identical with those which are observed by Peter himself, or (at his birth) by an observer beside him; the same is declared to be

true for observers at A' and P' with respect to Paul's and Zebedee's clocks.[11] It follows that since the observers both at A and A' find $t=0$ and $t'=0$ to be identical, the births of Paul and Peter will be *really* simultaneous. Moreover, the clocks on Peter's platform are also synchronized or synchronizable. This means that, by Peter's time-measurement on his own platform by means of light signals, at every point thereon some event can be shown to have occurred at a moment identical with the moment $t'=0$ at A'. But the moment $t'=0$, which is thus *a single moment for all points on Peter's platform,* is admittedly simultaneous at point A' with the moment $t=0$, which is similarly a single moment for all points on Paul's platform. But Zebedee is admittedly born at P at the moment $t=0$. And at $t'=0$ (*i.e.,* at $t=0$) some point on Peter's platform must be actually coincident with P, since we may suppose the platforms to extend as far as we like from A' and A respectively; and P' is simply the name we have chosen to give to that point. Hence at that moment an observer at P' will *see* that the reading of Zebedee's clock there is $t=0$, and that Zebedee is then just born there. Thus the two observers on Peter's platform, on pooling their information, will find that Peter, by their synchronized clocks, was born at time $t'=0$, and by Paul's synchronized clocks at $t=0$, and also that Zebedee was born at $t'=0$ by their clocks and at $t=0$ by Paul's; *i.e.,* they, as well as the observers on the other platform, will find that Peter and Zebedee were born at the same time.[12]

Now I am not contending that McGilvary's argument with respect to the non-simultaneity of the births of Peter and Zebedee has no basis in the theory; as I had already remarked, McGilvary is here "employing a genuine Einsteinian principle. . . . His reasoning, so far as it goes, is relativistically orthodox."[13] What I have maintained is that on equally good and equally relativistic grounds it can *also* be shown that Peter and Zebedee *would* be born at the same time; and this I have again demonstrated in the foregoing paragraphs. My contention throughout has been that there are *contradictions* in the theory—when the assumption is made that the readings or ages ob-

[11] McG. I, p. 368: "There is no disagreement as to what is directly observed by competent observers at any two coincident points"; p. 365: "Both observers at P and P' actually *see* the readings of both clocks."

[12] It is *not* assumed in this argument that Peter's observers will find the distances A–P and A'–P' equal. If it seems to any reader that there is a contradiction in this, I ask him to observe that it is not *my* contradiction, but is inherent in the theory.

[13] L. II, p. 559.

served and computed from one platform are the actual readings or ages on the other. This, the essential point of my argument, seems to have eluded McGilvary completely; for all that he has attempted to do in his reply (on this point) is to prove over again that *one* of two contradictory consequences of the theory follows from it. In so far as he has done this more elaborately than I had, he has simply strengthened my case. He does not spare protestations that I have completely misunderstood the special theory. I am able to reply, in this case, in more complimentary terms. He understands very well certain of the characteristic and familiar implications of the theory on the present question; but to other less advertised, though scarcely less obvious, presuppositions and consequences of it he remains persistently blind. And what he calls my misunderstanding consists, I venture to suggest, in a recognition of these latter implications and of the fact that they are in conflict with those which *he* recognizes. A proof that they are involved in it is not rebutted by showing—what is not denied—that the others are also, and that the two sets of consequences are inconsistent.

The logical situation here, then, is that (1) it can be shown on *certain* relativistic principles that Peter and Zebedee would not, in the accepted hypothetical case, be born simultaneously; and (2) that it can be shown on certain *other* relativistic principles that they would be born simultaneously. This, in itself, is obviously a contradiction. But let us develop the further consequences of these two implications taken together, using, for the case of the former, McGilvary's own arguments and computations. (1) Assuming that Peter and Zebedee are *not* born at the same time, what will be their ages, and the readings of their clocks, when they meet? McGilvary's answer is that by Zebedee's clock Peter meets Zebedee "at time $t = 70$, *i.e.*, at 70 seconds past 12 o'clock". But Peter's clock on Zebedee's arrival at P, "has the reading $t' = 21$".[14] Thus, since their ages "advance with the rhythm of their clocks", Zebedee at their meeting is of the age of 70 (seconds or years, according to the distances assumed) and Peter is 21. (2) Peter and Zebedee, as has been shown, *are* born at the same time; and, as McGilvary says,[15] Peter's ageing (whenever he was born) proceeds during his relative motion only 3/10 as fast as Zebedee's and Zebedee's only 3/10 as fast as Peter's. Thus—if the retardations are physical effects occurring on their respective platforms—each when they meet will be 3/10 as old as the other. From this consequence there can be no escape, when it is once understood that the two trav-

[14] McGilvary's proof of this is summarized at McG. I, pp. 370–371.

[15] McG. I, p. 371.

ellers could be born at the same time, and that the theory implies that they could (though it also implies that they could not). If we wish to assign specific values to these ages, it is necessary to have one age 'given' for the computation to start from. Let us, then, take the age of Peter as determined by McGilvary, *viz.*, 21, since we are assuming that this follows from one of the implications of the theory. Then Zebedee will, at their meeting, be 3/10 of this age, *i.e.*, a little over 6. But it is equally necessary that Peter be 3/10 as old as Zebedee, since otherwise he would be robbed of the benefit of *his* relative motion; *i.e.*, he will be not quite two. But in order to arrive at the conclusion that Peter is 21, McGilvary assumed Zebedee to be 70; therefore he must be 70. Thus, on combining the consequences of both of these implications of the theory, it follows that Peter will be 21 and 6, and Zebedee will be 70 and 2.[16] Each will be older and younger than the other. But even if we do not combine (1) and (2), the symmetrical paradox, as has been shown, follows from (2) alone. If, however, the comparative retardations are not taken as *real* retardations, all these fantastic consequences can be avoided.

This must suffice for the proof of Propositions V, 2*a* and V, 3*a*.

E. We proceed to Propositions V, 2*b* and 3*b*. There is, of course, in relativity theory no fundamental difference between unaccelerated motion of approach and of separation. Propositions I–III apply to both. Nevertheless it is, for the purpose of the present discussion, useful to distinguish them. For if, *e.g.*, the travellers Peter and Paul start their journeys from the same place, no question can be raised as to the possibility of their being born at the same time, or being of the same age when their relative motion begins; they may, as in the usual story, be supposed to be literal twins. McGilvary's argument for the non-simultaneity of the births has, therefore, here no relevance. Does

[16] The initially given age necessary for getting the computation started is here taken to be an age (of one of the travellers) which has already been retarded in comparison with the other's. This procedure is in accord with the assumption which I am combating, that the comparative lag (and its result, when both are born simultaneously) is a physical fact on each traveller's own platform. There must be a real retardation on S relative to *any* real age on S', and *vice versa*. The vicious circularity of this is a consequence of the assumption. Another and more orthodox procedure is to take each system in turn as at rest. When this is done, no retardation need be assumed for the resting system, and therefore, when the computation is reversed, the same initially-given time-interval (proper time) may be taken for S' as was first taken for S. In this case the results of the equal mutual lag (when both infants are born at the same time) must be equal; *i.e.*, each will be younger than the other, not only in the same ratio, but by the same amount. (*Cf.* the computation of Born cited below).

there, however, legitimately arise a corresponding question as to the possibility of common moments *after* they have ceased to be at the same place? Manifestly not. For if, after their motion had begun, there were no single moment with respect to which we could say, 'At this instant Paul, in consequence of the retardation, is (or to an observer in the other system appears to be) younger than Peter is at this same instant', we should be debarred from asserting that Paul's relative motion had been accompanied by any comparative retardation whatever. Throughout the period of increasing displacement no lag, whether mutual or unilateral, could be predicated. In this case, then, the implication in relativity theory of the postulate of common moments for the two distant bodies is especially evident. And the symmetrical paradox plainly follows. For (*a*) the travellers admittedly start at the same time with the same age; (*b*) the comparative lag during their relative motion is admittedly equal; (*c*) therefore, at any common moment after the start, the age of each will be less than the other's *i.e.*, Paul will be younger than Peter and Peter at the same instant younger than Paul.

A motion of separation, moreover, is the kind with which physicists usually begin when they tell us the tale of the twins. They do so, as I have previously remarked, because they habitually limit their consideration of the problem to a round-trip. In a return journey an acceleration (reversal of direction) occurs; and they are able to deduce from the general theory that there will, during the acceleration, be a compensatory speeding-up of the ageing of one of the twins (and of the rate of the clock beside him), and therefore that there will, at the end of *such* a trip, be no symmetrical disparity in age.[17] Nevertheless, since a round-trip starts with a motion of separation (outward journey), we can, by examining what the relativists tell us about this first phase of the total journey, show by their own testimony that, so long as they keep to unaccelerated motion, they explicitly admit Proposition V, *2b*, *i.e.*, the symmetrical paradox. This I have already shown in each of my previous papers;[18] but since McGilvary still declares that the conclusion is not justified by the equations, it is necessary to remind the reader of the fact that so competent a mathematical expositor as Born, in a passage incompletely cited by McGilvary, has deduced this consequence from them. Two observers, *A* and *B*, both carrying "ideal clocks which indicate their proper time" are supposed by Born to be originally at rest. The clocks start going at

[17] *Cf.* L. I, pp. 158–159; II, p. 552.

[18] L. I, pp. 158–159, 164–165.

the same time. Only one instance of relative motion is in question, but we consider each observer in turn as moving relatively to the other, beginning with B; and in B's case we assume that "the effect of acceleration [during reversal] may be neglected", *i.e.*, B's clock is to be regarded as in unaccelerated motion throughout. "Then the clock of the observer B must have lost time compared with the clock of A after B's return to O", from which he set out. When the computation is worked out, it results that "*the advance of A's clock with respect to B's clock is $\frac{\beta^2}{2} t_0$; and this holds good for every moment of the motion,* since the outward and the inward journey take place with the same velocity, and in particular it holds good for the moment of turning". We next consider A's clock as the one in motion. Then, "in the times when A is moving uniformly and the special principle of relativity must be applied", *e.g.*, at any time during the outward journey, "*A's clock must, conversely, be behind B's clock by the amount $\frac{\beta^2}{2} t_0$*".[19] What happens to A's clock, by the general theory, in consequence of its speeding-up during the reversal, does not concern us here, since we are interested only in the unaccelerated outward journey which precedes the reversal. What Born has told us about this, then, is that (a) there must be assumed common moments during this mutual displacement, with reference to any one of which the readings of the two clocks can be supposed comparable; and (b) that *at every such moment A's clock is in advance of B's clock and is also behind B's clock.* And the terms "in advance of" and "behind" refer to the instantaneous readings of A's clock in comparison with B's at a common moment, not simply to a mutual comparative lag during an interval. Born then applies the same conclusion to the case of the twins. He thus, for the case of the outward journey, expressly presents the symmetrical paradox as resulting from the equations of the special theory.

McGilvary's reply to this is that "by Born's logic" the A and B clocks "when they meet again at O" will not have mutually lost time relatively to one another, but "will have lost time equally compared

[19] *Einstein's Theory of Relativity*, pp. 214–215 and 282–283. It will be observed that for the motion up to "the moment of turning", *i.e.*, for the outward journey, the assumption that "the effect of acceleration is negligible" is unnecessary in the case of B's motion; for during this phase there is no acceleration that can affect the comparative behavior of the clocks. There is, indeed, an acceleration at the moment of starting; but this, Born points out, has "no influence on the relative rates of the clocks of A and B since they are at the same point" at that moment (*op. cit.*, p. 283).

with a clock left stationary at *O*". The confusions here are prodigious. (*a*) What *is* the point *O*, and with respect to what is it stationary? Obviously, not with respect to *A* or *B*. For both of them, by McGilvary's account, have moved away from and returned to it; and a clock there during the two phases of unaccelerated motion will have been retarded in comparison with each of them, and they in comparison with it, all three having been simultaneously set going at the same place with the same reading. It can only be said to have remained stationary with respect either (*i*) to some third relative system—which is irrelevant when we are considering the effects of the relative motion of *A* and *B*; or (*ii*) with respect to some absolute and universal system, *i.e.*, in a Newtonian space. (*b*) "Born's logic" and his equations are *not* concerned with the time-relations of the *A* and *B* clocks to a third "left stationary at *O*"; they are concerned with the time-relations of these two clocks to one another. *His O* is simply the origin of the *x*-axis of *A*'s system, which is at the start coincident with the origin of the corresponding axis of *B*'s system; and the clock which "remains at *O*" is *A's* clock.[20] (*c*) McGilvary's reply is, in any case, irrelevant, because it relates only to a round-trip; it "scrambles the eggs". As I had repeatedly noted, for such a trip the equations show that the two clocks at their reunion would not exhibit a mutual inversion of coincident readings. But (*i*) what they show is that the two clocks would then have unequal readings. They could therefore not "have lost time equally" compared with another clock which is then at the same place with both of them. (*ii*) The equations show this, however, only because, during the reversal, a speeding-up equal in amount to its total retardation is ascribed to *A*'s clock,[21] in accordance with the general theory (a fact which McGilvary ignores). (*iii*) But my argument did not relate to a round-trip. It related, as I have reiterated to the point of tedium, exclusively to the phase of unaccelerated motion prior to the reversal. And with regard to this, Born's equations show precisely what I have said they show,

[20] In relativity "the point *O*" has no meaning apart from a specified reference-system; "it is fundamentally impossible to find the same place a second time in space" (Born, *op. cit.*, p. 61).

[21] *Cf.* the passages already cited, L. I, pp. 157–159; also the analysis of Einstein referred to, *ibid.* Einstein also is referring only to the time-relations of *two* clocks; and he finds that during the outward journey of either it "goes more slowly" than the other. But, once more, since each started at the same time with the same initial zero-reading, this mutual lag implies a mutual inversion of the clock-readings at any subsequent moment of the outward, or unaccelerated, portion of the journey.

viz., that "at every moment" of that phase *A*'s clock will be "in advance of" and also "behind" *B*'s clock. Thus McGilvary's attempt to meet my argument for Propositions V, 2*b* and 3*b* fails completely; it is merely a combination of non-relativistic assumptions, of a highly imaginative account of Born's reasoning, and of irrelevancies to mine, which is identical with Born's actual reasoning.[22]

F. The foregoing shows that all the contradictory propositions under V necessarily follow from the theory, when a retardation is regarded as a "physical fact" on the platform to which it is referred by the observers on the other platform. The only remaining passage in McGilvary's reply which has any *prima facie* pertinency to the main issue is the paragraph about this distinction between "physical reality" and "appearances". But what he says on this is not directed to showing that this distinction is ambiguous or meaningless; it is directed only to showing that, in a passage on this point, I assume that Peter and Zebedee could have been born at the same time. I have above pointed out that I did not "assume" but proved this, and have for good measure proved it over again. The distinction is, in fact, obvious, whether we adopt a realistic or an idealistic metaphysics. If the former, the comparative retardation of Peter's age either actually occurs on his platform or it does not; the same is true of Zebedee's age. And since Peter and Zebedee (as shown under *D*) would be born simultaneously, the resultant mutual inversion of their ages, at the place where they eventually meet, will be either a physical fact or it will not. If it is, the contradiction pointed out follows; if it is not, the result of the

[22] McGilvary in his fifth paragraph finds it "difficult to understand why" I had said that he had "'failed to meet, or even mention, the consideration'" which I had "'presented as the decisive proof that . . . any . . . way of escape from the contradiction involved in the clock-paradox' is impossible." That proof is the one restated above. That he had "failed to mention it" will be evident to anyone who reads his first paper. He supposes, however, that he had "met it squarely", presumably by his argument about the affair of Peter and Zebedee. This, however, does not meet it, either squarely or circuitously; for that argument has to do with a motion of approach, it depends upon a supposed proof that Peter and Zebedee could not be born simultaneously, and it takes no account of the statement of Einstein and the computation of Born as to the mutual inversion of readings and ages during the first phase of a round-trip. It is therefore without relevance; for this final argument of mine concerns only the motion of separation of Peter and Paul, who admittedly are born simultaneously, and it rests upon the cited statement of Einstein and computation of Born as to the time-relations of the *A* and *B* clocks and as to the ages of Peter and Paul throughout this phase of unaccelerated motion. With the actual statements of these writers about an outward journey McGilvary's argument is completely at variance.

"mutual lag" can be only a "reciprocal disparity of appearances". If an idealistic view is assumed, the age experienced by Peter when he meets Zebedee and the (different) age ascribed to him by Zebedee are *both* mere appearances; so of the age experienced by Zebedee and that ascribed to him by Peter. If, however, the idealist supposes one to be judging about the age actually experienced by the other, he too would presumably regard it as a contradiction to say that Peter's experienced physiological age is at the same time both greater and less than Zebedee's, and *vice versa*.

I have, I fear, reached the limits of length generously allotted by the Editors to this discussion. I must therefore reluctantly refrain from examining most of the remainder of McGilvary's reasonings. As they are all irrelevant to the main issue, dealing chiefly with questions about accelerated motion, the philosophical outcome of the discussion is not affected; and therefore the confusions which appear to me to abound in them need not be pointed out.[23] I must, however, refer before concluding to the fact that McGilvary repeats the "charge" that I have "scrambled the eggs", *i.e.*, "mixed up the problem of a return-journey with that involved in unaccelerated motion". Obviously, the only way of justifying this assertion would be through analysis of the actual contents of my papers, with respect to this matter. He attempts no such analysis. He merely quotes a remark of mine that the eggs have been sometimes scrambled by some other persons, namely, by certain expositors of the theory of relativity; promises to show that this remark is inaccurate; and concludes that *I* must therefore have "scrambled the eggs". Comment on this, if it is intended as reasoning, would be superfluous; and the conspicuous fact of the matter, as I am obliged to repeat, is that both my previous papers insisted, above all things, that the pertinent question concerns exclusively the unaccelerated relative motion of two clocks or living organisms. But anyone who drags in the question of a completed return-journey, and who assumes—as relativistic writers usually do—that, by showing that at the end of such a journey the symmetrical paradox does not arise, he has thereby shown that it does not arise as a consequence of the principles of the special theory during an unaccelerated

[23] I recognize one error in my preceding paper, for which I am glad to make my apology to McGilvary. It occurs in L. II, n. 22. It is true, as he says, that in his previous paper "the distances *M–A* and *A–P*" were not, as I had supposed, "by hypothesis equal". The point was, however, in no way essential to the main argument given in the text.

motion (*e.g.*, during the first phase of a return-journey), is *eo ipso* guilty of "scrambling the eggs".

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A REINTERPRETATION OF RELATIVITY

THE theory of relativity is an undeniable achievement in physics and is a logical development of the theory of measurement; but it does not have the significance for the universe which is usually ascribed to it. Real time is *not* fused with space and *absolute simultaneity does have a definite and definable meaning.*

Physics as a science is concerned with measuring and dating and so getting numerical laws. All this gives knowledge about the external world. But it would be a mistake to project this mensurational knowledge into nature without interpretation. The rejection of absolute simultaneity as meaningless encouraged most relativists to do this.

My analyses have led me to make a distinction between *chronological time* and *real time.* Chronological time is an affair of dating and measuring in terms of some standard motion. Real time is the fact of change or eventness. I hold that absolute simultaneity has meaning for real time, while operational simultaneity, which is the kind that relativity stresses, is bound up with light-signaling. When two bodies are moving with respect to one another, their operational simultaneities are not identical. It follows that length (space) and t (chronological time) as numerical quantities always require a reference to the frame from which the measurements are made. It is for this reason that physicists speak of space-time. They mean that length and t as quantities are not separable.

I have no criticism to pass upon this so long as measurement is not confused with what is being measured. Measurement gives knowledge about nature. The philosopher—and I hope the physicist also—needs likewise to think clearly about the structure of nature. It is here that the question of the real nature of time appears. Is the Astronomer Royal's time—to use Eddington's expression—as basic as supposed? I take it that physicists like Jeans, Eddington, and Millikan are aware of this problem.

It is my thesis that real time is simply the fact of change or eventness in the universe and is always local. There is no change which runs instantaneously across the universe. The unity of the universe is spatial rather than temporal and is of the nature of substantial co-