DISCUSSION

THE TIME-RETARDING JOURNEY: A REPLY.

I AM glad to have, through the courtesy of the Editors, the opportunity to reply to Professor McGilvary's discussion [1] of my paper on "The Paradox of The Time-Retarding Journey".[2] I shall assume that the general nature of the issue raised in that paper, and the conceptions and terminology employed in it, are familiar to the reader. It will, unhappily, be necessary not only to attempt to clear up some, at least, of the numerous confusions in which McGilvary seems to me to have enveloped the subject, but also to correct some extraordinary misstatements of the contents of the paper under discussion.

McGilvary begins by imputing to me a fundamental confusion which, if I had fallen into it, would necessarily have vitiated the whole of my argument. "Lovejoy mixes up the problem of a return journey with that involved in unaccelerated motion"; and these "eggs", which I have "scrambled", must be "unscrambled", since "it cannot be too strongly emphasized that the Special Theory has *logically nothing to say* on retardation involved in a journey to and fro". The last remark is correct; the rest is a precise inversion of the facts. Those eggs were not scrambled by me; on the contrary, the paper in question was largely devoted to unscrambling them. The scrambling was done by relativists. It is they who introduced, and who continue to tell, the tale of a time-retarding return journey; and in some current and accepted expositions of relativity by distinguished mathematicians and physicists, this tale is presented as a corollary of the Special Theory and is defended from the specific standpoint of that theory.[3] This, I pointed out, is inconsistent with the relativist's own principles: since

[1] This REVIEW, Vol. XL, July, 1931, pp. 358–379.

[2] *Ibid.*, pp. 48–68, 152–167. This paper will hereafter be referred to as PTJ.

[3] *Cf., e.g.*, Born, *Einstein's Theory of Relativity*, p. 214; there are other examples, which I have not at hand where I write. It is true (as noted in PTJ, p. 159), that those who first deduce from the Special Theory alone both the implication of a time-retarding return journey, and also an ostensibly sufficient reply to the objection that the comparative retardation should be bilateral, usually add, when later setting forth the General Theory, that it is in this that the decisive reply to the objection is to be found. It was none the less necessary for the purpose of my paper to clear away the confusion inherent in the former argument before examining the latter.

the Special Theory is pertinent only to unaccelerated motion, while a round-trip involves three accelerations, "the Special Theory does not justify any inference as to what the effects of such a trip will be; and therefore the story of Peter and Paul, as usually told, has no proper place in expositions of that theory".[4] Throughout the paper the question of the implications of Einstein's doctrine in the case of unaccelerated relative motion was carefully distinguished from the question of its implications in the case of a return journey; and the most characteristic feature of my argument was the contention that the former question—not the latter, as relativistic writers usually imply—is the crucial one for the issue under discussion. McGilvary here, then, has merely repeated in substantially the same terms a consideration which I had especially emphasized and systematically applied. He has nevertheless given his readers to understand that I had overlooked and implicitly denied it.

Relativistic physicists have, however, sometimes attempted—still from the standpoint of the Special Theory—to meet the contention that the theory implies a bilateral comparative retardation, by remarking that *one* of the two systems assumed to be in relative motion is in the case of a round-trip accelerated, and that therefore the retardation implied by the theory will affect only the other one. In dealing with this argument I made a distinction of which McGilvary seems to have missed the point.[5] As it is essential to much of the argument, I will briefly restate it: The term 'acceleration' is ambiguous. It may refer (*a*) to a change of direction or velocity of the motion of Peter and Paul relative to one another, or (*b*) to a change of direction or velocity of either relative to a universal or cosmical reference-system, the 'totality of the stars'. The former may be called 'relative acceleration', the latter, for the sake of brevity, 'absolute'. An 'absolute' acceleration might be unilateral, *i.e.* Paul's motion *might* be accelerated while Peter's was not, in sense (*b*). But the assumption of such a cosmical frame of reference belongs exclusively to the General Theory, and its consequences can be discussed only in the light of that theory. So long as we keep to the concepts pertinent to the Special Theory, the only acceleration which the relativist is entitled to invoke, in the hope of thereby avoiding the symmetrical paradox, would be an acceleration in sense (*a*). But Paul's motion relative to Peter could not be accelerated without Peter's motion relative to Paul being accelerated.

[4] PTJ, p. 58; the point is amplified in Note 11, and reiterated at pp. 62, 63, 152, 160.

[5] PTJ, pp. 59–62, 161.

Hence *this* acceleration—introduced into the argument, not by me, but by the relativist—would be bilateral, and any inference drawn from it with respect to one of the travellers must also be drawn with respect to the other. If it is supposed to render the Special Theory inapplicable to Peter's case, it renders it inapplicable to Paul's, and so no retardation on either side could be deduced. If, on the other hand, "the acceleration in question were treated as theoretically negligible"—as it has been by eminent expounders of relativity, in the case of *one* of the travellers—it should be equally negligible in the case of the other; there would accordingly, by the Special Theory, be a comparative retardation, but on this supposition it "would necessarily be reciprocal". This argument McGilvary thinks absurd; he supposes it to be analogous to saying: "*If* Lovejoy were, along with all his present views, also an epistemological monist, he would be a most inconsistent philosopher." But the only logical similarity between this and the foregoing reasoning is that both contain hypothetical propositions; and I venture to suggest that not all arguments containing hypothetical propositions are formally equivalent.

But McGilvary, too, presently deals with the supposition of a return journey, *i.e.* of an eventual reunion of Peter and Paul. He makes this supposition in two forms; one of them is as follows: Each traveller—born simultaneously with the other "at the same place", *i.e.* at virtually coincident points on their respective moving platforms—after having moved "an equal distance along the other's platform" at a high speed is to be "transferred instantaneously to the other's platform, on the assumption that the transfers are to have no disturbing effects, and that they are made under similar kinematic conditions". This, it is evident, differs in two respects from the usual form of the story. It makes no difference, so far as the relative motion of Peter and Paul is concerned, whether the change of its direction is accomplished by setting each platform moving in the reverse direction or by allowing both platforms to continue the motion originally attributed to them and having the travellers exchange platforms. In either case the bodies of Peter and Paul reverse *their* relative directions and return to the "same place". But the assumption of an interchange of platforms while both are moving in opposite directions implies, of course, that the reversal of direction is instantaneous; and this, with the further supposition that the reversal will "have no physical effects", is not a part of the tale as commonly told. It is, indeed, McGilvary observes, "a violent supposition, as the transfer is a most violent transfer, but we are not responsible for either"; for it is "Lovejoy's supposition".

But no such supposition has ever been made by me. About a transfer from one moving platform to the other there is not a word in the paper which McGilvary is ostensibly discussing; nor is there any assumption that a reversal of direction, whether instantaneous or gradual, would "have no physical effects". McGilvary attributes it to me because my paper contains the words already cited: "If the acceleration [in the reversal] were treated as theoretically negligible". He has, that is, taken the protasis of a conditional sentence in the subjunctive, expressly referring to a supposition of certain writers whose reasoning I was criticizing; has combined with this an addition of his own; and has presented the whole as if it were a categorical proposition affirmed by me, despite the fact that the presumable falsity of this supposition was more than once remarked.[6] Moreover, all that is said by McGilvary about the return journey is irrelevant to the principal object of my paper, viz., to focus attention upon the consequences of the theory of relativity for the case of a journey *without* reversal of direction. It was, indeed, necessary to consider what relativistic physicists say about a return journey, since *they,* for certain reasons to which I shall recur, dwell chiefly upon it. When dealing with the effect of a reversal upon retardation I in general followed Einstein's account of the matter.[7] He concludes in the General Theory that such an acceleration would have one, and only one, result relevant to the problem under examination, namely, a speeding-up of the clocks, physiological processes, etc., of *one* of the travellers—it being assumed that one of them, but not the other, would at the time reverse his direction (*i.e.* be accelerated) relatively to the cosmical reference-system, and that this would produce a temporary difference in gravitational potential between the two moving bodies. Of this, the proper Einsteinian account of the physical effect of a reversal of direction, McGilvary, singularly enough, makes no mention, when telling us what would happen on a relativistic round-trip; the omission is the more remarkable in view of his having at the outset rightly noted that the Special Theory, taken by itself, has no relevance to the question of "retardation with reversal of direction". Since this is so, it would, surely, not have been amiss to take some notice of what Einstein actually says on this question in that part of his doctrine which *is* relevant to it.

Let us, however, consider what McGilvary does with the "violent supposition" which he gratuitously introduces. We shall first assume the reunion to be accomplished in the manner already mentioned. The

[6] *E.g.*, pp. 60, 61, 161.

[7] This has already been summarized in PTJ, pp. 157, 158.

reversal in question is evidently an acceleration only in sense (*a*), *i.e.* it is a reversal of the direction of the motion of Peter and Paul relative to one another; for McGilvary now proceeds to deduce the consequences of a return journey exclusively by means of formulas belonging to the Special Theory (though he has previously observed that they have "logically nothing to say" about such a journey). Assuming a certain velocity and distance for the motion of Peter and Paul, he concludes that each infant is, when transferred to the other's platform, "21 seconds old physiologically", and "when they meet, each is 42 seconds old physiologically".[8] What is the point of this conclusion? Though it rests upon the supposition for which McGilvary has disclaimed responsibility, it nevertheless seems to be presented in the end as if it were a true account, on relativistic principles, of the age of the travellers at the end of a return journey not theoretically impossible. If it is so intended, it is completely at variance with the orthodox relativistic doctrine. Einstein and (so far as I have examined their works) all other mathematical expositors of the theory agree that *in any possible case* of such a journey, if two travellers who had been born simultaneously at the same place should meet again after a period of relative motion at high velocity—supposing that the motion had, during its phases before and after the reversal, been rectilinear and uniform—they would *not* be "of equal age physiologically". If the physical reality of the retardations is assumed, as in the usual story, it is inferrible on Einsteinian principles, namely, those of the Special and General Theories together, that the final ages, clock-readings, etc., will be unequal; the equations showing this may be found in any mathematical treatise on the subject.[9] And it is apparently the orthodox version that McGilvary wishes to vindicate.

The argument here may, however, perhaps be meant to be taken in a different way. McGilvary's purpose may be merely to show that, if the supposition which he attributes to me were (*per impossibile*) admitted, while relativistic principles were followed on all other points, the implied consequence of the hypothetical return journey would not be what I have called the symmetrical paradox—viz., that each voyager

[8] For some reason, McGilvary here prefers to take seconds rather than years as the units in his calculation.

[9] There is one possible case for which this conclusion does not follow, even though the postulates of the General Theory are assumed; this special case was pointed out in PTJ, p. 163 (ii). It is not, however, recognized by Einstein; and the reasons for it are foreign to McGilvary's present argument. For the equations, cf. Born, *op. cit.*, pp. 282–284; or Kopff, *The Mathematical Theory of Relativity*, p. 127.

would be younger than the other—but that both, at their reunion, would be of the same age. If this is the argument intended, it is trebly fallacious. In the first place, it is irrelevant to the paper under discussion, because the initial supposition is not mine. In the second place, it is irrelevant because I have not maintained that the symmetrical paradox would necessarily result at the *end* of a *return* journey. In the third place—what is more important—if the suppositions here specified by McGilvary *are* premised, the consequence which he deduces from them is wrongly drawn. For the suppositions exclude any such physical effects of an absolute acceleration as the unilateral speeding-up of clocks, etc.; and we are expressly told that Peter and Paul undergo "equal" (and therefore merely relative) "accelerations"—which would, as we have seen, have either no effect or else bilateral and equal effects. But in that case the two infants when they met again should not be of the same age, but, by undisputably relativistic logic, should each be younger than the other. For it is in the end clearly admitted by Einstein and other mathematical writers on relativity that if there were not an *unequal, i.e.* a unilateral, acceleration, and therefore a unilateral speeding-up, in consequence of the reversal of direction, a comparative retardation *would* be deducible for both travellers and both clocks.[10] It is only because the acceleration relative to the cosmical system, which is assumed to be involved in a return journey, would produce an "overcompensation" on one side, that the admission of a reciprocal retardation is declared by Einstein to be avoidable. Exclude this consequence of the acceleration (as McGilvary in his present hypothetical case excludes it), and the symmetrical paradox would avowedly follow. It is true that McGilvary essays to prove the contrary by equations in his footnote 11; unfortunately, his equations yield a result precisely opposite to that to which the equations of Einstein and the other mathematical physicists lead them, on this point.

In the other form of his supposition of a reunion McGilvary gets Peter back to the same point with Paul by transferring him, "without disturbing effects", to a third platform, moving "relatively to Paul's in

[10] This is perfectly explicit in Einstein's own argument, cited in PTJ, 157–159: whichever one of the two clocks is in turn taken as in motion relatively to the other (in reality, of course, they are both in such motion at once), during the two phases of the total movement which Einstein distinguishes as "*Teilprozesse* (2) *und* (4)" that clock "goes more slowly than the other". These phases are respectively identical with the period during which Peter and Paul are moving apart and that in which they are approaching one another. If the reader wishes to see this in the form of equations, he will find them in the passage of Born's referred to, *loc cit.*

the direction opposite to that in which Peter's moves." The velocity of this third platform relative to Paul's is to be the same as Paul's original velocity relative to Peter's; in other words, Peter "retraces his movement with the same speed back as he had on his outward jouurney." In this way he will overtake Paul, who meanwhile has remained at his birthplace, *A*, on his own platform. And in this case McGilvary concludes that Peter will be younger than Paul on his arrival at *A*, and his clock-reading will be slow in comparison with Paul's. The supposition incidentally involves some additional paradoxes of its own, when the relation of Peter's time, etc., on his new platform to that on his original one is considered. Disregarding these, this second supposition does not differ in any relevant respect from the first. In both cases, the bodies of Peter and Paul, and their clocks, simply reverse the direction of their relative motion, though by means of different mechanical devices, and return to a common point, the motion being by hypothesis unaccelerated except during the reversal, and at uniform velocity in both directions. Yet McGilvary handles his equations in so original a manner that he gets opposite results for these two instances of precisely the same kinematic phenomenon: if Peter uses one vehicle for rejoining Paul he will be of the same age and their clocks will agree, if he uses another, moving at the same relative speed, he will be the younger and his clock will be slow.

McGilvary is unaware of his self-contradiction because he supposes there *is* a kinematically relevant difference between his two methods of reuniting Peter and Paul. As we have seen, when the travellers return to a common point simply by swapping platforms he regards *each* as reversing his direction, and therefore describes them as undergoing equal accelerations. But in the present instance, in which Paul remains on his own platform while Peter is transferred to a third by which he eventually catches up with Paul, McGilvary supposes that Peter alone reverses his direction; that he alone, therefore, is accelerated; and that this difference between them in the matter of acceleration will affect the retardations, so that in this instance, unlike the first, their eventual ages and clock-readings will be disparate. As he puts it, with the emphasis of italics, "*it is the clock that reversed its direction* [*sc.*, Peter's] *that is slow*". There is a dense mass of errors and confusions here; but it will suffice to notice four. (1) It is explicitly a part of the initial supposition in both instances that reversals are to have no "physical effects". Hence if there were a unilateral reversal in the second instance, it could not affect the rate of Peter's physiological processes or of his clock; and it could therefore produce no differ-

ence, with respect to retardation, between the first instance and the second. (2) But this reversal or acceleration could no more be unilateral in the second instance than in the first. For in both cases it is equally clearly indicated that only the relative motion of Peter and Paul is in question; and—to repeat yet again the fundamental consideration to which it appears impossible to persuade McGilvary to attend—relative motion is reciprocal, and there can be no reversal of Peter's motion with respect to Paul unless there is at the same time a reversal of Paul's motion with respect to Peter. Both clocks therefore will, *in this sense,* reverse their directions, whatever the means by which the return journey is managed. If, then, the result of a reversal of relative direction were (which it is not) that the clock which reversed would be slow in comparison with the other, then, once more, both would be slow. (3) If, on the other hand, there is—as McGilvary assumes—a unilateral acceleration which (contrary to the initial supposition) *does* have a physical effect on the retardations, this can only be what we have termed an "absolute" acceleration; Peter must be supposed, while reversing the direction of his motion relative to Paul, to reverse it also relatively to the cosmical reference-system, while Paul does not do so. And in such a case we have already observed that, by the General Theory, the comparative retardation at the end of the return journey would be asymmetrical. Only, the result to which McGilvary's equations bring him is again exactly opposite to Einstein's. The clock which had—in the only respect in which any differential effect could result—reversed its direction would, according to the General Theory, be *fast.*[11] (4) This unilateral effect would, moreover, according to Einstein, arise in the first instance as well as in the second, and would modify the speed of the same clock during the reversal. Peter could not prevent an absolute acceleration from occurring simply by taking a different celestial taxicab in order to meet Paul.

In whatever way, then, McGilvary's account of the consequences of a hypothetical return journey is construed, it invariably proves irreconcilable with the reasonings of the author of the theory of relativity on the same matters. Under these circumstances I find it a little odd that McGilvary should remind me in reproachful italics that "we are

[11] It would undergo during the reversal a speeding-up equal to twice the amount of its retardation during its two phases of unaccelerated motion, which latter amount would be precisely equal to the retardation of the other clock. *Cf.* the passage of Einstein cited PTJ, p. 159; and for the equations, Born, *Einstein's Theory of Relativity,* pp. 282–284, or Kopff, *The Mathematical Theory of Relativity* (English tr.), pp. 124–127.

dealing with *Einstein's relativity*", and should in general assume the tone of a faithful and authoritative expounder of the pure relativistic creed.

Since he in fact disregards the most essential principles of Einstein's argument on the matter under discussion—"the final and official form of the argument by which escape from the symmetrical form of the paradox is sought"[12]—McGilvary naturally fails to meet, or even to mention, the consideration which I presented as the decisive proof that this (or any other) way of escape is impossible. This is already implicit in what has been said above, and was, I should have thought, stated with sufficient clarity and emphasis in my previous paper;[13] but it seems necessary to recapitulate it briefly. It runs, then, as follows: To avoid the absurd conclusion that each clock will be equally retarded in comparison with the other, and that each twin will be younger than the other, the relativist is *compelled* to postulate a return journey, and then to invoke, in accordance with the General Theory, the aforesaid compensatory speeding-up of the clocks and physiological processes on one of the two systems, during the phase in which the direction of their relative motion is being reversed. But in doing so, he expressly assumes—as, for the rest, the Special Theory requires—that during the phase of unaccelerated motion *prior* to the reversal (not to speak of that after it) a *bilateral* comparative retardation has been taking place. In other words, he recognizes that on the outward one-way journey which is a part of the return journey, each clock *will* be increasingly slow in comparison with the other, and each twin will be younger than his brother.[14] If the relativist did not assume this, there would be no bilateral effect for the unilateral speeding-up to offset. Thus nothing is accomplished by assuming a return journey and thereby introducing an acceleration. If it is an acceleration merely in sense (*a*), it is irrelevant because, if it has any effect, it will have the same in the case of both travellers. If it is an acceleration in sense (*b*)—and this, according to Einstein himself, is the only one which need be considered[15]—it is still irrelevant, because it is introduced too late. The mischief has already been done before the acceleration occurs. Hence "the very absurdity which [Einstein's] reply is designed to eliminate is a necessary presupposition of the reply."[16] The symmetrical para-

[12] PTJ, p. 159.

[13] *Ibid.*, pp. 164, 165.

[14] *Cf.* once more Einstein's argument already cited (PTJ, pp. 158, 159), and the relevant passages in Born, Kopff, or any competent mathematical exposition of the General Theory.

[15] *Cf.* his remark in *Die Naturwissenschaften*, 1918, p. 700, cited in PTJ, p. 162.

[16] PTJ, p. 165.

dox, admittedly a self-contradiction, is, then, inherent in the theory, so long as the retardations are construed as physical phenomena affecting bodies in relative motion—though *this* paradox need not arise if they are construed merely as illusory appearances of such bodies experienced by external observers.[17] Unless or until this difficulty is disposed of, the crucial point of the argument against the physical reality of the retardations, and therefore one of the most essential points of the question concerning the philosophical significance of the doctrine of the relativity of time, remains untouched.

McGilvary deals at greatest length with a special hypothetical example which I propounded in order to illustrate the implications of the Special Theory (as that is usually interpreted) in a case of relative motion in which there is no acceleration whatever. Two parallel platforms of indefinite length, separated only by a distance sufficient to prevent friction, are supposed to be in motion relatively to one another with a constant high velocity. When the point A on one platform is passing the point A' on the other, two infants are born, Paul at A, Peter at A'. At A and A', and at intervals in both directions along either platform, observers are stationed, each provided with a clock which automatically records the number of revolutions of its hands. The clocks on each platform are synchronized in the usual manner. At a point P on Paul's platform, lying at a distance from A in a direction which we may, with McGilvary, call "eastward", another child, "Paul's cousin", is born, at a moment which, by the clocks on Paul's platform, is the same as the moment of Paul's birth at A: and at a coincident point P', on Peter's platform, "Peter's cousin" is born, simultaneously with the birth of Paul's cousin at P. All remain throughout at their birth-places on their own platforms. This is the "general set-up" which McGilvary and I agree in accepting. Given this situation, I assumed that all four babies would be born simultaneously. It will, however, shorten the argument to limit it to a single crucial proposition, viz.; that Paul's cousin could be born simultaneously with Peter. Since, however, the two platforms are in continuous motion in opposite directions, the point A' and P will eventually become (virtually) coincident, and Peter and Paul's cousin will then be in "the same place". But meanwhile, my argument continued, Peter and Paul's cousin have been in unaccelerated relative motion; each therefore will have undergone a retardation of his physiological processes, clock-readings, etc., in comparison with the other's; and thus when A'

[17] Another paradox would, it could be shown, arise even on this supposition; but that is another story.

and P are coincident, Peter and Paul's cousin will—if retardations actually occur as physical facts—each be younger than the other at the same place. To show that, in this particular hypothetical case also, this absurd conclusion follows from the customary relativistic assumptions it is, it must be borne in mind, necessary to establish only the two propositions (1) that Peter and Paul's cousin *could* be born simultaneously; (2) that, according to the Special Theory, the comparative retardation arising while they continuously approached one another would be bilateral and equal.

(1) The former proposition McGilvary regards as inconsistent with relativistic principles. It rests upon a "Newtonian postulate" which I have illicitly introduced. It is true that by "Paul's clocks", *i.e.* those on his platform, the four babies are all "born simultaneously". But by Peter's clocks (McGilvary finds) Paul's cousin could not be born simultaneously with Paul, nor, consequently, with Peter, for

> in order to understand relativistic time, . . . we must start with the fact that *at no time, registered by the clocks of either platform, does anyone of the clocks of the other have a reading equal to the reading of any other clock of that other platform.*

Now in this case McGilvary is employing a genuinely Einsteinian premise, though it is not happily expressed. By a familiar argument, though, as I have elsewhere attempted to show, a fallacious one,[18] Einstein reaches the conclusion that two events which are simultaneous from the point of view of one system cannot be simultaneous from the point of view of another system in motion relatively to the first. And this principle McGilvary correctly applies to the present case. Since, by hypothesis, Paul's birth at A and Paul's cousin's birth at P are simultaneous by the clocks on Paul's platform, they cannot be simultaneous by Peter's time-reckoning. But Peter's birth *is*, by admitted relativistic standards, simultaneous with Paul's, since they are both born at the same place at the same time. Therefore Peter's birth—being simultaneous with Paul's which is not for Peter, simultaneous with Paul's cousin's—cannot, for Peter, be simultaneous with Paul's cousin's. Though McGilvary's reasoning here, then, is, so far as it goes, relativistically orthodox, and though it seems to show that, on Einsteinian principles, my proposition (1) cannot be admitted, it in fact fails to disprove that proposition or the conclusion of which it was a premise. What McGilvary overlooks is that there are other presuppositions of the relativist about simultaneity-at-a-distance, and that

[18] "The Dialectical Argument against Absolute Simultaneity", JOURNAL OF PHILOSOPHY, 1930, pp. 617–632, 645–654.

these have implications contrary to that upon which he dwells. For these latent inconsistencies in the theory a critic of it is obviously not responsible. It can be shown by two distinct arguments that the relativist must admit that the births of Peter and Paul's cousin could be simultaneous, whether the judgment upon the question were that of an observer on Peter's platform, or of one on Paul's.

i. In the first argument Paul serves, so to say, as a logical intermediary between his cousin and Peter. For the observer at A', on Peter's platform, the births of Peter and Paul are simultaneous; he *sees* them both occurring together at the place where he is. He also *knows* that indefinitely many other events on Paul's platform must be occurring simultaneously with Paul's birth; *i.e.* that the synchronized clocks on that platform all show the same reading when that event occurs. It is true that McGilvary makes it a rule for his observers that they shall not think; but this seems to me a bad rule for observers, and in any case I trust the reader will not adhere to it. Since, then, Peter's birth is observably simultaneous with an event having a certain date (clock-reading) at A on Paul's platform, and since other events must have the same date on that platform, one of these events may conceivably be the birth of a cousin of Paul's at some point, P, to the eastward of A. It will perhaps be suggested that the sense in which Peter's birth is admittedly simultaneous with Paul's is not the same as the sense in which Paul's is admittedly simultaneous with his cousin's. On this matter Einstein's own utterances are somewhat equivocal. But the use of the same term should signify that the same relation is in question; and if it is assumed that the term is used in two wholly different senses, other paradoxes arise, which I have pointed out elsewhere.[19] In any case, McGilvary explicitly tells us that the senses are the same: "the difference between 'simultaneity at the same place' and 'simultaneity at a distance' is not a difference in the connotation of 'simultaneity'", but only in "denotation". If this is assumed, the first argument clearly follows from the admitted facts; it is an application to our hypothetical case of the principle: If two events are in the same sense simultaneous with the same event they are simultaneous with each other. There is no reason why an *intelligent* observer on Peter's platform should be supposed to be incapable of recognizing this.

ii. There is, however, a more direct argument to the same conclusion, in which Paul (though not his platform) may be left out of the affair altogether. For another assumption equally necessary to the relativist is that there may, and, indeed must, be *simultaneity between*

[19] *Ibid.*, pp. 625–632.

events on two different systems. He is logically debarred from assuming that no single event on, say, Paul's platform can ever occur at the same time with any single and distant event on Peter's; for such an assumption would be equivalent to denying the possibility of relative motion. To say that two bodies are in such motion is to say that they are moving at the same time—*i.e.* that *while* one is getting, say, nearer to the other, the other is getting nearer to the first. There must be a 'common while' if there is to be a relative motion; and this common while must contain common moments. No relativist, I suppose, has ever been guilty of asserting that when one of two originally distant bodies begins to approach another, the second does *not* begin to approach the first. Or again, in the customary story of the round trip, at the same moment at which Peter's direction relative to Paul is reversed Paul's direction relative to Peter is admittedly reversed. To suppose that these reversals happened at separate or incomparable moments, one in "Paul's time" exclusively, the other in Peter's, would be to deny that any reversal of Paul's motion *relative to Peter* had occurred. An 'intersystemic', though not necessarily a 'universal', simultaneity is thus invariably assumed by the relativist for any two systems of which he predicates relative motion. This is not an accidental relapse on his part into a 'Newtonian postulate'; it is necessary to give any sense to his discourse.[20]

The same assumption is obviously implicit in the notion of retardation. A comparative retardation presupposes a *period of time common to both systems* during which the two sets of processes which are to be compared are going on; and this period must have an initial moment and a terminal moment which are identical for both systems, and a series of intermediate moments, also identical for both, in the course of which the retardation of the clocks and physiological processes on the one, in comparison with those on the other, gradually takes place. Deny these presuppositions, and the proposition 'Peter has been ageing less rapidly than Paul's cousin' becomes entirely meaningless.

Since, then, there can—and, for the logic of relativity, imperatively must—be moments common to any two specified relatively moving systems, there is no reason why, at some such common moment, two infants should not be born, one (Peter) at A' on the one platform, the other (Paul's cousin) at P on the other platform. There is also no reason why, at the same common moment, recording chronometers of

[20] This has also been pointed out in the article mentioned, JOURNAL OF PHILOSOPHY, XXVII, pp. 653, 654. The assumption is evident in the argument cited from Einstein and Born, PTJ, pp. 157–159.

identical construction should not be set going beside both infants, each chronometer with the initial reading of 12 o'clock. We may therefore postulate that such simultaneous births, and such simultaneous beginnings of clock-movements, occur at the points mentioned.

(2) We have, then, Peter and Paul's cousin born "at the same time" at A' and P respectively. The moment of their birth is the moment of the beginning of their unaccelerated relative motion. What, according to the Special Theory, will be the effect of this motion while it is going on, and what will be its outcome when A' and P become coincident? No orthodox expositor of the theory, I believe, has ever denied that each traveller will undergo a retardation his 'time', in comparison with that of the other, during the period in which they are approaching one another. The meaning of this proposition is, it is true, often obscured by the use of what I have called the phraseology of 'alternating options'; *either,* we are told, will be thus retarded, according to our choice of reference-body. But it can hardly be necessary to repeat again that 'either' here necessarily means 'both', since the retardation is a function of the relative motion and both travellers at once, with their accompanying clocks, are in relative motion. The relativistic physicists declare plainly that the calculation of the retardations (so long as the motion remains unaccelerated) is perfectly reversible—that, in Kopff's words, "the only consequence that can be drawn from the Special Theory, in relation to the Lorentz transformation, is that during the uniform rectilinear motion of A with respect to B, both in the first and in the second process, the clock A lags for an observer at B, and the clock B does the same for an observer at A".[21] If McGilvary thinks that any different consequence can be drawn from the Special Theory, he has Einstein and the other mathematicians against him, and his real quarrel is with them. But it is implied by the usual version of the story of the two travellers born at the same time that the mutual lag will not merely *seem* to the observers at A and B—or, in our illustration at A' and P—to occur, but will *actually occur on the respective*

[21] *The Mathematical Theory of Relativity,* p. 50. The expression "first and second process" refers simply to the case in which A is "taken" as in motion relatively to B, and that in which B is "taken" as in motion relatively to A; but, as we know, both "processes" occur at once, if there is any relative motion between the two travellers at all. Kopff, *loc. cit.,* attempts to escape the resultant paradox in the usual manner earlier indicated: he shifts the argument to the case of accelerated motion, and then brings in the resultant speeding-up of one of the clocks, "according to their relative position" at the time of reversal. It will have been observed that Einstein, in the passage previously referred to, himself deduces from the Special Theory precisely the mutual lag above mentioned.

platforms of *A* and *B*—or of Peter and Paul's cousin. And therefore, when Peter and Paul's cousin presently meet, each traveller will *be* physiologically younger than the other and each clock will show a record of fewer elapsed hours than the other. And this is equivalent to saying that the situation when Peter and Paul's cousin meet will be precisely that which McGilvary says is not implied by the theory—"a mutual inversion of unequal coincident clock-readings" and physiological ages.

One of the most curious confusions in McGilvary's paper is to be seen in his remarks on the question of the "physical reality" of the retardations. This question is, he declares "ambiguous"; in fact, he suggests, it loses all meaning when we remember that "retardation, like largeness, is always a respective characteristic". An "Einsteinian clock" is "not retarded with respect to the time-system of its own platform"; it *is* retarded "with respect to the time-system of the platfrom relatively to which it is in motion"; one of these, "where it is", is as much a physical fact as the other, "where that is"; and beyond this, we are told, no question of physical fact arises. But in truth the meaning of the statement that a comparative retardation is a "physical reality" is perfectly unambiguous. It means precisely what McGilvary expresses when he correctly says that (assuming the same speed and distance as before) "it is agreed by all relativists that Peter's clock at *A'* ticked, and Peter's heart beat, 21 times from his birth till his arrival at *P*, and that the heart of Paul's cousin beat, and Paul's clock at *P* ticked, 70 times from *this* cousin's birth till Peter's arrival at *P*". The (supposed) fact that Peter's clock ticked and his heart beat 21 times instead of 70 *is* the retardation; if this were not a "physical reality" his clock would have ticked the same number of times as Paul's cousin's did while the same state of unaccelerated relative motion between the two of them subsisted. (It is, of course, illicitly assumed here by McGilvary that Paul's cousin's clock would not also be retarded; but that is beside the present point.) Now to say that Peter's clock thus ticked, or that his heart beat, only 21 times, is not exactly the same as saying that the clock did *not* tick, nor the heart beat, this number of times, but merely appeared to do so to observers on the other platform. Where, then, is the ambiguity in the former proposition, or in the distinction between it and the latter? Wherein, also, lies the respectivity of the assumed retardation? Peter's clock did not tick 21 times "with respect to the platform relatively to which it was in motion", and 70 times "with respect to its own platform"; it just ticked 21 times. This is elsewhere recognized by McGilvary: ques-

tions of number are, he grants, "absolute"; "there is no difference" between observers "in any count", and therefore the number of ticks will be the same for the observers on both, or all possible, platforms; and each traveller's "physiological age advances in the rhythm of his clocks". If, now, we suppose the journey lengthened, and substitute years for seconds as our units, the proposition on which "all relativists agree" means that when Peter arrives at P he will have the physical attributes of a youth of 21 (with the mental characteristics conditioned by this physiological age), while Paul's cousin will have those of an old man of 70. About this also there is nothing respective. A man is not physiologically 70 with respect to one moving object and physiologically of some other age with respect to another. Now this situation in which Peter and Paul's cousin are 21 and 70 at the same time and place is precisely the outcome of the orthodox relativistic story, when a one-way journey rather than a return journey is assumed. The critic of the story merely points out that, by the relativist's premises, the argument can equally legitimately be reversed so as to prove Paul's cousin the younger; which would mean that he must, by relativistic principles, have two physiological ages at once.

While much more that ought to be said about McGilvary's reasoning must be left unsaid for want of space, I must refer briefly in conclusion to his equations. The disastrous outcome of his calculations in the case of the return journey, under the specified hypothetical conditions, has already been shown. In the case of the unaccelerated motion of Peter and Paul's cousin, his computations concerning their respective ages at the time of their meeting prove nothing whatever about the question at issue, for a very obvious reason: *They are made from the point of view of only one of the travellers.* In order to employ the Lorentz transformations it is, manifestly, necessary to have first something to transform; in other words, the values of x and t must first be determined by measurements (actual or hypothetical) made on one platform by observers travelling with it. Such measurements on Paul's platform would, *e.g.*, give the distance, for him or his observers, from the assumed origin A to the point P (which is initially coincident with P'), and the time taken, as measured by his clocks, for Peter to move from P to A. With these values as data, the equations can then be applied; they will show that on *Peter's* platform, the corresponding length will be shorter and the time less, the disparity varying with the value assigned to v. This computation McGilvary correctly makes; assuming that $v = c\sqrt{91}/10$, that P, as measured on Paul's platform, is $7c\sqrt{91}$ miles from A, and that Peter, by Paul's clocks, arrives at P (where he meets Paul's cousin) at time $t = 70$, it follows

that, by Peter's clocks, he arrives there at time $t' = 21$. But though this gives us the times of that event for both platforms, it gives them *only from the point of view of Paul or his cousin.* But Peter is equally entitled, in a world of relativity, to make measurements on his own platform, and, taking these as data, to apply to them the same transformation equation, in order to determine how much time elapsed, *by Paul's clocks,* and by how many heartbeats, etc., Paul's cousin's physiological age advanced during the relative motion which brought him and Peter to the same place. Since, by the equation, the time on the *other* platform is always slow in comparison with that on which the basic measurement is made—the so-called stationary system—Peter will find that the time, by Paul's clocks, is less than by his own—not more, as it was from Paul's point of view, *i.e.* when Paul's measurements were taken as the initial data; and the retardation which each can thus determine by calculation it is supposed that he also sensibly perceives. The computation of Paul's cousin's age from Peter's point of view McGilvary fails to make. This astonishing omission is apparently due to a confusion between computing times *for* both platforms and computing them *from* both platforms. With all his equations—which are merely different applications of the one familiar formula for computing retardation—McGilvary has too few by half, since in none of the specific cases with which he deals does he apply this formula bilaterally.[22] This is to say that he has forgotten the principle of

[22] To show this by quoting and analyzing all McGilvary's equations would require more space than is available. I refer the reader to McGilvary's text and ask him to observe that in footnotes 3, 4 and 5, which show, by means of the Lorentz formulas, the times of certain events on Peter's platform, the values assigned to x and t are those assumed to be directly measurable on Paul's platform, and that the subsequent statements concerning Peter's age, in comparison with Paul's cousin's, are based on these calculations. In notes 6 and 7, which introduce a "cousin" of Peter's born at a "point M' *west* of A' and distant from it by $7c\sqrt{91}$," the values for the time, t', and the distance x', are assumed to be determinable, *apart* from the transformation, on *Peter's* platform, and the time required for Paul to reach Peter's cousin at M' is thus computed from Peter's (or his cousin's) point of view; but in this case there is a corresponding omission to compute the time required from Paul's point of view for Peter's cousin to move from M to A. If, however, the computations in notes 4 and 5 are combined with those in note 7, they establish my contention, as a little analysis will show. If we start with Peter's measurements, Peter's cousin's meeting with Paul occurs at time $t' = 70$ by Peter's clock and at time $t = 21$ by Paul's clock (note 7). This is the reverse of the result obtained (notes 4 and 5) for Peter and Paul's cousin. But the distances M–A and A–P, from whichever platform determined, are by hypothesis equal, as are the distances M'–A' and A'–P'. Thus two persons, *e.g.* Peter and his cousin, travelling on the same platform with the

relativity itself. It is true enough that, as he remarks, the Einsteinian theory "is not just the classical theory of the relativity of motion dressed up in mathematical clothes". But it is also true that the Special Theory includes the classical principle of the relativity of motion as its indispensable presupposition, that the denial of any 'privileged' frame of reference is of its essence, and that consequently, where two bodies in unaccelerated motion are concerned, all relativity estimates are reciprocal, that is to say, reversible. This is the first thing taught the children in *really* "relativistic nurseries". A discussion in which it is forgotten can hardly be said to be pertinent to the theory of relativity at all; *Hamlet* with Hamlet left out is an inadequate parallel. When the omission is repaired and the transformation equation for time is applied from both sides, there manifestly results a proof of the conclusion which McGilvary has endeavored with elaborate irrelevance to controvert.

But he does not even adhere consistently and unequivocally to his denial of it. For in one passage (pp. 371, 372) he reaches (even though by devious reasoning) the result that, "measured by the synchronism of Peter's clocks", Paul's cousin's clock during his transit from P' to A' "runs only 3/10 as fast as Peter's clock", and that, "measured by the synchronism of Paul's clocks", Peter's clock during his transit from A to P (where he meets Paul's cousin) "runs only 3/10 as fast" as Paul's cousin's clock. This, obviously, *is* a reciprocal comparative retardation. Moreover, Peter is by hypothesis born when A and A' coincide, and Paul's cousin is born when P and P' coincide; and their transits from A and P' respectively until they meet constitute their

same velocity, cover what for them are equal distances in the same direction in unequal times, one being 21 and the other 70 at the end of his journey, by the clocks on their own platform—when the readings of those clocks are determined in the first case from the other platform, in the second case from their own. But on the other hand, nothing in the Special Theory permits us to suppose that Peter can take a longer time than his cousin to travel the same distance at the same velocity when distance and time are determined from a *single* platform, *sc.*, their own. Peter, therefore, having been born and having started on his travels when $A = A'$, will arrive at P at the same time as his cousin, starting from M $(= M')$, arrives at A, namely, at $t' = 70$, by his own clock *as he observes it;* whereas by the same clock *as Paul's cousin observes it* Peter's time of arrival at P will be $t' = 21$ (note 5). And this—if the retardation is taken as a physical reality—is our symmetrical paradox once more. Peter will be of two ages at once; and he will be both older and younger than Paul's cousin, who will, by the corresponding inference, also be of two ages at once, when Peter meets him at P. But McGilvary fails to bring these two sets of equations together and to deduce the consequence which they jointly entail; and he therefore remains unaware that he has mathematically confuted himself.

entire lives. And, once more, "the physiological age of each advances in the rhythm of his clocks." Now if *all* this is to be taken as referring to physical facts (in the sense already defined in McGilvary's own terms), Peter will be 3/10 as old as Paul's cousin and Paul's cousin will be 3/10 as old as Peter, at the same instant at the same place. If this absurd consequence is not the one intended, the result stated by McGilvary can only mean that each *seems* to the other to be younger than himself, but that their actual ages are the same—*i.e.* that the retardations are not physical realities. And it was to establishing the inevitability of this dilemma, in which McGilvary is himself here clearly involved, that my entire argument was devoted.

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