

## THE PARADOX OF THE TIME-RETARDING JOURNEY

### (II)

**I**N the preceding instalment of this paper \* I have examined a number of attempts to show that the celebrated “paradox of the clocks”, or—which is the same thing in a more striking form—the “paradox of the twins-of-unequal-ages”, follows from the relativistic premises usually accepted only in its asymmetrical form, *i.e.* that it implies that one of the twins, on his return from a journey at a very high relative velocity, will be younger than his brother, but not the absurdity that each will be younger than the other. None of these attempts seemed to achieve its object. Upon the principles of the Special Theory, and upon the (inconsistent) assumption that the retardations of duration implied by it would occur in the case of a round trip, and that these are actual physical effects, such as would manifest themselves in differences of clock-readings and of physiological ages when Paul, the “travelling twin”, rejoined his brother Peter, the symmetrical form of the paradox—in other words, a formal self-contradiction—has thus far appeared unescapable; and it has also been shown that the same contradiction results even more obviously when the twins are assumed to be in uniform relative motion without any reversal of direction.

4. There is, however, another way of reasoning about the whole matter, essentially different from those we have thus far considered, which is nevertheless often set forth as an equally valid interpretation of the Special Theory of Relativity. We have hitherto seen the orthodox expositors of the theory zealously though unconvincingly maintaining that the Lorentz retardations (whatever be true of the contractions) will really affect only Paul, the “travelling twin”; this seemed to be necessary in order to avoid the self-contradictory implication of a reciprocal comparative retardation. But the same expositors are accustomed to tell us that the properties of physical things are “relative” or

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“respective” to the space-frames to which they are referred; that one frame is as “real” and valid as another; and that therefore these discrepant properties can equally rightly be predicated of the same object, each, of course, in the proper frame of reference. Thus, with regard to the contractions, d’Abro, while insisting that they are not “illusions”, significantly qualifies this negation: “In Einstein’s theory the rod is contracted, but we must add that this contraction holds only in the space of the frame with respect to which the rod is in motion. In the space of the frame accompanying the rod in its motion, no contraction would occur.” Now in a world of relativity “there exists no privileged space, only ‘my space’, ‘your space’, ‘his space’”.<sup>21</sup> The equal ‘rightness’ of all frames of reference is a theme upon which physicists, when explaining these matters for the general reader, especially like to dilate. “‘Right’ as applied to frames of space”, as Eddington puts it, “is a blank label”; it can with as much (or as little) justification be attached to one as to another. For in the case of characters which are respective the question of rightness does not arise—until the *relatum* is specified. And the (quantitative) properties ascribed to physical things, being themselves ‘relative’ to frames, share this immunity from the troublesome question of rightness. “Distances, lengths, volumes—all quantities of space-reckoning which belong to the frames—are likewise relative. A distance as reckoned by one observer on one star is as good as the distance reckoned by another observer on another. We must not expect them to agree; the one is a distance relative to one frame, the other is a distance relative to another frame.”<sup>22</sup> A corresponding conclusion should hold for the retardations. Consequently Paul’s time will be retarded in Peter’s space and not retarded in his own, and *vice versa*. And of the two different durations between any two definite events taking place in Paul’s projectile, neither can be considered the ‘true’ or ‘real’ duration in preference to the other.

While this way of looking at the affairs of Peter and Paul implies—contrary to the arguments which we have previously reviewed—that the retardations (and contractions) will be

<sup>21</sup> *Op. cit.*, p. 220; italics mine.

<sup>22</sup> *The Nature of the Physical World*, p. 21.

symmetrical, it may also appear to exempt that implication from the charge of self-contradiction which I have been hitherto urging, and, further, to show that the conception of these effects as analogous to perspective appearances (in either system or both) is inadmissible. That conception presupposes that Paul's body—or his clock—is one entity, present on his projectile; and that the appearance or observed counterpart of his body or clock on Peter's system is another entity. But if we keep clearly in mind—it may be argued—the concept of frames of reference, we shall see that two entities are not involved, but only one with two quantitative attributes. Paul's body *is* related to both frames; it has, accordingly, one shape, or one physiological age, in, or with respect to, the one, another in, or with respect to, the other. Its two shapes, or its two ages, are, indeed, disparate. But there is no contradiction in asserting its possession of both shapes or both ages, for it possesses them with respect to different *relata*. Thus the situation, in the matter of the shapes and clocks and ages of Paul, and of Peter, is at once perfectly symmetrical and yet entirely free from contradiction.

This reasoning, it will be observed, transfers to such characters as the lengths and shapes of bodies, and to time-lapses, the logical property of respectivity which undisputably belongs to position and motion. The position of a body can, of course, be defined with respect to two or more sets of coördinates at once; and if the frames themselves are conceived to be in motion—*i.e.* to be set up on relatively moving bodies—then a given body may be truly at rest with respect to one frame and in motion with respect to the other. And it appears to be inferred, or tacitly assumed, that since this is true, and offers no difficulty to the mind, there is likewise no real difficulty in supposing that the other characters mentioned are respective to frames of reference—that, *e.g.*, Paul is truly six feet tall in, or 'relatively to', one frame, and just as truly three feet tall in, or 'relatively to', the other; and that the period between his departure and return is (given sufficient velocity) truly 200 years in, or 'relatively to', the one frame, and truly 2 years in, or 'relatively to', the other. Lengths, shapes and durations being by hypothesis 'relative to', *i.e.* functions of, rates of unaccelerated relative motion, are naturally, it is as-

sumed, also 'relative', *i.e.* respective, to the same *relata* to which the postulated states of motion or rest are relative.

In this argument—which manifestly has had a great part in the usual reasoning about the Special Theory and its philosophical consequences—there appear to me to be a number of logical defects; but I shall here mention only the simplest and the most decisive one.<sup>23</sup>

As Eddington, in attempting “to make clear the distinction between absolute and relative quantities”, observes, “number (of discrete individuals) is an absolute quantity. It is the result of counting, and counting is an absolute operation. If two men count the number of people in this room and reach different results, one of them must be wrong”.<sup>24</sup> But the retardation of time-lapses is—among other things—essentially a question of number. This Eddington has himself remarked in another context:

If the speed of travel is very great we may find that, whilst the stay-at-home individual has aged 70 years, the traveller has aged one year. He has only found appetite for 365 breakfasts, lunches, etc.; his intellect, clogged by a slow-moving brain, has only traversed the amount of thought appropriate to one year of terrestrial life. His watch, which gives a more accurate and scientific reckoning, confirms this.<sup>25</sup>

Now the proposition that number is an absolute quantity presumably means—at all events it should mean—that it is a *non-respective* quantitative attribute. It would therefore be absurd to say that Paul, between his departure and return—or, in the case of the moving platforms, between leaving A and arriving at P—has eaten two different numbers of breakfasts, say 365 and 25550, the one number “relatively to” his own frame of reference, the other “relatively to” Peter’s. If Peter thinks that Paul has eaten either more or fewer than 365 breakfasts during his absence, then (assuming for the moment that there has been a

<sup>23</sup> The application of the notion of respectivity to the contractions I have briefly discussed in *The Revolt against Dualism*, pp. 142–147.

<sup>24</sup> *The Nature of the Physical World*, p. 23.

<sup>25</sup> *Ibid.*, p. 39. Here it is once more forgotten that Peter is as much a traveller as Paul; and it is assumed that the retardation will affect, *not* the time of the frame of reference with respect to which the chronometers are in motion, but that of the frame with respect to which they are at rest.

real retardation, as Eddington supposes) Peter is wrong. Paul, according to what is implied by Eddington's assumptions, is equally wrong if he thinks that Peter has eaten any other number than 25550 breakfasts. And if Paul actually has a body, then each of these meals has consisted in a certain type of complex bodily event—definable in terms of chemical changes, electronic movements, or what you will. The number of *these* events, then, will likewise be 365—not relatively to a frame, nor to Peter's observations, but just absolutely 365. Also the number of complete circles described by the hour-hands of Paul's watch will be  $24 \times 365$ , irrespective of frames of reference.

'Retardation', then, is a name for the determination of certain quantities which, since they consist in the number of times of occurrence of certain specific classes of event between two given events, are absolute or non-respective quantities; and they consequently cannot benefit by that measure of exemption from the question of exclusive rightness which is legitimately claimed by respective attributes. This alone does not imply that Paul's age is not *conditioned* by his relative velocity; it means only that the contradictions previously pointed out cannot be escaped by applying the notion of dual 'reference' to ages. Of course, if we suppose—taking the case assumed in the passage last quoted from Eddington—that the fact that Paul, within a given period, eats only 365 breakfasts is a real consequence of his relative velocity, then Peter, having the same relative velocity, should, during the same period, eat 365 breakfasts. So considered, the two would at their reunion be of the same age. But this, as we have seen, is not what the hypothesis of real retardations in the Special Theory asserts. "All relativity estimates are reciprocal." If we start with the duration, or the number of events of a periodically recurrent kind, experienced in the one system, we must, using the Lorentz formula, compute from that a different number of such events for the other system. If we find the number in the one case to be 25550, we shall find it in the other case to be 365. But this should, according to the usual suppositions, be equally true from whichever end we start the computation. Paul and Peter, then, are once more required to have eaten, within the same period of their respective lives, both 365

and 25550 breakfasts, *i.e.* each of them must not only have eaten more (and fewer) breakfasts than the other has eaten, but also more (and fewer) breakfasts than he himself has eaten. But in view of the admitted absoluteness of number, the absurdity of the thesis of the physical reality of the retardations (so long as they are regarded as consequences of relative motion) is, if possible, more patent than before.

5. The proposed solutions thus far examined have all been formulated from the standpoint of the Special Theory. But it is usually upon an argument based on the General Theory that those who maintain the actual time-retarding effect of relative motion ultimately rely. This argument was presented by Einstein himself in a "*Dialog über Einwände gegen die Relativitätstheorie*", in 1918.<sup>26</sup> With the assertion of the critic of his doctrine, in that dialogue, that "not even the most faithful adherent of the theory can maintain that, of two correctly regulated clocks at rest beside one another, each can be slower than the other", Einstein agreed. If this were a consequence of the relativity asserted, the theory would imply an absurdity. The reply turns upon two principles of the General Theory: first, that during an acceleration there arises a gravitational field in the region occupied by the body undergoing acceleration (principle of the equivalence of inertial and gravitational forces); second, that "a clock goes faster, the higher the gravitational potential of the region in which it is". By assuming that the journey is a round trip, by employing the method of alternating options, and by applying the aforesaid two principles to the case in which Peter is regarded as the traveller, and not to the case in which Paul is so regarded, it is shown that in the former case there will be a speeding-up of Peter's clocks, during the time when his velocity is being accelerated; and that this speeding-up—by a lucky chance—is just sufficient to offset the retardation which his clocks undergo during the portion of his motion which is unaccelerated. Thus when he is the traveller there will, on balance, be no comparative retardation of his time at the end of his journey, but in the other case, *i.e.* when the motion is credited to Paul,

<sup>26</sup> In *DIE NATURWISSENSCHAFTEN*, VI, 1918, pp. 697 ff. For another exposition of it *cf.* Born, *op. cit.*, pp. 256–257, which I follow in the main in the next paragraph.

there will be no such offset, and his time, therefore, will be retarded, as set forth in the Special Theory. Hence the comparative retardation discoverable at the end of the journey will be unilateral only. "By this consideration", declares Einstein, "the paradox is entirely cleared away."

Less summarily stated, the argument runs thus (I follow with some abridgment, and with omission of the mathematical development, Max Born's exposition): We may, in accordance with the principle of relativity, suppose either Peter's system or Paul's to be our stationary system of reference.

*Supposition 1.*—Peter is at rest "in an inertial system, . . . in a region of space in which the measure-determination is Euclidean and gravitational fields are absent"; Paul, as before, makes his journey out and back; his time is therefore retarded, and on his return his clock will be slow, in comparison with Peter's, by an amount which it will be sufficient for the present purpose to designate as  $m$ .

*Supposition 2.*—"We regard Paul as at rest, and Peter as making a journey in the reverse direction." But, observes Born, "of course we cannot simply infer that Peter's clock must now be fast in comparison with Paul's by exactly the same amount" as Paul's was in comparison with Peter's under *Supposition 1*. For Paul "is not at rest in an inertial system but is experiencing accelerations". And, according to the General Theory, "when the system of reference is altered definite gravitational fields must be introduced during the periods of acceleration". In the present case, "Paul is at rest in a system of reference in which, at the moments of Peter's departure, of his reversal of direction, and of his return, gravitational fields briefly occur, in which Peter falls while Paul is held fixed by external forces. Of these three gravitational fields, the first and last have no effect on the relative rates of the clocks", since at the moments when they arise Peter and Paul are at the same points, and there is therefore no difference in gravitational potential. But at the moment of Peter's reversal of direction a difference of potential arises, in consequence of which his clock is speeded up so that it becomes fast in comparison with Paul's by an amount which computation shows to be (approximately) equal to  $2m$ . But during the two

periods of Peter's unaccelerated motion—*i.e.* the journey out and back—"the special principle of relativity must be applied", and therefore Peter's clock will during this motion be retarded by the amount  $m$ . Consequently, on the whole, Peter's clock will be fast in comparison with Paul's by the amount of the speeding-up minus the amount of the retardation, *i.e.* by  $2m - m = m$ . In other words, upon this supposition as upon the other, Paul's clock will in the end be slow in comparison with Peter's by the amount  $m$ , and Paul will still be younger than Peter. Thus the General Theory opportunely comes to the rescue of the Special Theory, and averts the self-contradictory consequences which—it is now apparently admitted, though it has hitherto been stoutly denied—would flow from the latter taken alone. "The clock-paradox", says Born, "is due to a false application of the Special Theory of Relativity to a case in which the General Theory should be applied." <sup>27</sup>

Such, then, is the final and official form of the argument by which escape from the symmetrical form of the paradox is sought. The argument, however, contains three fallacies, any one of which is sufficient to invalidate it.

(a) In order to make the General Theory applicable (in Supposition 2) it is necessary to assume that the motion in question is accelerated at a time when the two brothers, with their clocks, are at different places, *i.e.* that the traveller—whichever he be—makes a round trip. But the inference under

<sup>27</sup> Born, *Einstein's Theory of Relativity*, 1924, pp. 282-284. In quoting I have slightly condensed and clarified the translation at one or two points. It is perhaps advisable to quote Einstein's own statement (*op. cit.*) of the crucial part of the argument. He distinguishes the total event, in Supposition 2, into five periods: (1) that of the initial acceleration of Peter's clock  $U^1$ , until it assumes the velocity  $v$  along the negative  $x$ -axis of the coördinate system; (2) the outward journey at the constant velocity  $v$ ; (3) the period during which Peter's clock undergoes an acceleration (reversal of direction); (4) the return journey, again at the uniform velocity  $v$ ; (5) the arrest of Peter's motion at the point occupied by Paul. "Während der Teilprozesse (2) und (4) geht zwar die mit der Geschwindigkeit  $v$  bewegte Uhr  $U^1$  langsamer als die ruhende Uhr  $U^2$ . Aber dies Zurückbleiben wird überkompensiert durch einen schnelleren Gang von  $U^1$  während des Teilprozesses (3). Nach der allgemeinen Relativitätstheorie geht nämlich eine Uhr desto schneller je höher das Gravitations-Potential an dem Orte, an dem sie sich befindet, und es befindet sich während des Teilprozesses (3)  $U^1$  wirklich an einem Orte höheren Gravitations-Potential als  $U^2$ . Die Rechnung ergibt, dass dies Vorausseilen gerade doppelt so viel ausmacht als das Zurückbleiben während der Teilprozesse (2) und (4)."

Supposition 1, as to what will happen if Paul is taken as the traveller, is based wholly upon the Special Theory. That theory, however, is—as we have already noted, in the discussion of Argument 1, above—inapplicable to systems in accelerated motion; in other words, no inference can be drawn from it as to what will happen to Paul in the case of a round trip. But the inference from Supposition 1 is an essential part of the supposed “solution” as stated; therefore this solution is invalid.

(b) The “solution” not only employs the method of alternating options, but employs it in a peculiarly arbitrary and preposterous way. In its ordinary use, that method consists in first “taking” one of the two systems as at rest, and then the other, and arguing that the same conclusions follow in each case in turn (but not in both together), except that the rôles of the systems in question are reversed. What upon the one supposition is true of S is upon the other supposition true of S'. But in the present argument it is assumed that we can “take” a single physical event—the relative motion of S and S'—in two ways, and deduce *differing* kinds of physical consequence, according to the way in which we “take” it. When we pass from Supposition 1 to Supposition 2, which is ostensibly only another way of describing the same physical event, we do not merely transpose to S (exclusively) the attributes which we had formerly inferred for S', and *vice versa*—which would be bad enough—, but we assume that events of *quite dissimilar* types occur in nature in consequence of this shift in our arbitrarily chosen reference-body—our choice as to which system we shall “regard” as stationary. This criticism has been well put by Gawronsky:<sup>28</sup> “When it is a question of the motion of U<sub>2</sub> [Paul’s clock], it is, quite simply, an ‘external force’ by which its motion is initiated, reversed, and finally arrested. But when, on the contrary, it is U<sub>1</sub> [Peter’s clock] that is supposed to be moving, the humble ‘external force’ is set aside, and a whole troop of gravitational fields march upon the scene. . . . But how can the essential character (*innere Gehalt*) of an object be altered by our way of looking at it (*Betrachtungsweise*); and even if this were possible, how could two *Betrachtungsweisen*, which affect an object in two such radically different ways, nevertheless

<sup>28</sup> *Der physikalische Gehalt der speziellen Relativitätstheorie*, pp. 35–36.

be said to be 'equivalent'? . . . What can under no possible circumstances be justified is the assumption which Einstein makes the basis of his argument, *viz.* that when one of the clocks 'moves' the event has one definite physical character (*Gehalt*), and when the other 'moves' the selfsame event, the identical relative motion of the two clocks, has a wholly different physical character." So long as the method of alternating options is employed in this peculiar manner in the argument, this criticism must, I think, be regarded as fatal to it.

But we must here pause to consider whether the argument might not—while still employing the same considerations from the General Theory—be so amended as to escape this objection. And in fact it is not impossible to reconstruct it in a form different from that in which it is usually put by physicists (in consequence of their inveterate addiction to alternating options); and in this form it is at least not open to objection (b). And it is perhaps this, rather than the reasoning quoted, that expresses what, at bottom, the physicists have in mind. As amended the argument would run as follows. It is one of the postulates of the theory that while motion and velocity are relative, acceleration—as it is commonly phrased—is absolute. This phrasing, however, is not altogether fortunate. If referred to the systems in question, *i.e.* to two systems conceived merely as in relative motion *inter se*, it is doubtless true that 'acceleration' cannot have an absolute significance. If there is a uniform relative motion of the systems, and then a reversal of direction, both are accelerated. What clearly *is* absolute, however, as we have seen, is the kind of effect accompanying the acceleration of one of the systems but not that of the other—namely, the effect of so-called inertial forces. *These* must always be unilateral; you cannot, by merely choosing a different frame of reference, shift *them* from the one body to the other. In the case under consideration it made no difference (in view of the relativity of motion) which brother we called the traveller. But upon either supposition, it could be only in one system, not in both, that inertial forces would manifest themselves. There must, consequently, (according to the General Theory) be a difference of gravitational potential between the region occupied by system S and that occupied by

system  $S'$  during the reversal of the direction of their relative motion; hence only one set of clocks, physiological processes, etc., will receive the compensatory speeding-up resulting from this difference in potential. This disparity in the effects, however, must have some ground. Since it is not due to the acceleration of  $S$  with respect to  $S'$  (inasmuch as the effect of such acceleration would be bilateral), it must be due to the fact that  $S$ , when the direction is reversed, is accelerated with respect to some *third* reference-body, while  $S'$  is *not* accelerated with respect to this third body. But to what body is *this* acceleration to be referred? Einstein replies: "It cannot be affirmed that there are no masses present to which the generation of the gravitational field (in the case of an acceleration) can be ascribed. Certainly the accelerated coördinate systems cannot be regarded as real causes of the field—as a humorous critic once supposed me to assert. But it is possible to conceive of all the stars which exist in the universe as taking part in the generation of the gravitational field, for during the phase of acceleration of [Peter's] coördinate system they are accelerated relatively to the latter, and can therefore induce a gravitational field."<sup>29</sup>

Thus in the General Theory a species of cosmical reference-system, consisting of the totality of the stars, reappears; and it is this which makes the fact of absolute acceleration intelligible. Of two bodies which are in motion relatively to one another, one may be, and the other not be, in motion relatively to this system; and thus the acceleration (reversal of direction) of their motion with respect to one another may be also, in the case of one of them, an absolute acceleration, accompanied by the manifestation of inertial forces (Einsteinian 'gravitational fields'). We need not, then, make two opposite assumptions about the state of motion or rest of Peter or of Paul; we need only recognize that they may simultaneously, while having equivalent kinematical relations to one another, have different spatial and kinematical relations to the totality of the stars. And it is this, we may assume, that is really postulated in Supposition 2. But from this supposition—according to the argument cited—it follows that

<sup>29</sup> DIE NATURWISSENSCHAFTEN, VI, p. 700. The conception is, of course, taken over from Mach.

one, and only one, of the twins (*viz.* whichever one has during the acceleration been in the region of higher gravitational potential) will, when they come together again, be the older, and will find his clocks faster than his brother's.

While this argument would not be chargeable with the above-mentioned arbitrary and onesided use of the method of alternating options, it has nevertheless some logical peculiarities of its own. It will suffice to call attention to two of these.

(i) The introduction of a single cosmical reference-system, to render conceivable the absoluteness of acceleration, seems an abandonment of the principles of the relativity of position and motion. If "the totality of the stars" defines a universal frame of reference in which everything else has at any moment a univocal position and velocity and a unique spatial relation to every other thing, we are virtually back again in a world of absolute space and time; and unless such a universal frame is assumed, the assumption of a unilateral acceleration, and consequently of a unilateral speeding-up of the clocks, in the case of Peter and Paul, seems inadmissible. I shall not, however, develop this consideration, for it is not essential to my argument.

(ii) The acceleration would be unilateral *in all cases* only if in all instances of relative motion of two systems out-and-back with respect to one another (*i.e.* in all round trips) only one of the two were accelerated with respect to the "totality of the stars". But it is conceivable that, in their accelerated motion with respect to one another, *both* systems may likewise be accelerated with respect to the cosmical system. This might, for example, be the case if Peter and Paul both moved away from and returned to a point which, relatively to the cosmical system, remained at rest. In such a case their clocks (and ages) would presumably, according to the present amended form of the argument, undergo during their respective phases of acceleration no comparative or differential speeding-up; but their respective retardations during their phases of unaccelerated motion would still occur, in the amount  $m$ , since the retardations are supposed to be functions, not of their motions relative to the cosmical system, but of their relative motion *inter se*. Hence each would once more, at their reunion, be younger than the other by that amount. Not even,

then, by outdoing Joshua and invoking *alle Sterne welche im Weltall sind* can the relativist escape the conclusion that, if any retardation at all can, upon his principles, be inferred in the case of a round trip, the retardation must at least in some possible instances be reciprocal, *i.e.* the symmetrical or contradictory form of the paradox must follow from his premises.

(c) The final objection to the supposed refutation of the paradox which has been based upon the General Theory is, no doubt, evident from what has already been pointed out more than once. The General Theory—however you apply it—becomes pertinent only when an acceleration of one or both systems, at a time when Peter and Paul are at a distance from one another, is assumed, *i.e.* it is pertinent only to the case of a supposititious round trip. But we have seen that no round trip need be supposed. The whole paradox, in its symmetrical form, presents itself *during the outward journey*, before any such acceleration occurs. At any instant of this phase the retardations must, on relativistic principles, be both comparative and reciprocal, *i.e.* each twin must be physiologically as well as chronometrically younger than the other. And, as we have already seen, there is no difficulty about conceiving of comparable instants on the two systems; we need only suppose the brothers to be moving on contiguous parallel platforms, along which clocks originally synchronized are placed at intervals. When any two such clocks pass each will, according to the logic of the accepted doctrine, be slow in comparison with the other. It is, indeed, assumed by the relativist himself, even in his present reply to the objection—and must be assumed even in the amended form of the reply—that two phases of unaccelerated motion occur (phases 2 and 4 in Einstein's analysis) and that *during these* precisely the sort of reciprocal comparative retardation which *constitutes* the objection will arise; both in Supposition 1 and in Supposition 2 we are told that the "traveller's" clocks, etc., will during these periods be retarded (which can only mean "retarded in comparison with those of the other") by the amount *m*. But both brothers are travellers at one and the same time, since their travelling simply consists in their state of relative motion; both consequently undergo the comparative retardation, and thus, during the first

of these phases, each grows increasingly younger than the other. Thus the very absurdity which the reply is designed to eliminate is a necessary presupposition of the reply. We must conclude, then, I think, that the solution of the difficulty offered by the eminent author of the theory is no more successful than those previously examined.

The conclusion which I would here draw from all this is a limited and hypothetical one; it is essentially the conclusion set forth by Bergson in his *Durée et simultanéité*. I do not contend that there are no retardations and contractions resultant from relative motion; the physicists appear to have certain reasons, which I do not now attempt to examine, for thinking that there are. But it is clear, at all events, that these cannot be real physical effects on either moving system, such as are asserted in the story of Peter and Paul, as the physicists usually tell it; they can only be, as Bergson has maintained, reciprocal quasi-perspective distortions of objects on the one system as those objects are perceived—or perhaps are merely calculated on the basis of certain relativistic assumptions—by observers on the other.<sup>30</sup> Bergson's interpretation—which, he insists, really leaves the theory of relativity "intact"—has met with small applause from the physicists, and it is, of course, implicitly denied by all who tell us that it is an accepted part of orthodox science that Paul would actually be younger than Peter on his return, *i.e.* that the assymmetrical form of the paradox is valid. "It cannot", d'Abro declares, "be stressed too strongly that this interpretation of the problem is worse than incorrect. . . . The comparative retardation of the travelling twin's clock is an inevitable consequence of the Lorentz-Einstein transformations. To deny, therefore, that these transformations correspond to what would actually occur, to what would be detected and measured by the most accurate instruments, would be to deny that the transformations possess any physical basis. . . . The logical outcome of this attitude will be to deny the legitimacy of the entire theory. . . . If the Lorentz-Einstein transformations failed to yield the results that would actually be measured, Einstein's theory as a means of physical discovery would be not

<sup>30</sup> *Op. cit.*, 2nd. ed., pp. 243-246.

only useless but entirely incorrect, and we should have to abandon it." But "every precise physical experiment that it has been possible to accomplish has verified the anticipations of the Lorentz-Einstein transformations."<sup>31</sup> This utterance is entirely typical; yet it is also rather odd. For few, if any, physicists seriously maintain that the contractions of rods have the sort of physical reality which is thus ascribed to the retardations; yet, of course, the same status should be ascribed to both. It is not generally supposed that Paul's footrules, when he came to earth, would still prove, on direct comparison, to be shorter than Peter's, and that Paul himself would be a dwarf among his great-great-grandchildren; how can it be, then, that his clocks would still prove slow, or his physiological age be less than Peter's? The contractions, in short, admittedly are *not* physical facts, but only, in Born's phrase, *Betrachtungsweisen*—or, as Eddington puts it, they are "real" but not "truly real". Rods on a rapidly moving body, lying along the axis in which the relative motion is occurring, will, it is assumed, *look* contracted to observers on another body, and objects which seem to be globes to one who travels with them will seem to be oblate spheroids to one who does not share that motion; and it is merely this reciprocal disparity of appearances that the transformation-equations are, in the case of the contractions, supposed to express. The equations are "means of physical discovery" in so far as they predict what Peter would infer or experience when observing Paul's system, and *vice versa*; they not only need not be anything more, but they could not, in a relativistic universe, be anything more. Hence, in asserting that rapid travel would have a time-retarding or age-postponing effect *on the traveller*—in other words, in affirming the fifth of the earlier-enumerated propositions<sup>32</sup>—physicists have introduced into their doctrine a sort of absolute wholly incongruous with the fundamental principles of the theory; the strange tale they usually tell ought to be recanted by them on that ground alone, even if it did not lead—as it does—to self-contradictory consequences.

Yet from another point of view this inconsistent behavior of the physicists is by no means unintelligible. For it is in the

<sup>31</sup> *Evolution of Scientific Thought*, p. 227; cf. A. Metz, *La relativité*, pp. 65 ff.

<sup>32</sup> This REVIEW, Vol. XL, p. 56.

interest of the doctrine of the relativity of time that this assertion of an absolute, unilateral, physical retardation has been introduced. If Paul's clocks are *not* actually retarded, nor his senescence delayed, in consequence of his relative motion, then on his return—or, indeed, without any return—he and Peter will be able to determine that their times have been perfectly in accord—to whatever illusion about one another they may have for a time been subject. They will both, other circumstances remaining the same, have lived the same number of days, eaten the same number of breakfasts, had the same number of heartbeats, and so on. They will, in short, find reason to conclude that there is, in Bergson's words, "an indefinite multiplicity of fictitious times" but "only one real Time". And the physicists are quite right in feeling that this does *not* leave the theory of relativity "intact".

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