THE PARADOX OF THE TIME-RETARDING JOURNEY

(I)

NUMEROUS physicists and other men of science of high authority have set forth as clear and indubitable deductions from the Special Theory of Relativity certain paradoxes which turn upon an interpretation of the meaning to be assigned in that theory to the Fitzgerald-Lorentz contractions and retardations.[1] The more striking of these paradoxes have to do with the effects upon a hypothetical traveller of relative motion at any extremely great velocity. These deductions have been expressed in a variety of forms, all embodying the same principle. The most piquant of them may be called the Paradox of the Time-Retarding Journey. Though this pleasing relativistic *Märchen* is an oft-told tale, it is necessary for our purpose to repeat it here.

Once upon a proper time there were—or let us rather say, in the indefinite present, 'there are', since, after all, these events have not yet really happened—twin brothers, Peter and Paul. Peter is a home-keeping youth, but Paul has the soul of an adventurer and busies himself with the construction of a machine for flying through space at an enormous speed. When he and Peter are twenty years of age he completes this contrivance and sets forth upon his journey. Attaining almost at once a velocity amounting to a large fraction of that of light, he continues his celestial joy-ride at a uniform velocity, moving always in a straight line. At the end of a year, as shown by the clocks he has carried with him, he is able to reverse his motion and return to the earth at the same speed. Landing in his home-town, he discovers that in it decades

[1] *E.g.*, Einstein, "Die Relativitätstheorie", in VIERTELJAHRSSCHR. D. NATURFORSCH. GESELLSCHAFT IN ZÜRICH, 1911, Jahrg. 56, p. 12 (quoted in Gawronsky, *Der physikalische Gehalt der speziellen Relativitätstheorie*, 1925, p. 30); *cf.* also "Dialog über Einwände gegen die Relativitätstheorie" in DIE NATURWISSENSCHAFTEN, VI, 1918, pp. 697 ff.; Eddington, *Space, Time and Gravitation*, 1921, p. 26; *The Nature of the Physical World*, p. 39; J. Becquerel, *Exposé élémentaire de la théorie d'Einstein*, 1922, p. 58; Max Born, *Die Relativitätstheorie Einsteins*, 1922, p. 194; Birkhoff, *Origin, Nature and Influence of Relativity*, 1925, p. 103; G. N Lewis, *The Anatomy of Science*, 1926, pp. 24–7.

have passed and that his stay-at-home twin is now far older than himself. Nor would this be a conventional difference of calendars, a mere matter of bookkeeping. Paul will—such, in a relativistic universe, we are told, are the hygienic benefits of a fast life—be *physiologically* younger than Peter; indeed, if his speed was less by only 1/20000 than that of light, he will, returning as a youth of twenty-two, find that 200 years have elapsed on the earth, and consequently that his twin brother has long since died of the infirmities of age. For not only the clocks—in the colloquial sense of the term—which Paul has carried on his voyage, but all his physical processes, the motions of the atoms composing his body, will have been retarded in proportion to the rapidity of his flight, while no such effect will have been experienced by Peter. "In truth, a marvellous conclusion", says Max Born, "but one which no quibbling can escape. We must simply adjust ourselves to it as, some centuries ago, our ancestors adjusted themselves to the idea of men standing on their heads at the antipodes."[2] This "spectacular consequence of the Special Theory", says Gilbert Lewis, "has now been accepted into orthodox physics"; it is, indeed, "simply a more startling form" of the conclusion "that there is no absolute measure of the flow of time".[3] In short, we are given to understand, it is a part of the established body of scientific truth that—though it happens to be as yet technologically impracticable for Paul to make the flight in question—if he did make it the result indicated would be literally predictable; he would, in proportion to the speed and duration of his journey, become younger than his brother born on the same day. The retardation which Paul's time underwent would confer upon him, it is true, only a somewhat equivocal benefit; as André Metz has put it: "The traveller has really lived only two years; he therefore has, from this point of view, no superiority over other men. If his stomach allows him to take only two meals a day, it is $2 \times 2 \times 365$ meals that he will have been able to take in his projectile, and not $2 \times 200 \times 365$. If he is an author accustomed to write a book every six months, he will have written four books, not four hundred, . . . for his

[2] *Die Relativitätstheorie* (1922), p. 194.

[3] *Anatomy of Science*, pp. 124–7.

literary production is also equivalent to a clock! The only advantage (if it is one) which M. Paul will have will be that of finding himself [on his return] in the midst of his great-great-grandchildren."[4]

Now the difficulty with this conclusion is not, as writers on relativity usually appear to suppose, that it is, as thus far stated, extraordinarily paradoxical from the point of view of common sense. After all, what is supposed to have happened to Paul is, when expressed in concrete empirical terms, of a not wholly unfamiliar sort. First, the interval between his departure and his return has, as the phrase goes, 'passed more rapidly for him' than for those at home; it has subjectively 'seemed shorter' to him than to them. And, as the basis for this, his physiological functions have been retarded in the sense that fewer of all of them have occurred during the interval. But that the period between two events may have a very different subjective duration for two persons—say the period between the beginning and end of a philosophical lecture for the lecturer and for his audience—is not a new discovery; nor is it contrary to familiar analogies that a difference in physical conditions should make a difference in the mode or rate of occurrence of physiological processes. True, the difference in the effect is in this case supposed to be very great, and to be due to a physical condition—namely, relative velocity—which has not usually been believed to have effects of the sort in question. But the difference in the conditions is also supposed to be very great; and there is, to common sense, no obvious reason why velocity should *not* be a factor affecting senescence. True, again, that for Paul the actual number of units of time, as measured by his chronometers, would, according to the argument, be fewer than for persons remaining on the earth, and counting time by their chronometers, which, before Paul set out, were synchronous with his. But the variations of chronometers with varying conditions are likewise not wholly foreign to ordinary experience; and though, once more, unaccelerated relative motion has not commonly been assumed as one of the causes of such variations, Paul, when he empirically discovered (as in the usual version of the tale he is supposed eventually to do) that such

[4] *La relativité*, p. 26 n.

motion has an effect on clocks, might simply conclude that not all the causes of variation had previously been recognized by the watchmakers. The congruence with one another, during the voyage, of *all* these usual indicia of time-lapse, within Paul's own moving system, would, no doubt, lead him (if uninstructed in the theory of relativity) to attribute a sort of absoluteness to the time thus indicated, and he would therefore feel a natural surprise on learning that in no respect had time-measurements on the earth kept in accord with his. Yet even this would not amount to a contradiction—not even if Paul retained a prejudice in favor of Newtonian time; and he could therefore, with no excessive difficulty, accommodate his mind to the conception.

It is true that if we carry out certain further deductions from the same premises some rather striking consequences follow. If, for example, the voyager, instead of returning, travelled continuously with the speed of light, no time would elapse for him at all; he would be, in a sense, immortal, but it would be an immortality in which nothing happened, and therefore somewhat difficult to distinguish from death. But we are here concerned only with the supposition that his speed is less than that of light, which in any case, according to the theory, it always must be. And so long as we confine ourselves to this case, the real, or at least the major, paradox does not appear until we note a further implication of the Special Theory which most narrators of the tale of Peter and Paul omit. That theory is primarily a rigorous generalization of the principle of the relativity of motion; if the latter be false, the former cannot be true. And this principle asserts that the unaccelerated motion between any two systems is reciprocal, and therefore that if there *were* any modifications in durations and linear magnitudes *attributable solely to such motion*, these must affect both systems. It is, then, on relativistic principles just as true to say that the earth, and therewith Peter, has been moving away from Paul, as to say that Paul has been moving away from Peter. Hence the conclusions previously drawn for Peter must now be drawn for Paul. He too will find his time retarded, and in the same measure. As Professor Montague has put it, with a kindly sense of what Peter's feelings must be in the case hitherto assumed:

Suppose that the stay-at-home brother should feel piqued by the youthful appearance of the other who has returned from abroad. He would only have to remember that all motion is relative and that consequently he had a perfect right to regard himself as the traveller and his brother as having remained stationary. In that case it would be *he* who looked and really *was* younger. But that could happen only in a universe in which squares were round and the *principium contradictionis* had been put to sleep.[5]

This, however, does not quite express the full implication, for the case supposed, of the classical principle of the relativity of motion. Many of the critics of this part of the Special Theory have followed its defenders in employing what we may call the method of alternating options; they argue that (on relativistic principles) the reasoning can be equally well worked from either end, and that therefore, as compared with his brother, *either* twin in turn can be shown to be younger.[6] But in fact *neither* Peter *nor* Paul "has a right to regard his brother as having remained stationary" and himself alone as the traveller. It is, of course, true that each is at rest in the frame of reference which he sets up on his own system, but even in his own frame neither is at rest relatively to the other. Along one of the axes of Peter's coördinate-system the distance between himself and Paul is continually changing, and along the corresponding axis in Paul's coördinate-system the distance between himself and Peter is continually changing; and this change in the relation of distance along an axis is what is meant, and all that is meant, by unaccelerated relative motion. A change of this relation between two terms cannot be unilateral; it must be predicable of both terms at once or of neither. There is, no doubt, a certain convenience, at least for the imagination, in conceiving of one reference-body as fixed and the other alone as moving, relatively to the first, but it is

[5] PHILOSOPHICAL REVIEW, XXXII (1924), p. 156. Professor Montague regards the absurdity of this apparent consequence as one of the reasons for rejecting the Special Theory, at all events pending a further experimental testing of the Ritz hypothesis, which proposes to explain the negative result of the Michelson-Morley experiment on the supposition that the velocity of light is affected by the motion of its source.

[6] *Cf.* D. Gawronsky, *Der physikalische Gehalt der speziellen Relativitätstheorie*, 1925, p. 30, for the same objection to the Special Theory and a review of some of the replies to it by German physicists. Gawronsky's review, however, does not seem to me to include all of the significant attempts to deal with the difficulty, nor to deal satisfactorily with all of those he mentions.

a convenience gained at the cost of a temporary relapse into the conceptions of absolute space, position and motion. And the alternation of options in taking one system *or* the other as in motion can be legitimate only because both are *actually* in (relative) motion, however you 'take' them. An arbitrary choice of one way rather than another way of describing an objective fact cannot alter the nature of the fact.[7] If, then, either twin has been in relative motion, both must have been, and with the same relative velocity; and therefore—since the direction of motion is, in the Special Theory, irrelevant to the contractions and retardations—all the benefits of rapid travel supposed hitherto to have been gained by Paul between his separation from and reunion with Peter, must equally accrue to both. Their ageing will, therefore, have been retarded in the same ratio.

Retarded, it must be asked, in comparison with what? It is upon this question that it is especially important to have a clear understanding, for much confusion has arisen in consequence of vague or wavering ideas on this point. In order to answer the question correctly we must recall the logical function in relativist physics of the Lorentz transformation-equations. They are (in Einstein's words) formulas "for determining the space-time magnitude of an event when changing from one [that is, from *either*] body of reference to another"—a state of relative motion being assumed to exist between the two reference-bodies and the invariance of the velocity of light being taken as an established fact. The formula for the contractions gives the lengths of the rods etc. on Paul's system when those lengths are observed (or computed) on Peter's system; and the formula for the retardations gives the time-interval between two readings of the clocks on Paul's system as that interval is observed (or computed) on Peter's system. It is by comparison with the clocks on the earth that Paul's clocks will be slow on his return; and it is

[7] This is recognized by Einstein himself when he remarks, *à propos* of the familiar illustration of the 'moving' train and the 'stationary' embankment: "From the point of view of the theory of relativity the fact cannot be expressed by saying that 'it may *possibly* be the surroundings of the train (rather than the train) that have undergone an acceleration.' It is not a question of two different, mutually exclusive hypotheses about the locus of the motion, but of two ways of describing the same fact (*Sachverhalt*), both equally valid in principle." DIE NATURWISSENSCHAFTEN, VI, 1918, p. 701).

commonly assumed that, if Peter had sufficiently powerful telescopes, he would, during Paul's voyage, sensibly observe similar discrepancies between the readings of Paul's clocks and the readings of his own at corresponding instants. (To determine the corresponding instants it would be necessary for Peter in the case usually supposed, that of a journey out and back, to make the proper allowance for the time of transmission of the light from Paul's projectile to the earth.) The contractions and retardations are predicated *of* Paul's rods and clocks, but it is on Peter's system (the earth) or from Peter's point of view that they manifest themselves; and the other term of the comparison is a magnitude on Peter's system. In short (to quote a typical statement of the conclusion): "If in any two Galilean frames originally at relative rest, two identical clocks and two identical cubes are placed, and if then the two frames are set in relative motion, *either* observer would discover as a result of his measurements that *the other man's clock* had slowed down and that his cube had become flattened in the direction of the relative motion. *Nothing would have happened to the cubes and clocks;* only the conditions of observation would have been changed".[8]

Here, it is true, no change of a physical nature seems to have actually befallen either brother or anything that is his—except in the appearances which the other brother and *his* objects present. But if this were all, the orthodox asymmetrical paradox of the twins would not arise; for if nothing had happened during Paul's voyage to his body, or to anything that he carried with him, as a consequence of his relative motion, then manifestly no *effect* of anything's having thus happened would be apparent when he and Peter again came to rest on the earth. It would follow that each brother's observations or measurements of the objects on the other's system would have *no physical validity, as descriptions of objects on the other system;* the only physical facts (if any) corresponding to them would be of the nature of perspective distortions manifested in the mirrors, camera-plates, or time-recording instruments carried by each external observer. But the orthodox version of the tale of Peter and Paul implies (even though writers who tell it often disclaim this implication) that *the retarda-*

[8] D'Abro, *The Evolution of Scientific Thought*, 1927, p. 145; italics mine.

tions observed by Peter have really or physically happened to Paul—though, until his return, he would not be aware of them through any measurements made on his own system, since all his measuring-instruments would be affected equally with the things they measured. Something physical assuredly *has* happened to Paul in consequence of his relative motion if (as is assumed) at the end of it he is younger than he would have been if he had stayed at home—in other words, younger than Peter who did stay at home; or if, supposing his relative velocity to have been very great, he is alive whereas Peter is long since dead, as he himself, but for his travels, would also have been. Between being dead and being alive there is, one can't but think, a real difference; and this difference between the effects of relative motion and non-motion (for Paul) implies that the retardations have not merely *appeared to Peter* or, after his demise, to other terrestrial observers, to be taking place on Paul's system, but that they truly have been taking place on it.

But, since relative motion is reciprocal, all of these things must hold good also of Peter. His lengths and durations will seem shortened and retarded to Paul; that is to say, his rods will seem shorter than rods originally (*i.e.*, when in a state of rest on the earth) of the same length, on Paul's system, and his clocks will seem slower than clocks on that system with which they were originally synchronous. But, by the presupposition of the orthodox version of the story, these differences observed by Paul from his projectile will actually hold good of Peter and the rods and clocks on the earth. They will not only seem but be shorter and slower than Paul's. Thus, according to the logic of the customary argument about the twins—when we keep in mind the reciprocity of relative motion—Paul's clocks will be slower than Peter's and Peter's slower than Paul's; and Paul on his return, or at any time during the voyage, will be physiologically as well as chronologically younger than Peter, and Peter will be physiologically as well as chronologically younger than Paul; and, in the extreme case, when the projectile comes back to earth, Paul will be alive while Peter is dead, and also Peter will be alive while Paul is dead—that is to say, Paul will simultaneously be both dead and alive, and Peter will simultaneously be both dead and alive.[9]

[9] Some interesting further complications would result if we should introduce a third brother, a 'triplet', setting out and returning at the same time as Paul, but

This consequence being absurd, it is certain that there is something wrong somewhere in the premises from which it follows. Let us, then, recapitulate briefly those premises. They consist of the following propositions: (1) the retardations and contractions, in the amounts specified by the Lorentz transformation-equations, are *observable* under the conditions theoretically assumed; (2) they are *comparative, i.e.*, they characterize the lengths, durations, clock-readings, and ages of organisms on one system in contrast with those on the other; (3) the motion of the two systems is relative and therefore *reciprocal;* (4) the contractions and retardations, being by hypothesis consequent solely upon this motion, are also reciprocal, *i.e., each* observer finds that they hold good for objects and time-lapses on the other system, in contrast with his own; (5) the comparative contractions and retardations thus noted or computed by the observer on each system, in consequence of relative motion, are *physical facts in the other system, i.e.*, are actual modifications of the shapes of objects and the behavior of chronometers in the system relatively to which the objects and chronometers are at rest. This last, we have seen, is necessarily implied by the usual assumption, in the asymmetrical version of the paradox of the twins, that Paul's bodily processes will be so affected by his high relative velocity that there will be a difference between his physiological condition and Peter's on his return, a difference *then* observable by both of them—unless it is so great that Peter is already dead. Now, though the physicists who have told us of the age-deferring effects of truly rapid transit have not always recognized quite the full enormity of the paradox which results if all five of these propositions are accepted, they have usually felt it needful to avoid the slightly milder, though not fundamentally different, paradox resulting from the application of what I have called the method of alternating options—the conclusion, that is, that *either* Peter or Paul could equally well be proved, on relativistic principles, to be younger than the other. They have therefore sought to show that certain of the propositions above mentioned are false. And the ones which they have usually chosen to eliminate are precisely those

moving, relatively to Peter, with half of Paul's velocity; but I will not lengthen the argument by inquiring into the consequences of such a supposition. They are partly suggested by Guggenheimer, *The Einstein Theory*, p. 180.

which are most certainly true, namely (3) that relative motion is reciprocal, and (4) that any effect attributable solely to such motion must be symmetrical, occurring equally on both systems.

I shall now review the methods employed by numerous physicists or mathematicians of distinction in their attempts to justify the denial of the two last-mentioned propositions, or by other means to escape the implication of the paradox in its symmetrical form. The first four of these are efforts to accomplish this end from the standpoint of the Special Theory alone.

1. What has come to be the stock method for meeting the difficulty, so long as the argument is conducted within the limits characteristic of the Special Theory, is exemplified in Max Born's authoritative exposition. He first affirms the indubitable truth of the paradox of the twins in its asymmetrical form: "if A and B are twin brothers, then B must, on his return from the journey, be younger than A"; precisely this is the *wunderlicher Schluss* to which we are to habituate our minds. But to the argument that, since the motion of A and B is relative, the same conclusion must apply to A, Born answers: "The principle of relativity has to do only with systems in uniform rectilinear motion with respect to one another; to accelerated systems it is in the form thus far set forth (*i.e.*, that of the Special Theory) not applicable. System B, however, is accelerated. It is therefore *not* equivalent to A. A is an inertial system, B is not." [10] Hence the symmetrical paradox does not arise.

Now, disregarding for a moment the premises of this reply, there is plainly something peculiar about the logic by which the conclusion is derived from the premises. For the reasoning, when fully stated, appears to run thus: the occurrence of a retardation is inferrible only from the Special Theory; that theory applies exclusively to inertial, *i.e.*, unaccelerated, systems; system B (Paul's) is *not* an inertial system; *ergo*, it *can* be inferred that a retardation will occur in this system. Peter's system (A), however, *is* an inertial system; *ergo*, a retardation will *not* occur in Peter's system. This, surely, is a topsy-turvy sort of logic. If the other postulates of the physicists are admitted, it seems plain that they have attributed the age-deferring effect of Paul's

[10] *Die Relativitätstheorie Einsteins*, 1922, p. 194; English tr., 1924, pp. 214 ff.

adventure to the wrong twin; it will not be the traveller in the projectile but the (so-called) stay-at-home brother who will be the younger at their reunion—since he alone conforms, so to say, to the requirements for the applicability of the Special Theory. It is not, however, difficult, to see how physicists may have been led into this extraordinary reversal of the conclusion which follows from their premises. They 'take' Peter's system as 'at rest', *i.e.*, as not in relative motion at all (though it is, of course, upon relativistic principles, preposterous to take it so); they infer therefore, that Peter's rods or clocks will not be contracted or retarded; they know, however, that according to the theory contractions and retardations must take place when relative motion occurs; and they therefore attribute these to the rods and clocks on Paul's projectile—forgetting, for the moment, that, if that system is accelerated, as they in fact assume, the theory does not apply to it. If they wish to make all three assumptions, *viz.*, (a) that Peter is 'at rest', (b) that Paul's motion is accelerated, and (c) that the acceleration is not theoretically negligible, the physicists surely ought to infer that there will be no retardation on either side and no eventual disparity in age—in other words, that there is no such thing as a time-retarding *round-trip*, or at all events that the Special Theory does not justify any inference as to what the effects of such a trip will be; and therefore that the story of Peter and Paul, as usually told, has no proper place in expositions of that theory. It will not, however, follow that the theory does not imply that there will be time-retarding effects of a one-way (*i.e.*, an unaccelerated) journey.[11] Into this possibility we shall inquire presently.

[11] There would, of course, be three phases of acceleration in the motion of Paul's projectile: when he starts, when he reverses his direction relative to Peter, and when he lands on the earth. Physicists who dwell upon the acceleration usually appear to have the second in mind and to ignore the first and third. But the first alone would suffice to make the Special Theory inapplicable to Paul's journey—and therefore to render the deduction of a retardation uninferrible—*so long as the starting-phase is reckoned a part of the journey.* On the other hand, after this phase is over, and before his reversal of direction, Paul *will* be in a state of uniform rectilinear motion relatively to the earth, and *with respect to this phase*, if the Special Theory is true, a retardation of clocks, physiological processes, etc. must be deduced—and would be observable if the conditions specified in the supposition were realized. But it must be deduced for both of the participants in the relative motion; hence it

This argument, then, draws the wrong conclusion from the premises. But we must also examine more closely the premise from which the asymmetrical character of the retardation is inferred. That premise consists in the supposition that *only* Paul will undergo an acceleration when he turns his projectile homeward. Now it seems a paradox not less extraordinary than that of the twins that one of two systems in relative motion can have *that* motion accelerated while the other does not. Yet we are told that this is precisely the essence of the special principle of relativity; it "upholds the relativity of velocity and the absoluteness of acceleration".[12] Because of this, "the special principle *does not state that . . . when two bodies are in relative motion we can consider either one of them as being at rest"*; it asserts that "reversibility of this kind is possible *only when the the motions of both bodies are Galilean*". Now "in the present case the state of the twin remaining on earth is truly Galilean (if we neglect the earth's rotation and the curvature of its orbit); but this Galilean condition no longer applies to the twin who reverses his course. . . . In this case there is acceleration, and the well-known symptoms of accelerated motion, namely, forces of inertia, would make their appearance and would bring about well-defined physical disturbances."[13]

The paradox of absolute and unilateral acceleration, does not, however, serve to eliminate the paradox of the twins-of-unequal-age. It does not even substitute the one for the other; it merely adds a new difficulty while leaving the first unaffected. If you mean by acceleration a change either of direction or of velocity of motion, and if you are talking about the *relative* motion of two designated systems, S and S′, then it is a flat contradiction to say that S alters its velocity or its direction relative to S′, while S′ retains the same velocity or direction of motion relative to S as before. Relativistic physicists have, however, been led into this

will be a retardation of the clocks, ages, etc., of each in comparison with those of the other. It will appear below, however, that the relative motion of Peter and Paul can be conceived in a way in which *none* of the three accelerations need be supposed to occur at all. And in the General Theory we shall find that the first and third accelerations are supposed not to affect the retardations in any way.

[12] D'Abro, *The Evolution of Scientific Thought*, p. 104.

[13] *Ibid.*, p. 236.

joint assertion of two seemingly incongruous theorems—that velocity and direction of motion are relative, and acceleration, which is merely a change of velocity and (or) direction, is absolute, by a notorious empirical fact—namely, that the *dynamic effects* of acceleration, so far as they are observable by us, are *not* identical in the case of two systems in relative motion. If Paul reverses his direction suddenly, he, his clocks, and all material objects which he carries with him, will receive a terrific jolt. They will manifest their inertial forces by a tendency to continue to move in the same direction, which it will require the action of powerful contrary forces to overcome; and a dislocation of their relative positions in the system will take place. But nothing of this sort, it is admitted, will happen on the earth. Peter will feel no jolt, and the mechanism of his chronometers will not be disturbed, when Paul reverses his direction. This disparity in the dynamic effects is then, by the relativist, read back (illicitly) into the purely kinematic concept of acceleration itself.

There is, however, a conceivable argument based upon the empirical fact in question which has a certain *prima facie* plausibility. That the acceleration as such is reciprocal may be (since it must be) admitted by the relativist; but since this reciprocal acceleration does not produce the same dynamic effects in the two systems, it may be urged that it is possible that a reciprocal acceleration will likewise produce asymmetrical effects upon the lengths and time-lapses on the two systems. One reply to this argument has been presented by Bergson.[14] By a 'system' in kinematics is meant a set of points or of bodies which are at rest relatively to one another. Theoretically, two moving 'systems' are equivalent to moving points; if we regard them as such, the acceleration of relative motion will obviously be in all respects symmetrical. We may, however, attribute to the two systems any dimensions we like; if we still conceive each of them *as* a system, *i.e.*, "a collection of points which invariably maintain the same positions with respect to one another, and if we take account only of translatory motion, it is evident that we can treat them as if they were two material points, and the acceleration will, again, be perfectly reciprocal".[15] But when physi-

[14] *Durée et Simultanéité*, 2nd ed., pp. 259–265.

[15] *Ibid.*, p. 261.

cists speak of phenomena such as dislocations resulting from reversals of direction as occurring "within a system", they are confusing the concept of a system with that of a multiplicity of separate systems; if, for example, a traveller on a train is jolted off his seat by the sudden stoppage of the train, "the material points composing his body manifestly do not preserve invariant positions relative to the train, nor, in general, to one another. They therefore do not constitute *inter se* a single system; they are a number of systems, S^{II}, S^{III}, etc., which show themselves, when the 'jolt' occurs, to be animated each with a motion of its own. Hence, in the eyes of the physicist on the system S, each will have its own proper time, t^{II}, t^{III}, etc. The reciprocity will still be complete between S and S^{II}, between S and S^{III}, as it is between S and S^{I}. So long, then, as we consider only the relative motion of two systems properly so called, every consequence of such motion will be symmetrical.

This reply however, is purely formal and hardly sufficient. After all, it will be said, Paul's clocks, his body, etc., are systems, not single points; and they and all the other objects carried in Paul's projectile *will* be jolted, in consequence of the acceleration, while the objects on the earth will not; and it therefore is still possible to argue by analogy that effects on chronometers, etc., other than those connected with the jolting, which are also consequences of the acceleration, may likewise be unilateral. The fundamental objection to this argument is not, then, that given by Bergson. It is that the jolting could not conceivably be confined to Paul's system *if it were assumed that the acceleration (change of direction) is merely relative to Peter's system*, and therefore reciprocal, and that the jolting is purely a consequence of *that* acceleration. To say that it is so confined would (upon these assumptions) be to say that precisely the same phenomenon—*viz.*, a reversal of the direction of the relative motion of a body—produced a given effect in one case and no effect in the other; but science cannot proceed upon the hypothesis that an identical cause sometimes produces an effect and sometimes does not, when other conditions remain invariant. If the effect *is* unilateral, then one or the other of the assumptions mentioned must be false: either the two systems are not in merely relative motion, *i.e.*, the so-called inertial system (the earth) is in a state of rest in an absolute space or with respect to some absolute reference-body; or else the dislocations of objects on Paul's system are *not* consequences solely of a reversal of direction of relative motion with respect to the earth. The former alternative would take us back to the Newtonian universe; the latter might conceivably be brought under the well-known hypothesis of Mach about the nature of inertial forces. Paul's system would be affected differently from Peter's because its relations to the matter composing the rest of the universe would be different.[16] It is unnecessary to ask here which alternative is preferable, since either would destroy the analogy between the ordinary dynamic effects of acceleration and the effects relevant to the Lorentz contractions and retardations. For in the latter case it is assumed (in the Special Theory) that the only motion in question *is* the relative motion between the two systems, and also that any effects (of this sort) of the acceleration (if there are any such effects) will be due solely to the change in this relative motion, *viz.*, to the change of the direction of the motion of each system with respect to the other. Any *such* effect, then, resulting from the acceleration should be the same on both systems. There is, it is true, a tendency among physicists, even when expounding the Special Theory, to drag in a third (and essentially an absolute) reference-body, as the explanation of the postulated absoluteness of acceleration. This conception, however, has a place—if anywhere —only in the General Theory; its use in that theory will be considered later. Until such a conception is explicitly introduced and justified, we are concerned only with the motion which is a private affair between Peter and Paul; and from this point of view, *i.e.*, that of the Special Theory, we must still conclude (a) that no comparative retardation whatever can be deduced for the case of a journey in which there is a reversal of direction; but (b) that if there *were* any retardation—*i.e.*, if the acceleration in question were treated as theoretically negligible—the comparative retardation inferrible would necessarily be reciprocal.

But quite aside from these considerations, the invocation of the notion of the absoluteness of acceleration is beside the mark

[16] *The Science of Mechanics*, tr. by T. J. McCormack, 1907, pp. 231–238.

for a simpler and more obvious reason. The paradox of the twins—in its symmetrical form—arises even though no reversal of Paul's motion, and no acceleration whatever, is supposed to occur. This is sometimes recognized even by those who begin by telling us that the paradox disappears when the absoluteness of acceleration is recognized. Thus d'Abro observes that "as a matter of fact we can amend the problem of the two twins so as to eliminate the acceleration by assuming that the so-called travelling twin" proceeds "without stopping or reversing his course. In this case the motions of both twins are Galilean; and, true enough, the special principle of relativity permits us to consider either one of the twins as at rest and the other in motion." The supposition of an actual return (which was introduced by the relativistic physicists themselves) makes the story more striking, but it is not necessary in order to show the consequences which result from conceiving either twin to be physically affected by the relative motion. Instants simultaneous (even according to Einstein's definition) can be taken on the two systems, without assuming that there is any cessation of the motion or reversal of its direction. This may perhaps be more clearly seen if we imagine Peter to be on a flat platform extending as far as we please in either direction, and Paul to be on a similar platform immediately adjacent to Peter's and in uniform unaccelerated motion relative to it and parallel with it. If, while the two were at rest, synchronized clocks and automatic cameras were placed at intervals along the inner edges of both platforms, the event of any reading of any one of Peter's clocks will be simultaneous with the reading of any clock of Paul's which may be passing, for both clocks will be in this case virtually in the same place. Nevertheless their readings, according to the usual interpretation of the theory, will not agree; and their disagreement will be of the peculiar sort already mentioned. Peter's clocks, after the motion has begun, will be slower than the corresponding clocks of Paul as they pass them, and *vice versa;* and Peter's cameras will, after 200 years have elapsed by his clocks, show Paul passing them with the appearance of a youth of 22, though on his own platform Paul will long have been dead; and Paul's cameras will similarly show Peter as a youth of 22 after he has, on his own platform, long been dead. And, according to this "accepted part of orthodox physics" each set of cameras will correctly record the facts as they are on the other platform. Or we may alter the supposition a little, in order to avoid any complication involved in getting the motion started. Peter and Paul, not now brothers, must be supposed to have been born simultaneously at points A and A′ when these points on the two platforms were passing each other, and each remains throughout at the place of his birth on his own platform. In both directions from A and A′, on both platforms, observation-posts are placed at wide intervals; at each of these assistant observers are stationed, duly provided with clocks originally synchronized. It is the law on each platform that no one can be appointed an assistant observer unless he was born at the same time as Peter and Paul. Assume that 70 years have elapsed on Peter's platform up to the moment when he passes observation-post P′ on Paul's platform. Given sufficient velocity, he, an old man of 70 gazing at his coeval, the assistant-observer at P′, will seem to that observer to be a young man of 21; and assuming, as is done in the customary story, that a retardation observed from one system is a physical fact on the other, Peter will *be* twenty-one as well as seventy. At the same time his coeval at P′ will appear to Peter to be 21, and will therefore be of that age, as well as of the age of seventy. We thus eliminate any acceleration, and therewith all attempts to evade the symmetrical form of the paradox by invoking accelerations. The fact that it does not happen to be practicable to construct two such platforms is no more relevant to the question before us—*viz., What is implied by the Special Theory with regard to the effects of relative motion in any theoretically possible case of such motion?*—than is the fact that it is not practicable to construct the projectile in which Paul is supposed to move, in the more usual version of the story.[17]

[17] I have initially assumed in the argument above that two persons born at the same time, and in unaccelerated relative motion, will, on their respective platforms (other conditions also remaining the same), age equally between the moment of their birth and a subsequent moment identical for both; and that therefore the observer at P′ must, from this point of view, be 70 when Peter is 70 (though when reciprocal comparative retardations are assumed, each must also be of another age). Any other initial assumption would obviously conflict with the principle of

2. Yet we are frequently told that, even when the relative motion of the two systems is assumed to be unaccelerated, no real contradiction is implied by the Special Theory. One supposed proof of this has been offered by d'Abro. We are to assume that Peter, the earth, and a distant star, are all at rest *inter se*, and that Paul travels without a stop past earth and star; and are to suppose further that a rigid rod extends from the earth to the star. A certain relative velocity being assumed, "calculation would show that the duration of the passage of the travelling twin from earth to star would be twenty years in the opinion of earth observers and only ten years in the opinion of the wanderer". Now the reason for this difference (we are told) is that

> Since a relative motion exists between the so-called wanderer and the rod, the distance from earth to star, or the length of the rod, is contracted to half its length when computed by him. . . . If the wanderer were to consider himself at rest and the earth-twin, together with the earth and star, travelling past him, exactly the same conclusions would endure; since once again the rod, or earth-star distance would be animated with exactly the same relative velocity in respect to the so-called wanderer, and it is relative velocity, and not absolute velocity, that counts. We see therefore that no inconsistencies can arise whether we consider one twin or the other as at rest.[18]

What this appears to mean is that Paul's 'opinion' as to the duration, on his own system and on Peter's respectively, of his voyage would be exactly the same, whether he expressed the fact of relative motion between himself and the rod and its appendages by saying that he was moving past the rod or by saying that the rod was moving past him; and that Peter's 'opinion' would be likewise unaltered, whichever form of expression *he* used.

causal uniformity. The relativist may, however, prefer to reason differently, as follows: When Peter is 70 on his own platform he will find the observer at P′ to be, in consequence of the time-retarding effect of his unaccelerated relative motion, 21 years of age. But the retardations are actual physical effects on the platform of the observed; therefore the person at P′ will really be 21. He, however, will correspondingly find Peter, who has also been in unaccelerated relative motion, to be, at the moment when they pass one another, younger than himself, *i.e.*, to be *less* than 21; and this will likewise be a physical fact about Peter on his own platform. Peter will thus, once more, be of two ages at once, and will be both older and younger than the observer at P′, who was born at the same moment as himself. Some further complications of paradox which would follow I omit. If the relativist finds this way of arriving at self-contradiction more pleasing than that indicated in the text, it is equally open to him.

[18] *Evolution of Scientific Thought*, p. 237.

This *should*, it might seem, be true on relativistic principles; the motion being relative and therefore reciprocal, it should apparently make no difference from which side you calculated its effects. But it is not generally admitted by relativistic physicists that "exactly the same conclusions" would follow. According to the usual interpretation everything would depend on which system was *taken as* being at rest. If Paul assumed that Peter's system was at rest and his own in motion, and proceeded to make the calculation, he would arrive at the conclusion that it was his own cubes that were flattened and his own time that was dilated. The effect of changing your option as to which system you will consider at rest is, according to the orthodox reasoning on the matter, that set forth by Professor Eddington:

> An exceptionally modest observer might take some other planet than his own as the standard of rest. Then he would have to correct all his measurements for the Fitzgerald contraction due to his own motion with respect to the standard, and the corrected measures would give the space-frame belonging to the standard planet as the original measures gave the space-frame of his own planet.[19]

But it is not necessary, in the present connexion, to dwell upon this disagreement. Let it be supposed that, so far as d'Abro's account of Paul's computation goes, no inconsistency arises. The fact would, in any case, be irrelevant. For the inconsistency pointed out by those who cavil at the story of the time-retarding journey lies beyond; it inheres—it is necessary to repeat— in the following joint implications of the usual version: that, given the velocity and period mentioned, Peter's opinion—supposing it to accord with the Einsteinian interpretation of the Lorentz transformation-equations—will be that Paul's age, in a physiological as well as a chronological sense, when he is in the neighborhood of the star, is ten years less than his own; and that Paul's opinion will—on the same supposition—be that, when he is in the neighborhood of the star, Peter's age is ten years less than *his* (Paul's) own; and *that both opinions will be correct*—that is to say, that the comparative retardations will 'actually occur', and will not merely 'actually appear', from the point of view of the external observer, to occur. To this, the real argument of those who are sceptical about the advantages of rapid relative motion, the considerations thus far adduced by d'Abro manifestly offer no reply.

[19] *The Nature of the Physical World*, p. 15.

3. A definitive and summary proof that the hypothesis of the time-retarding potency of relative motion is free from any really paradoxical implications is offered by the same writer in the following:

> To what does the example of the two twins finally reduce? Simply to this: The departure of our twin brother and his return to earth constitute two definite events. The duration separating the events being robbed of any definite value by the theory of relativity, there is no cause to be surprised that this duration should manifest different magnitudes to different observers.[20]

This brief piece of reasoning is doubly irrelevant. It is irrelevant, first, because the paradox asserted by the critics of the usual relativistic version of the tale does not primarily consist in the different magnitudes, on the two systems, of the *duration between* the two definite events of Paul's departure and return; it consists in the fact that, according to the presuppositions of the orthodox view, Paul would at the end of that duration, and therefore *at a given moment*, be older than Peter and Peter older than Paul (and that each also would be younger than the other) in a physiological sense—and that in fact Paul, if he had travelled far and fast, would be at a given moment on the earth both alive and dead—as would also Peter. In the case of the unaccelerated or one-way relative motion of the parallel platforms, an equivalent absurdity would follow from the premises of the theory; when point A, where Peter stands, passed P′, Peter would, for reasons already given, be both 21 and 70 years of age; or, if the motion of the platforms had been fast enough, he would be both alive and dead. In this case, moreover, it would be obviously inadmissible to say that the "duration separating the two events" of Peter's leaving the immediate vicinity of A′ and arriving at the immediate vicinity of P′ would have no meaning. A countable series of events simultaneous and virtually identical (because occurring together at virtually the same places) would connect, *on both systems*, the two terminal events in question. The argument cited is irrelevant, in the second place, because the conclusion maintained by the proponents of the usual relativistic view—including the author of the argument—is that the retardations do *not* merely "manifest themselves to *different* observers", but are effects, on at

[20] *Op. cit.*, p. 235.

least one of the systems observed, which would manifest themselves by persisting differences, discoverable by *both* twins, in the physical conditions of their clocks and of their bodies at the end of the duration in question—unless the difference were so great that one of the brothers was no longer in a condition to observe anything.

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(To be concluded)