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THE DIALECTICAL ARGUMENT AGAINST ABSOLUTE SIMULTANEITY. II[1]

2. We shall now provisionally disregard the considerations hitherto set down in criticism of Einstein's argument, and assume that simultaneity-at-a-place and simultaneity-at-a-distance may and must be actually different relations, and therefore need to be defined in different terms. What, then, is the precise nature of the logical procedure in which Einstein is engaging, when he frames his definition of the latter term? Is he arbitrarily making up a new complex concept "out of his own head," subject to no objective requirement except that the result shall not be self-contradictory? Or is he assuming that there is a relational concept, or category, of distant-simultaneity, which is already employed by all men, and which his definition merely makes explicit? Our judgment of the significance of the ulterior argument to the relativity of such simultaneity will depend partly upon the answers to these questions. A proof that a notion of which we have all of us been habitually making use is what I shall call "respective" in its import, and therefore capable, without contradiction, of being both affirmed and denied of the same subjects of discourse, may be a highly important logical discovery; but the artificial construction of a definition of a term so that the "relativity" of the thing defined follows from the definition may be nothing more than a private—and not at all difficult—exercise of ingenuity, of no importance either for factual science or for philosophy.

Now if we closely scrutinize Einstein's procedure while he is putting together his definition of distant-simultaneity, we find, I think, that the definition is not quite wholly of the one kind nor wholly of the other, though, in his own conception of it, it is mainly a free or arbitrary definition—a verbal and not (as some of his interpreters suppose) a "real" proposition. In three points, two implied, one explicit, it is not arbitrary. It is assumed that the term to be defined must designate a *relation between events* and, of course, one that can conceivably subsist between them; and that this is something distinguishable from the purely spatial relations of distance and direction between the events in question. These are the restrictions on his freedom of definition which Einstein tacitly implies; and, of course, he expressly accepts the restrictions of the gen-

[1] Concluded from article in preceding issue, No. 23.

eral experimental theory of meaning. In his own words (not, presumably, to be taken quite literally), "there is only *one* demand to be made of the definition of simultaneity, namely, that it must supply us with an empirical decision as to whether or not the conception that has to be defined is fulfilled." [2]

But if these are the only antecedent limitations to one's freedom of definition, in the present case, why should Einstein's be regarded as *the* definition of distant-simultaneity, all others being spurious? Obviously it can not be so regarded unless he shows that his definition is the only one which conforms, or can conform, to these requirements, and specifically to the one which alone he makes explicit, *viz.*, the last. But Einstein offers no reason whatever for supposing that no "empirical definition" except his own is possible; and in fact many others are manifestly conceivable. One such, for example, has been pointed out by M. André Metz in his vigorous and able defense of the theory of relativity against its critics. It would, suggests M. Metz, be an equally "empirical" definition "if one substituted the term 'sound' for 'light' in the formula"; if, that is, one said: "Any two distant events are simultaneous, if sound-signals from the points where they occur reach jointly a mid-point between them occupied by an observer." This would assuredly seem to common sense, as it does to M. Metz,[3] a wrong definition, because the propagation of sound is by them supposed to be already known, in advance of the definition, to be not isotropic; but this objection arises only because both M. Metz and common sense assume an already given "natural concept of simultaneity," the fulfilment of which would not be shown by the joint-arrival of the signals at the mid-point *M* unless their velocities over the distances *A-M* and *B-M* were known to be uniform and their times of transit over these distances therefore equal. In other words, common sense, supposing itself to be already well acquainted with what is meant and implied by "simultaneity-at-a-distance," finds therein reasons for holding that light-signals are at least *more* serviceable than sound-signals as means to inferring, from a given point, whether two events remote from that point did or did not occur simultaneously. But Einstein accepts no such ready-made *a priori* conception; his definition is, at this stage of his procedure, not yet reached; and it is not because he already *knows,* or can even meaningfully assume, that light is isotropic and sound is not that he defines simultaneity in terms of the joint-arrival of light rather than of sound-signals at the point *M*. This he is himself careful to point out. He imagines a critic who raises the objection that his definition presupposes a previous knowledge of the fact that light always traverses equal distances in equal

[2] *Relativity*, etc., p. 27.

[3] Metz, *La Relativité* etc., 1923, pp. 21, 224.

times; that "an examination of this supposition would be possible only if we already had at our disposal the means of measuring time"; and that it would therefore "appear as though we were moving here in a logical circle." But Einstein replies that his "definition assumes absolutely nothing about light. . . . That light requires the same time to traverse" the two paths "is in reality neither a *supposition* nor a *hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own free will in order to arrive at a definition of simultaneity." What this presumably means is that the proposition about the time required for light to traverse the two equal paths is verbal, not factual; the *meaning* of the term "equal-times-of-transit" is by arbitrary definition to be "times-occupied-by-any-two-light-rays-in-passing-from-two-points-to-a-midpoint-between-them." In short, in order to avoid circularity in his definition, Einstein finds it necessary, not merely to define "simultaneity" itself arbitrarily, but also to define arbitrarily one of the terms entering into the former definition. In a theory containing the "stipulation" Einstein mentions, no question of experimental fact could ever arise as to whether light traverses equal distances with respect to a given reference-body in equal times; to ask such a question would imply some *other* sense of "equal times" than that specified in the definition of that term. This, be it incidentally remarked, is a somewhat odd consequence of the attempt to arrive at a rigorously experimental concept of simultaneity.

We are to understand, then, that Einstein's definition of simultaneity-at-a-distance is, and is by its author intended to be, one adopted "of his own free will"—subject only to "the one demand," or at most the three demands, already mentioned. It is arbitrary not only in that it rejects the conceivable "empirical" alternative mentioned by Metz, but also in that it rejects the indeterminately numerous other definitions which might be framed in conformity with these requirements. The subsequent argument to the relativity of simultaneity is relevant, therefore, *only to the particular sense of simultaneity which Einstein has arbitrarily created by definition.* It is evident that he has realized that—if "simultaneity" is not an undefinable logical primitive, having, however, necessary or *a priori* connections with certain other concepts—the definition of it must inevitably be *either* circular or arbitrary. Of these alternatives, he is determined to avoid circularity, and therefore frankly accepts the other horn of the dilemma. It does not follow from this that the argument to relativity is not valid, *for* the defined sense of simultaneity, nor yet, necessarily, that the conclusion of the argument is not important; but that it *is* important—that, indeed, it is anything more than tautological—requires to be shown. To this question we

shall shortly return. Meanwhile, the supposition of most expounders of the Special Theory, that Einstein has proved the relativity of simultaneity in general—or that *his* "simultaneity" is something more than a logical artefact—must manifestly be given up.

3. The arbitrary character of the definition becomes further apparent when we note that two alternative "stipulations" were equally possible with respect to one relevant point not hitherto made explicit. Two events at A and B are, we are told, to be considered simultaneous if light-signals emitted from those points at the instants when the events occur there reach simultaneously an observer midway between A and B. But *when* must the observer be midway between them? Is it sufficient that he *shall have been* at the midpoint before the light reached it, or is it required that he *shall be still* in that position, with respect to A and B, when either signal (or both) shall arrive at the point of observation? Einstein's answer to this is, of course, evident. The criterion of simultaneity or non-simultaneity is so defined by him that if the two observers (or their recording instruments) *have been* at the midpoint at a time previous to the arrival there of either signal, then they are both applying the same criterion, even though one of them is no longer at that point. To make quite sure that there may be no mistake about this, let me quote Einstein's own expression of the crucial part of the argument. He is using the familiar illustration of the train moving along a railway track, and is asking whether "two events (e.g., two strokes of lightning at A and B) which are simultaneous with reference to the railway embankment are also simultaneous relative to the train"; and he is to show "that the answer must be in the negative." The points A and B at which the lightning strikes the embankment are assumed to "correspond to"—which apparently means, to be virtually coincident with—"positions A' and B' on the train." "Let M' be the mid-point of the distance A'-B' on the travelling train. Just when the flashes of lightning occur (as judged from the embankment), this point M' naturally coincides with the point M, i.e., the mid-point on the embankment, but it moves towards the right with the velocity of the train. If an observer sitting in the position M' in the train did not possess this velocity, he would remain permanently at M, and the light rays from A and B would reach him simultaneously, i.e., they would meet just where he is situated. Now in reality (considered with reference to the railway embankment) he is hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightning flash B took place earlier than the lightning flash A. We thus arrive at

the important result: Events which are simultaneous with reference to the embankment are not simultaneous, with respect to the train, and *vice versa* (relativity of simultaneity.)"[4] Now this "important result" follows only if the same test of simultaneity or non-simultaneity is applied by both observers, and only if it makes no difference to the applicability of that test that the observer in the train has *not* "remained permanently at *M*," while the observer on the embankment has done so. But it was not necessary to formulate the test—and therefore the definition—of simultaneity or its opposite in this way. It would have been equally possible to say: The events at *A* and *B* are to be considered simultaneous if the light-signals from them arrive together at a point which *is* midway between *A* and *B* at the moment of their arrival; and the separateness of the moments of arrival of the signals at the position of an observer who has been but no longer is midway between *A* and *B* is not to be considered evidence of the non-simultaneity of the events.

If the criterion had been formulated in the second way, the "relativity" of simultaneity would not have followed. The observer on the train would in that case simply say: "These signals reach me separately; but since I am not *now* midway between *A* and *B*, that fact does not show that the events at *A* and *B* were not simultaneous." Not only would this alternative definition be equally possible, but it would obviously accord, as the other does not, with our "natural" idea of simultaneity: i.e., it would express the implications, with respect to events at a distance, of that fundamental concept which we empirically acquire through the experience of simultaneity at a place. The only reason for preferring the first statement of the criterion to the second is that from it only can the relativity of simultaneity be proved. The logical procedure is that of choosing arbitrarily that definition of the crucial term in the argument which will permit the conclusion desired to be drawn.

Possibly it will be said in reply to this that the second statement of the criterion of simultaneity would not meet the requirement that a definition of that concept must "enable us to decide by experiment whether or not" two events, e.g., the lightning strokes, "occurred simultaneously." The observer on the moving system, e.g., the train, must make his measurements on his own system. When the point *M'* on that system coincided with *M* on the other system, the fact known to him was that he was midway between *A'* and *B'*, the points on his system which "corresponded" to *A* and *B* on the other. And he is still, when the first signal reaches him, midway between *A'* and *B'*, consequently, from his standpoint, he is still midway between the sources of the signals. But this reply would be

[4] *Relativity*, 1920, p. 31.

unconvincing. For, in the first place, it is not necessary to suppose that the observer on the train is unaware that he is moving relatively to the embankment in the direction *A-B;* and if he is aware of this, he knows that if *A* and *B* were equidistant from *M'* when *M'* and *M* were coincident, they can no longer be equidistant from *M'* after that point has moved from the position of *M* (which is still midway between them), towards *B*. He has therefore sensible means of determining that the requirements of the second criterion are not, for him, fulfilled; which means, not that he will judge the two events to be non-simultaneous, but that he will not (so far as *this* evidence goes) judge them to be simultaneous. If he *also* knows that the signals *do* arrive at *M* simultaneously, he will judge that the events *were* simultaneous. In the second place, if it is still insisted that each observer can have empirical information only about events on his own system, what follows is that the signals will arrive simultaneously *both* at *M* and *M'*, and that both observes will consequently reach the same conclusion, namely, that the events reported by the signals were simultaneous. For, once more, upon the present assumption, all that the man on the train could have known when his point *M'* in fact coincided with *M* was that he was midway, not between *A* and *B*, since these were not points on his system, but between *A'* and *B'* on his system; and the lightning strokes about the simultaneity of which he would have to judge would occur at *A'* and *B'*, not at *A* and *B*. But signals from *A'* and *B'* must reach *M'* simultaneously, since, upon the principles of relativity physics and the apparent evidence of the Michelson-Morley experiment, the velocity of light is constant over equal distances on any given system, irrespective of its relative motion. The consequence that both observers would judge the events *about* which they judged to be simultaneous would hold, whichever criterion of distant-simultaneity were adopted, so long as it was assumed that each was judging only about events happening on his own system at places equidistant from a midpoint determined on that system. But in this case the two observers would not in fact be judging about the same pair of events; the one would be judging about events occurring at *A* and *B*, the other about events occurring at *A'* and *B'*. Hence, if their conclusions did disagree in the case supposed (though they would not), the fact would have no significance, since the two observers would be talking about different things. Finally, the assumptions on these matters actually made by Einstein himself, in formulating his own statement of the criterion of distant-simultaneity, are that the observer on the train as well as the one on the embankment *is* judging about events occurring at *A* and *B*, i.e., at points on the embankment; that at a moment before the signals

arrived, M' was coincident with M, and was then midway between A and B; that it ceased to be midway between them before any signals reached it; and that because of this actual displacement, relatively to the embankment, the signal from A reached M' later than that from B. Now these suppositions are either of an empirically verifiable nature or they are not. If they are not, they have no meaning, according to the radically experimental theory of meaning, and it is therefore not permissible for Einstein to use them in formulating his definition. If they are so verifiable, they may legitimately be used by the observer on the train, in arriving at his conclusion about the simultaneity of the two events. But if they may be so used by him, there can be no objection, from the standpoint of the experimental theory, to adopting the second formulation of the test of simultaneity and non-simultaneity instead of the first; for they are the only presuppositions required to make that formulation intelligible and legitimate.

4. One may go farther than this in justification of the proposed alternative definition of the criterion of distant-simultaneity. In strictness, it alone, and not Einstein's definition, makes it possible for both observers to have the *same concept* of simultaneity and therefore to judge of the same matter. For simultaneity is to be defined in terms of an experimental test; and the same test is not applied unless the relevant experimental conditions are the same, at the time when the experiments are actually performed. The experiment here in question is that of determining the togetherness or non-togetherness of the arrival of the beams of light at a point equidistant from their sources. Now, *when* is this experiment performed by the observer on the train? Not, obviously, before either beam reaches him. But, by hypothesis, before either reaches him he is no longer at the same distance from the light-sources as the observer on the embankment. But this distance is one of the relevant experimental conditions. The two therefore do not perform the same experiment; and therefore they are not judging about simultaneity in the same sense. If, under these circumstances, the one declares that the two events are simultaneous and the other that they are not, the discrepancy between their conclusions does not prove the relativity of simultaneity; it only proves that they are talking at cross-purposes. The fact that two observers disagree concerning the simultaneity of a pair of distant events, when one of them uses as the definition or the criterion of that attribute the joint-arrival of light-rays at a point midway between their sources at the moment of their arrival, and the other uses as *his* criterion their joint-arrival at a point *not* midway between their sources at that moment, is no more significant than would be a disagreement between two illiterate persons over

the question whether a whale is a fish, when one of them meant by "fish" any free-swimming animal that lives in the water and the other meant gill-breathing and cold-blooded animals.

5. Let us, however, assume that both observers *are* using the same criterion, and that this requires merely the initial coincidence of M and M'—i.e., requires that the observers be equidistant from A and B at a moment *before* the light reaches either of them. Even so, the conclusion drawn does not follow. For that conclusion is general—it asserts the relativity of simultaneity, and therefore of time-intervals, to *any* state of relative motion between systems. But the argument (aside from any other difficulties) shows this only for a special class of cases, namely, those exemplified by Einstein's illustration of the train and the embankment, in which the motion of the two systems is parallel, or at all events not perpendicular. It is, however, possible to suppose that the positions of the observers at a given instant coincide at M, but that the subsequent uniform rectilinear movement of S' is along a line perpendicular to S at M. In this case, though S and S' will be in relative motion, the two light-rays from A and B should (according to Einstein's assumptions in the present argument) reach the observer on S' jointly, and therefore he, as well as the observer on S, will judge them to be simultaneous. Thus, even if all other objections to the argument are waived, it would clearly be insufficient to justify the *wholesale* relativization of time which is inferred from it.

6. It was apparently, as we have seen, in order to avoid circularity in his definition that Einstein chose the alternative of making it a "free" or arbitrary definition. Let us now ask whether he has in fact completely succeeded in avoiding circularity. Does he not presuppose certain meanings of both "simultaneity" and "time of an event," "in order to arrive at" his own definitions of those terms; and—what is more curious—are not these tacitly presupposed meanings at variance with those in the formulation of which he employs them? These questions, I think, must be answered in the affirmative. For though he *makes* the time of transit of the light-rays over the paths A-M and B-M *equal* by "stipulation," i.e., by pure definition, the stipulation relates only to the equality and not to the notion of "time of transit" itself. That we already know what is meant by saying that a duration elapses during the transit of light over one or the other path, he seems quietly to assume; but in doing so, he also assumes that we are already familiar with concepts which would ordinarily be expressed by the distinction between "simultaneity" and "non-simultaneity." To put the matter in another way: in "stipulating" that the times of transit of light from A to M and from B to M shall by definition be equal, Einstein is applying the quantitative predicate of equality to something

which, in order to be capable of having that predicate, must be assumed to possess number or magnitude, and this something is, explicitly, "time." He is, therefore, "begging" the idea of time-interval, before defining simultaneity. But the idea of time-interval —and still more obviously, that of motion during a time-interval— contains that of succession, in which is implicit the distinction between successiveness and non-successiveness, i.e., simultaneity, of events. And this distinction is not that between simultaneity and successiveness at a place; if it were, it would be inapplicable to the concept of motion, i.e., of successive occupancy of a series of increasingly distant places. It appears impossible for Einstein to formulate his definition of simultaneity-at-a-distance without first accepting both the meaningfulness and the truth of the law of the finite velocity of light, which itself presupposes that the meaning of simultaneity and non-simultaneity-at-a-distance is already known. His so-called "definition" of time is in terms of relative motion, which, plainly, already contains the idea of time and of the two characteristic types of temporal relation.[5]

7. It seems evident, finally, that Einstein himself does not adhere to the relativistic conclusion which he has deduced in this questionable way from his definition, and that he can not do so without vitiating the rest of his argument. For, after he has ascribed to each moving system a "time" of its own, in which events simultaneous on any other such systems are of differing date, he continues to speak of periods of time during which the two systems are in motion with respect to each other, and during which, also, some determinable number of events is occurring on each. *While* such and such things are happening on S, we are told, such and such other things are happening on S'. Now this sort of proposition can have no meaning if there is no *common* duration with a common measure, to which the motion of both systems with respect to one another, and the two series of events, are referred. You can not say that while S moves to the right S' moves to the left, if there are in reality simply

[5] Substantially the same point has been neatly put by La Rosa in an article which I have come upon since writing the above. After quoting Einstein's remark that he merely "stipulates" that the time of transit over the paths A-M and B-M shall be equal, La Rosa asks: "But how does it happen that this free choice finds its free manifestation only when it chooses as point of observation exactly the point M, midway between A and B, rather than any other point? Perhaps because it is more natural and convenient to suppose that the light takes the same time to traverse the two paths. But what sense can be given to the words 'the same time' when we do not possess the notion of contemporaneity indispensable for the measurement of time? And how, further, can it be that in the necessary choice of M as point of observation, the knowledge of the equal velocity of the light in the two directions is not already implicit? But what sense is to be given to the word 'velocity,' when we have not yet arrived at the notion of time?" (*Scientia*, Vol. 34, p. 297).

two distinct "whiles," one definable solely with reference to S and the other solely with reference to S'; for in that case, you will have isolated the two systems so completely from one another that the relation called "motion" can no longer be conceived to subsist between them. There must, in short, be a common "while" between any two systems of which relative motion is asserted. But if there is a community of "whiles" there must also be a community of "whens," unless motion, change, and time-lapse are assumed to be discontinuous. All changes in relative position or otherwise, asserted with respect to S, must be in one-to-one correspondence with *some* events in S'; otherwise there would be intervals during which there was motion and time-lapse for one system, but not for the other—which would be a contradiction, since the motion is reciprocal and therefore never predicable of the one at any instant when it is not predicable of the other. The assumption of this common time is not less necessary even if (in accordance with another portion of the Special Theory) the "time" on S' is said to be "slower" or more "dilated," than that on S. This can only mean that, between two of the instants identical for both, fewer physical changes of a given kind—e.g., complete revolutions of the hands of a clock—occur on the one system than on the other. The very expression of the relativity of clock-times is impossible without the presupposition of an inclusive duration of which the measure is the same for both reference-bodies and is therefore independent of their relative motion. To say literally and exclusively that a given "reference-body has its own particular time" is simply to deny that it is in motion at all with respect to any actual external system.

This argument of Einstein's, then, seems to be one more example of a type of phenomenon familiar enough to the student of the history of philosophy; it is another case in which the very premise which is unheeded and even denied may, by the attentive ear, be heard the while softly murmuring

When me they fly, I am the wings.

The whole of the Special Theory, in spite of its ostensible conclusion, is pervaded by the assumption that, in Whitehead's phrase, "an instantaneous event is nothing else than an instantaneous and simultaneous spread of the universe." We arrive, in fine, by a different road, at the conclusion already expressed by Bergson: *nous prétendons que le Temps unique subsiste dans l'hypothèse d'Einstein à l'état pure; il reste ce qu'il a toujours été pour le sens commun.*[6]

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[6] *Durée et simultanéité*, p. 33. Bergson adds that the same is true of "l'étendue indépendante de la durée," a matter with which the present paper is not concerned.