ELECTRICITY AND GRAVITATION.

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SYNOPSIS.

Electric Doublet Theory of Gravitation.—It is suggested that gravitation may be an effect arising from fluctuations of the electric charges associated with the electrons and positive nuclei of atoms. It is assumed (1) that the ether contains an enormous number of electric doublets moving in all directions with the speed of light; (2) that each charged particle is continually both absorbing and emitting doublets at a rate proportional to its mass; and (3) that during the absorption and emission of each doublet the charge on the particle fluctuates. If these fluctuations exist, it is shown that the mean value of the force exerted by one charged particle on another includes, in addition to the ordinary electrostatic force, an attraction proportional to the product of the masses.

Electric Doublet Theory of Radiation.—If we suppose that the doublets emitted by a particle possess available energy only when the energy of the particle changes, and that the effect of changing the energy is to produce periodic gaps in the emission of doublets with a frequency proportional to the amount of energy lost, we have a theory of radiation which is said to be compatible with the theories of Bohr, Planck and Einstein.

1. *The Field of an Electric Pole with a Fluctuating Electric Charge.*—We shall consider a type of field in which the electric charge associated with a pole varies on account of the emission of electrified light-particles. Expressions for the components of the field vectors when the pole moves with a velocity less than c and the direction of projection of the light particles varies in an arbitrary manner have already been given.[1] It will be sufficient to give here the components of the field vectors E and H for the simple case in which the pole remains stationary at the origin of coördinates and the light-particles are emitted in the direction of the negative portion of the axis of z. We then have

$$E_x = \frac{x}{r^3} f\left(t - \frac{r}{c}\right) + \frac{xz}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right), \quad H_x = -\frac{yr}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right),$$

$$E_y = \frac{y}{r^3} f\left(t - \frac{r}{c}\right) + \frac{yz}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right), \quad H_y = \frac{xr}{cr^2(r+z)} f'\left(t - \frac{r}{c}\right),$$

$$E_z = \frac{z}{r^3} f\left(t - \frac{r}{c}\right) - \frac{r-z}{cr^2} f'\left(t - \frac{r}{c}\right), \qquad H_z = 0,$$

[1] Electrical and Optical Wave Motion, Cambridge University Press (1915), p. 128, Messenger of Mathematics, May (1915), p. 1.

where $f[t - (r/c)]$ is an arbitrary function which determines the instantaneous value of the electric charge associated with the pole O.

A line of electric force can be regarded as the locus of light-particles projected in directions specified by the successive values of a unit vector s which is a function of $\tau = t - (r/c)$. Assuming that the vector E is proportional to the vector $rds - csd\tau$, we see that if (l, m, n) are the 3 components of s they must be supposed to vary with τ according to the equations

$$\frac{dl}{d\tau} = -\frac{ln}{1 + n}\frac{f'(\tau)}{f(\tau)}, \quad \frac{dm}{d\tau} = -\frac{mn}{1 + n}\frac{f'(\tau)}{f(\tau)}, \quad \frac{dn}{d\tau} = (1 - n)\frac{f'(\tau)}{f(\tau)},$$

Solving these equations we obtain

$$(1 - n)f(\tau) = (1 - n_0)f_0 = a,$$

say,

$$\frac{lf}{\sqrt{2f - a}} = \frac{l_0 f_0}{\sqrt{f_0(1 + n_0)}}, \quad \frac{mf}{\sqrt{2f - a}} = \frac{m_0 f_0}{\sqrt{f_0(1 + n_0)}},$$

where l_0, m_0, n_0, f_0, denote initial values of l, m, n, f, at some instant τ_0. These equations indicate that l, m and $1 + n$ all vanish when $2f = a$. This means that a line of electric force may end at some position of an electrified light-particle. If $f(\tau)$ is initially zero and $f'(\tau)$ has always the same sign, the lines of force all start from O and end at some position of an electrified light-particle, but if $f'(\tau)$ changes sign so that $f(\tau)$ fluctuates in value some lines of force go from one electrified light particle to another, while some of the lines of force issuing from O bend outwards to avoid the region occupied by the lines of force just mentioned. It is easy to see in fact that if f fluctuates, l and m do so also, and a line of force assumes a wavy form.

The fact that lines of force seem to avoid the region occupied by an emitted electric doublet indicates that the presence of doublets emitted in different directions at different times, so as to cover nearly every direction, may give the lines of force the characteristics of separate physical entities. The unit tube of force may thus be more than a mathematical fiction.

When $f(\tau)$ is not initially zero some of the lines of force issuing from O may go to infinity instead of ending at an electrified light-particle. The total amount of electricity that has been projected from O may, in fact, not be sufficient to compensate the charge at O at the time under consideration and so it is natural to expect that some lines of force will go to infinity.

The properties of these moving lines of force seem to be perfectly analogous to those of the stationary lines of force used in electrostatics. When $f(\tau)$ fluctuates for a short time and then remains constant a pulse travels outwards and leaves an electrostatic field behind. The flow of energy in the field is naturally of some interest. A simple calculation indicates that

$$E_yH_z - E_zH_y = -\frac{xz}{cr^4(r+z)}ff' + \frac{x}{c^2r^3}\left(\frac{r-z}{r+z}\right)f'^2,$$

$$E_zH_x - E_xH_z = -\frac{yz}{cr^4(r+z)}ff' + \frac{y}{c^2r^3}\left(\frac{r-z}{r+z}\right)f'^2,$$

$$E_xH_y - E_yH_x = \frac{x^2+y^2}{cr^4(r+z)}ff' + \frac{z}{c^2r^3}\left(\frac{r-z}{r+z}\right)f'^2.$$

In addition to the radial flow of energy specified by the second terms there is a tangential flow whose direction depends upon whether $[f(\tau)]^2$ is increasing or decreasing. If $f(\tau)$ fluctuates and returns to its original value, so that the emitted light-particles form an electric doublet, energy may be supposed to flow out tangentially from one constituent of the doublet and flow in tangentially to the other so that on the whole the energy associated with the doublet remains constant.

2. *Theory of Gravitation.*—We shall now suppose that an electrified particle is continually absorbing and emitting doublets which cause its charge to fluctuate. The reason why the charge appears to fluctuate is that when an electric charge travels with velocity c there are no terms in its field of order r^{-2} at an ordinary point of space, but when the charge is stopped for a short interval of time these terms appear as soon as a point is reached by a pulse which travels outwards from the point where the charge is stopped or deflected. If now the constituents of a doublet reach a charged particle at slightly different times the charge on the particle will appear to fluctuate. Similarly if the constituents of a doublet are emitted from a charged particle at slightly different times the charge on the particle will appear to fluctuate.

We shall now suppose that a charged particle absorbs doublets one by one and emits them one by one, the emission and absorption of individual doublets being regarded as separate events separated by an interval of time. The direction in which a doublet is emitted is supposed to vary at random so that on the whole doublets are emitted equally in all directions.

The number of doublets emitted per second is supposed to be enormous being large, perhaps, in comparison with the number of electrons in the universe. We shall suppose, however, that this number is proportional to the mass of the particle.

Let us now consider two particles A and B whose charges and masses are normally e, m and e', m' respectively. The number of doublets absorbed by B per second is supposed to be proportional to m. Of these doublets a certain percentage may be supposed to have been emitted by A. Now any particular doublet may have come from any one of the charged particles in the universe or it may have come from the ether. The number which comes from the ether may, perhaps, be large in comparison with the number which come directly from charged particles, for the doublet from the ether may have really come from a charged particle by a zigzag path. It seems reasonable, however, to assume that the percentage of doublets which have come directly from A to B in unit time is proportional to the number emitted from A per unit time, *i.e.*, to the mass of A. The number of doublets which B receives from A per unit time, when A and B are stationary, may thus be supposed to be proportional to the product of the masses of A and B. It is possible that this statement may need alteration when there are material particles between A and B but for simplicity we shall disregard this case. Let us now consider the very short interval of time during which the charge on B fluctuates on account of the arrival of a doublet emitted from A. If at an instant t of this interval the charge on B is $e' - F(t)$ the charge on A at the corresponding instant $t - 1/c(AB)$ may be supposed to be $e + F(t)$. The mean value of the force exerted by A on B is thus

$$\int_{t_0}^{t_0+T} [e' - F(t)]E\, dt,$$

where

$$f(\tau) = e + F\left(\tau + \frac{r}{c}\right).$$

Now if F returns to its initial value at the end of the interval T and if the mean value of F is zero, the mean value of the force is

$$\frac{ee'}{r^2} - \frac{1}{r^2 T}\int_{t_0}^{t_0+T} [F(t)]^2 dt.$$

Taking the mean value for a long period of time such as a second, the number of terms of the second type is proportional to the product of the masses of A and B, so in addition to the ordinary electrostatic force ee'/r^2 we obtain an attraction proportional to the product of the masses of A and B and inversely proportional to the square of the distance between A and B. It is thought that this force represents the gravitational attraction of A on B.

If in a short interval of time T the charge on B fluctuates on account of the absorption of a doublet, but the charge on A does not fluctuate during the corresponding interval $t - (r/c)$ to $t + T - (r/c)$ the mean value of the force exerted by A on B is simply ee'/r^2.

Again, if the charge on A fluctuates during the interval τ to $\tau + T$ on account of the emission of a doublet, while the charge on B does not fluctuate at all during the corresponding interval $\tau + (r/c)$ to $\tau + T + (r/c)$, the mean value of the force exerted by A on B is again ee'/r^2. The emission by A of a doublet during the interval τ to $\tau + T$ and the absorption by B of a different doublet during the corresponding interval $\tau + (r/c)$ to $\tau + T + (r/c)$ is supposed to be a rare event which results only in a small correction to the ordinary force of gravitation. As shown in § 1 the emission of doublets is accompanied by the emission of pulses but when the mass of the source remains constant these pulses follow one another so rapidly that the equivalent frequency of the radiation is much higher than that of light or X-rays and so the radiation remains ordinarily undetected.

3. *Theory of Radiation.*—When the mass of a charged particle is altered by the emission or absorption of a doublet, the doublet may be supposed to carry available energy equal to the amount of energy gained or lost by the charged particle. If the charged particle loses energy the doublet carries it away and may give the energy up to some other charged particle. Since the energy of the other charged particle is less than before, the number of doublets emitted per second becomes smaller than before but the number emitted in a very short interval of time does not change immediately. The result is that sooner or later there is a gap in the emission of doublets and this gap may be supposed to occur periodically with a frequency proportional to the amount of energy lost. This makes the number of doublets emitted per second proportional to the new mass.

This state of affairs may be supposed to continue for some time until a new steady state is reached. The light emitted by the charged particle thus consists of a long train of waves.

This theory of radiation seems to be quite compatible with Bohr's theory for it depends on the assumption that the amount of energy lost by an electron in a discontinuous change is proportional to the frequency of the light which is emitted after the sudden change of energy. The energy of the electron may be supposed to remain practically constant in spite of the emission of light. See, however, the note at the end.

When a particle absorbs energy from a doublet possessing available energy the number of doublets emitted per second increases but the number emitted in a short interval of time does not increase immediately. The result is that more doublets than usual are emitted at certain intervals and we again get light of a frequency proportional to the amount of energy gained. In this case, however, no doublet carrying available energy is emitted from the charged particle.

It should be mentioned that the present theory of radiation agrees in some respects with that which has been proposed by W. H. Bragg It also agrees with the theories of Planck and Einstein.

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When an electron is adjusting itself to a new steady state of motion it may be supposed to oscillate with the frequency at which gaps occur in the shower of doublets. The lines of force of the electron at this time may end at one or more of the doublets that have been emitted and it may be that the available energy, of which we have spoken, is associated with undulations produced by the electron on a doublet's lines of force after the doublet has been emitted. The final transfer of the energy of these undulations to the doublet may be a result of the tangential flow of energy and of the change in form of the lines of force when the doublet is being absorbed

The number of doublets which hit a charged particle of mass m every second may, perhaps, be given by the formula $h\nu = mc^2$, where h is Planck's constant; at any rate it would seem, if the above ideas are correct, that ν must be a lower limit to the number of doublets. This number ν is of order 10^{20} in the case of an electron.