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CXXIX. *The Structure of Light.* By Sir J. J. THOMSON, *O.M., F.R.S.**

IN the Philosophical Magazine for October 1924 I gave a sketch of a theory of the Structure of Light; the object of this paper is to expand and illustrate that theory and to discuss some effects of the interaction between electrons and light which seem to be characteristic of the theory.

On this theory Light has a two-fold structure: one part of it consists of waves of the same character as those postulated in the Electromagnetic Theory of Light. These waves may be regarded as a periodic distribution of closed lines of electric force similar to those represented in diagrams of the lines of force around a Hertz oscillator. Each portion of a line of force travels at right angles to itself with the velocity of light. If we follow the movement of any one of these lines, we see that since it has a velocity in its own plane it must expand as it travels through space and the density of the energy associated with it get smaller and smaller. We call this assemblage of closed lines of electrie force Maxwellian waves of light, or more shortly light-waves.

The other part of the structure of light is of a corpuscular character: the corpuscles are also closed lines of electric force and so will travel through space with the velocity of light, but the lines of force in the corpuscles are so arranged that they travel through space without change of shape,

* Communicated by the Author.

size, or energy. The size of the corpuscles depends upon the frequency of the light, the circumference of the corpuscle being equal to the wave-length of the light; thus for Röntgen rays the corpuscles are of atomic dimensions, for visible light they are much larger than the atoms. For light whose wave-length is not greater than that of visible light, by far the greater part of the energy of the light is in the corpuscles. We shall call these particles light-rays. On the theory we are discussing there is a very intimate connexion between the rays and the waves, the two constituents of light. The rays, like the waves, are supposed to consist of lines of electric force moving at right angles to their direction; the rays are supposed to be systems capable of vibration, the time of vibration of the rays being the same as that of the waves. Though the waves are not supposed to give to or take from the rays any energy, yet it is the waves which determine the path of the rays as the rays have always to travel in the direction of the Poynting vector of the waves.

The rays on this theory consist of rings formed of closed lines of electric force, each line of force being a circle with its plane parallel to that of the ring and its centre on the axis of the ring. The thickness of the ring (measured at right angles to its plane) being small compared with its dimensions in that plane. Each line of force in the ring moves in accordance with the principles of electromagnetism at right angles to the plane of the ring; thus it has no radial velocity, and so the shape and size of the ring and the energy associated with it remain unaltered as it travels through spaces. The ring can vibrate about its circular form, the time of vibration being the time light takes to travel over the circumference of the ring. The ring is in tune with the waves, so that the circumference of the ring is equal to the wave-length of the light. The rings on this theory are the quanta of light, so that Röntgen rays are supposed to have much smaller quanta than those of visible light. These rings possess the most striking properties of the quantum, for—

1. Since they consist of closed lines of electric force, they must, from the electromagnetic theory, travel with the speed of light; this explains why all quanta, whatever may be their energy, travel with the same velocity.
2. Since the rings are closed, their electric fields will be confined to the inside of the ring: thus (with the exception of a case to be considered later) they will not

give up energy to atoms or electrons near which they pass. Again, the mass of the quantum is its energy divided by the square of the velocity of light, the energy is hn, where h is Planck's constant $6{\cdot}55 \times 10^{-27}$ and n the frequency of the light. It follows from this that unless the frequency of the light is as great as that of very hard γ-rays, the mass of the quantum will be exceedingly small compared with that of an electron; hence a collision between a quantum and an electron can only give rise to the transference of a very small fraction of the energy of the quantum to the electron. To transfer the bulk of the energy in the ring forming the quantum to an electron a radical change is necessary—the ring must break, its lines of force must cease to be closed and become part of a tube of force connecting an electron with a positive charge, in some such way as that described in the former paper. The energy in the ring will be thrown into the circuit connecting the electron and positive charge and will be available for dragging them apart. If the energy is sufficient to drag the electron to an infinite distance or to a place where it is in stable equilibrium, the ring will be permanently destroyed and its energy transferred to the electron. If the energy is not sufficient for this, the electron, after being dragged some way from the atom, will, under the action of the restoring force exerted upon it by the atom, retrace its path, the ring will be reformed, and there will be no absorption of energy. Thus the energy of the quantum is transferred wh lly or not at all, and for each quantum absorbed a high-speed electron is liberated. Thus, when a given number of quanta are absorbed in a gas, the same number of high-speed electrons will be produced whatever may be the nature of the gas—this result was obtained long ago by Barkla.

3. If the rings forming the quanta for different kinds of light are geometrically similar to each other, *i. e.* differ in size but not in shape, the energy in the quantum will be inversely proportional to the circumference of the ring (Phil. Mag. [6] xlviii, p. 737). As the circumference of the ring is equal to the wave-length of the light, the energy in the ring will be directly proportional to the frequency of the light.

The ring of lines of electric force forming the quantum produces a strong electric force inside the ring but no force outside, the electric force is thus discontinuous at the surface, and the surface of the quantum is thus a surface of discontinuity in the electric field. The problem of the propagation of surfaces of this kind has been discussed by Love in his paper on "Wave Motions with Discontinuities at Wave Fronts," Proceedings London Mathematical Society, series 2, vol. i, p. 37, 1903. He shows that the conditions for propagation of a surface of discontinuity are

α. That the surface travels at right angles to itself with the velocity of light.

β. The electric and magnetic forces must be tangential to the surface of discontinuity.

The electric and magnetic forces due to the ring itself, if it is a thin annulus, will satisfy these conditions. Hence, when the ring is surrounded by the field of electric and magnetic forces due to the Maxwellian waves, the electric and magnetic forces in these waves must also be tangential to the surface of the ring, *i. e.* the normal to the surface, which is the direction along which the ring moves, must be at right angles to the electric and magnetic forces in the Maxwellian waves. Since the Poynting vector of these waves is at right angles to the electric and magnetic forces, it follows that the ring must travel along the Poynting vector. Thus, on this theory, the Maxwellian waves act as guides to quanta of the same period; when such quanta are surrounded by the waves, they are constrained to travel along the Poynting vectors of the waves. The waves do not give up any energy to the quanta or take any away from them. Thus the effect they produce does not depend upon any definite amount of energy being available from the waves, and though the momenta of the quanta as, for example, in reflexion, refraction, and diffraction may change, yet all these phenomena involve the presence of matter, and this can receive the momentum lost by the quanta.

It is not necessary to suppose that one set of Maxwellian waves can guide only one quantum—we should expect it to be able to control any quantum of the right period liberated whilst the wave was passing over the place of origin of the quantum.

The polarization of light on this view arises from the polarization of the Maxwellian waves and not to any change in the quanta.

Relation between the Energy in the Quantum and that of the Maxwellian Waves.

The measurement of the amount of energy in the Maxwellian waves is beset with great experimental difficulties. The energy in these waves is spread throughout a large volume, and the amount (except in cases of resonance) which could be given to an individual electron or molecule would be exceedingly small; *e.g.*, the energy given to an electron by the waves themselves is of the order of the energy contained in a volume of the wave equal to $\lambda^2 d$, where λ is the wave-length, d the radius of an electron. Thus we could not expect these waves to produce any effect when the intensity of the light is measured by ionization, by photoelectric effects, or by chemical changes—a calorific effect is about the only one they could produce. If the energy in the Maxwellian waves is appreciable, the heat produced when light containing N quanta of frequency n are completely absorbed will be greater than Nhn. Again, if we look at the question from another point of view and consider the energy required to excite light of a definite wave-length, the evidence as to the amount of energy in the Maxwellian waves is ambiguous. Thus, when there is resonance radiation, as, for example, when sodium light is absorbed by sodium vapour, the resonance light has the same energy in the quantum as the incident light; thus, if a quantum of the incident light had to find the energy for the quantum of the resonant light, and for its Maxwellian waves as well, the energy in the latter must be negligible. There are, however, other sources from which the energy of the Maxwellian waves might be derived. The energy in the Maxwellian waves of the incident light is available, and again there are not so many quanta emitted as absorbed; some of the energy of the incident light appears as an increase in the temperature of the absorbing vapour, and in the transformation from the energy in the quanta to the energy of translation of the molecules of the vapour some energy may be spent in exciting Maxwellian waves.

As the density of the energy in the Maxwellian waves diminishes as the waves recede from their source, while the energy in the quantum is invariable, there can be no fixed relation between the energy in the quantum and the density of the energy in the Maxwellian waves. Again, one set of these waves may, as we remarked before, guide more than one quantum; where this is the case, the optical effects will be the same as if these quanta started from the origin of the

Maxwellian waves and not from the places where the quanta were liberated.

There is, however, one case at any rate where the energy of the Maxwellian waves must come from the energy of the quantum: this is when there is the change discovered by Compton in the frequency of Röntgen rays when their quanta give up a little of their energy to electrons; here, since there is a change in the wave-length, there must be a change in the Maxwellian waves and new ones will have to be produced by the quantum. Thus the change in the energy of the quantum would be equal to the energy communicated to the electron plus the energy required for the Maxwellian waves. Thus from measurements of the change of frequency corresponding to the deflexion of the rays through a known angle we might get data for determining the energy in the Maxwellian waves.

The relative amounts of energy in the quanta and that spent in producing Maxwellian waves would from our point of view depend to a large extent on the character of the waves; there must be certain features attending the production of the light to cause quanta to be emitted at all. There does not from this point of view seem any reason for supposing them to exist in long waves such as those used for wireless telegraphy, which have frequencies very small compared with any known frequencies associated with atoms or molecules, and which might conceivably be generated by moving a magnet to and fro. In waves of this kind we should expect all the energy to be in the Maxwellian waves and the quanta to be absent, while in waves at the opposite end of the scale, such as Röntgen rays, we should expect the greater part of the energy to be in the quanta.

Method of Production of Quanta.

Let us take as a typical example the production of radiation by the falling in of an electron E towards a positive charge P. We suppose that the electrostatic potential energy when the charges are at P and E is located in the space round PE and is bound up with the lines of force stretching between P and E. If the electron acquires kinetic energy by approaching P, energy that was previously distributed between P and E must have moved close up to E and be concentrated round it in a volume comparable with the volume of the electron. This transport of energy requires time, and if the electron were to move very rapidly from E to another point E′ there might not be time for the energy in the outlying field to move to E′. The result would be that the region would be surcharged with energy, and the excess of energy would have to escape; one way of escape would be for it to be radiated as a quantum of light, or, if there were any free electrons in the region, some of the energy might run into them and increase their kinetic energy. If this effect took place simultaneously with the emission of radiation, the frequency of the quanta emitted would diminish as the energy given to the electrons increased. The various quanta would produce a continuous spectrum with a bright edge on the high-frequency side, at a frequency corresponding to the conversion of all the excess energy into radiation.

We may visualize the production of the quantum by supposing that owing to the rapid motion of the electron the tube of force connecting it to the positive charge is thrown into a loop, and that the loop, forming as it does a closed line of force, breaks away and travels out as the quantum.

As the circumference of the quantum is equal to the wave-length of the light, and as for Röntgen rays the wave-length is of atomic dimension while for visible light the wave-length is much greater, it follows that for Röntgen rays the energy of the quantum is concentrated into a volume comparable with that of the atom, while for visible light the energy in a volume of the quantum equal to that of the atom is but a small fraction of the whole energy. This difference in the sizes of the quanta accounts for the very conspicuous difference between the absorption bands for visible light and for Röntgen rays. The absorption bands for visible light are often exceedingly sharp; the absorption seems to be practically entirely confined to the light, with frequencies between very narrow limits. For Röntgen rays, however, though the absorption begins quite abruptly at a particular frequency, light of a higher frequency is also absorbed to an appreciable extent, so that there is a wide absorption range with a well-defined edge on the low-frequency side. This is what we should expect from the view we have taken of absorption, which requires that if the energy given to the electron exceeds that corresponding to the ionizing potential absorption may take place. In this case the absorption is not dependent upon resonance; in the case of ultra-violet light, however, it is. Let us consider what resonance does. Dr. Horace Lamb and the late Lord Rayleigh have shown that when resonance is sharp the amount of energy streaming into the resonator is not measured by the cross-section of the resonator, but that all the energy from an area of the wave-front equal to λ^2/π, where λ is the wave-length, is focussed on the resonator. Poynting vectors starting from anywhere in this area all pass through the resonator.

Thus when, as in the case of Röntgen rays, the dimensions of the resonator are comparable with the wave-length of the radiation resonance will not materially increase the amount of energy it receives. The conditions are entirely different with visible light, for here the wave-length is very much greater than the dimensions of the resonator. So that resonance produces an enormous concentration of the energy reaching the atom. The energy in the quantum whose area is proportional to λ^2 is by resonance concentrated into the bulk of the atom.

Effects due to Collisions between the Quanta and Electrons.

On the theory we are discussing, there are strong electric forces inside the quantum. Thus the passage of the quantum over an electron at rest may be expected to result in the increase in the energy of the electron at the expense of that in the quantum; the loss of energy by the quantum would reduce the frequency of the light.

Let us suppose the quantum to be a ring whose thickness measured at right angles to its plane is a, and that the difference between the external and internal radii of the ring is b. If the quantum overtakes a free electron at rest, the electron will be inside the quantum for the time a/c, where c is the velocity of light.

If F is the electric force in the quantum, v the velocity communicated by it to the electron, e the charge on the electron, and m its mass,

$$mv = \mathrm{F}e\frac{a}{c}. \quad \ldots \ldots \quad (1)$$

The product of the electric force into the cross-section of the ring is constant; hence

$$\mathrm{F}ab = 4\pi se, \quad \ldots \ldots \quad (2)$$

where s is a fraction and e the charge on an electron.

Both a and b are proportional to λ; let $a=k\lambda$, $b=k_1\lambda$, λ being the wave-length,

$$\mathrm{F}kk_1\lambda^2 = 4\pi se, \quad \ldots \ldots \quad (3)$$

E, the energy in the ring, is equal to $F^2/4\pi$ times the volume of the ring; hence

$$E = \frac{F^2}{4\pi} kk_1\lambda^3 = \frac{4\pi s^2 e^2}{kk_1\lambda} = \frac{4\pi s^2 e^2 n}{kk_1 c},$$

where n is the frequency of the light. Thus

$$h = \frac{4\pi s^2 e^2}{kk_1 c},$$

h being Planck's constant.

Substituting the value of F given by equation (2) in (1), we have

$$mv = \frac{4\pi s e^2}{k_1\lambda c}$$

$$= \left(\frac{4\pi e^2 h}{c}\right)^{\frac{1}{2}} \left(\frac{k}{k_1}\right)^{\frac{1}{2}} \frac{1}{\lambda}. \quad . \quad . \quad . \quad . \quad (4)$$

Thus the momentum communicated to the quantum is proportional to the frequency of the quantum. Substituting the numerical values for e, h, c, m, we find

$$v = \cdot 86 \left(\frac{k}{k_1}\right)^{\frac{1}{2}} \frac{1}{\lambda}.$$

k/k_1 will be less than unity, so that the velocity communicated by a quantum of light of wave-length 10^{-8} cm. will be considerably less than 10^8 cm./sec.; the energy corresponding to this velocity will be less than that represented by a volt, and will only be a small fraction of the energy of the quantum.

The momentum of the quantum is hn/c, while that of the electron is mv; the ratio of the two is equal to

$$\frac{mv \times c}{hn} = \left(\frac{4\pi e^2}{hc}\right)^{\frac{1}{2}} \left(\frac{k}{k_1}\right)^{\frac{1}{2}}$$

$$= 1\cdot 2 \left(\frac{k}{k_1}\right)^{\frac{1}{2}} \times 10^{-1}, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and is constant.

The energy acquired by the electron and lost by the quantum is $mv^2/2$: this by (4) is equal to

$$\frac{4\pi e^2 h}{2mc} \frac{k}{k_1} \frac{1}{\lambda^2}.$$

If δn is the change in the frequency of the quantum, $h\delta n$ is equal to the change in energy. Hence

$$\delta n = -\frac{2\pi e^2}{mc} \frac{k}{k_1} \frac{1}{\lambda^2}.$$

But

$$\delta n = -\frac{c\delta\lambda}{\lambda^2}.$$

Hence

$$\delta\lambda = \frac{2\pi e^2}{mc^2}\frac{k}{k_1}$$

$$= 1\cdot 7\times 10^{-13}\frac{k}{k_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is the increase in wave-length a quantum would experience if it passed over a free electron at rest. The area of the surface of the quantum is $k_1\lambda^2$. If N is the number of free electrons per c.c. in the region through which the quantum is passing, the number of electrons a quantum will pass over in a distance δn is $k_1\lambda^2 N\delta x$. Hence from the preceding expression we get

$$\frac{d\lambda}{dx} = \frac{2\pi e^2}{mc^2}k\lambda^2 N$$

$$= 1\cdot 7\times 10^{-13}k\lambda^2 N \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\frac{dn}{dx} = -\frac{2\pi e^2}{mc}kN$$

$$= 5\cdot 1\times 10^{-3}kN. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Thus on this view the wave-length of the radiation would increase slowly as the radiation passed through a medium containing free electrons. Compton has shown that when Röntgen rays are scattered the wave-length is increased, but I am not aware of any experiments which show an increase in wave-length due to the passage of the radiation through an absorbing substance.

Scattering of Quanta by Electrons at rest.

We have seen (equation (4)) that the momentum given to an electron at rest by a quantum passing over it is proportional to the momentum of the quantum and is at right angles to this momentum. Thus the quantum will be deflected by the collision through an angle α, where by equation (5)

$$\alpha = \left(\frac{4\pi e^2}{hc}\right)^{\frac{1}{2}}\left(\frac{k}{k_1}\right)^{\frac{1}{2}} = 1\cdot 2\left(\frac{k}{k_1}\right)^{\frac{1}{2}}\times 10^{-1}.$$

This angle is constant if, as we have supposed, F the electric force in the quantum is constant. If this force depends on the distance from the centre of the quantum, then the angle through which the quantum is deflected by collision with an electron will depend upon the part of the quantum ring struck by the electron.

Multiple Collisions.

The direction of the deflected particle may be any generator of a cone whose axis is the undeflected direction and whose semi-vertical angle is α. Hence, by the theory of probability, after a quantum has made a large number p of collisions, the probability that it has been deflected through an angle between θ and $\theta+\delta\theta$ is

$$\frac{2}{p\alpha^2}\epsilon^{-\frac{\theta^2}{p\alpha^2}}\theta\,.\,d\theta.$$

If ϕ^2 is the mean value of θ^2,

$$\phi^2 = p\alpha^2.$$

If the quantum has made p collisions, the change in wave-length is by (6) given by the equation

$$\begin{aligned}\delta\lambda &= \frac{2\pi e^2}{mc^2}\frac{k}{k_1}p \\ &= \tfrac{1}{2}p\alpha^2\frac{h}{mc} = \tfrac{1}{2}\phi^2\frac{h}{mc} = 1\cdot 2\times 10^{-10}\phi^2.\end{aligned}$$

The scattering we are considering arises from the superposition of a large number of minute deflexions. This must be distinguished from the normal scattering to which attention has usually been confined; the intensity of this normal scattering in the simplest cases varies as $1+\cos^2\theta$, so that large angles of scattering are not very much less probable than small ones. This normal scattering is due, not as in our case to the addition of a very large number of small deflexions, but to the production under certain circumstances of one—it may be a large—deflexion of the quantum. The origin of these large deflexions, produced by atoms or molecules in the path of the primary beam, is due to the secondary electrical waves started by the electrons in the atom under the action of the primary, producing in the neighbourhood of the atom electric waves, where the Poynting vector makes large angles with its direction in the primary beam. Thus there is a finite chance that a quantum following the Poynting vector may be deflected through a large angle. If it is deflected its momentum is changed and the change in momentum must be communicated to the atom, or to the electrons in it. If the momentum is communicated to an electron, the electron will absorb a finite amount of energy and the quantum will lose energy and correspond to a longer wave-length. If the momentum is communicated to the positive core of the atom rather than to an electron, very little energy will be given to the core in consequence of its large mass, and there will not be an appreciable change in the frequency of the quantum. Thus the rays which are scattered by "single" scattering may or may not show a change of frequency. In Compton's experiments the scattering was the normal scattering, and he found part of the scattered light showed a change in frequency and part did not.

The amount of multiple scattering in a direction making an angle θ with the direction of the incident light due to the collisions between the quanta and the electrons and showing a change of wave-length is proportional to $\epsilon^{-\frac{\theta^2}{pa^2}}\theta$, while the normal scattering to $1+\cos^2\theta$. Thus, unless $\sqrt{pa}$ is a large angle, the intensity of the scattered light with smaller wave-length will fall away very rapidly as θ increases and very little of this light will be scattered backwards, while the normal scattering is as great in the rear as in front; hence the best place to look for the light of altered wave-length would be in a direction making a small angle with the incident light.

We have in the preceding calculations supposed that the electron could be treated as free; if the electron forms a part of an atom, the results will only apply when the time taken for the quantum to pass over the electron is short compared with the time of vibration of the electron.

Since all the dimensions of the quantum are proportional to the wave-length, the time taken for a quantum to pass over an electron will be inversely proportional to the frequency; hence electrons which behave as if they were free when acted on by quanta of high frequency, such as those of Röntgen rays, need not do so for quanta of visible light, which have much smaller frequency.

The effect produced by collisions on the frequency of a quantum depends essentially on the size of the quantum; it will vanish when the quantum is large compared with the molecule against which it strikes. For if, as in the case of visible or ultra-violet light, the quantum is very large compared with the molecule, it will envelope the whole of the molecule, the positive core as well as the electrons. Thus as the molecule is an electrically neutral system it will not gain any momentum from the electrical forces in the quantum; thus no energy will be given to the molecule and none taken from the quantum, so that no change of frequency will be produced by the collision. This is in accordance with experience, for though changes in frequency have been observed with Röntgen rays whose wave-length is less than 10^{-8} cm., and whose quanta are therefore smaller than the atoms or molecules against which they strike, no such changes have been observed with ultra-violet or visible light when the wave-lengths are hundreds or thousands of times greater than the diameter of the atoms, and the quanta therefore vastly greater than the atoms.

Though the collision of a quantum with a free positive particle would result in the particle gaining as much momentum as an electron, and therefore deflecting the quantum through the same angle, the energy given to the particle and therefore the change of frequency would only be m/M of that for the electron—here m is the mass of an electron, and M that of a positive particle.

The transference of energy between electrons and quanta in the way we have been considering might lead to very important consequences. We can see, for example, that if the free electrons instead of being at rest were in rapid motion, a collision between a quantum and an electron might increase instead of diminishing the frequency of the quantum. Thus, suppose that the electron had a component of its velocity parallel to the plane of the quantum in such a direction that when the quantum passed over it the electric force in the quantum tended to stop the electron.

Then if the initial velocity of the electron is greater than half that which the quantum would communicate to an electron at rest, the electron will lose and the quantum will gain energy, so that the frequency of the quantum will be increased. The greater the velocity of the electron the greater the change in frequency. The passage of quanta of Röntgen rays through a dense swarm of high-speed electrons would, even if the rays were homogeneous to begin with, cause their frequencies at the end to spread over a finite range and thus seriously modify the character of the radiation. One way of producing high-speed free electrons is by means of hard Röntgen radiation, so that a very intense beam of hard Röntgen radiation might affect the average velocity of the free electrons in the medium through which it was passing, and thereby the character of a beam of soft radiation passing through the medium at the same time.

The frequency of the quantum is increased when the electron struck by the quantum is moving in the direction in which the electric force in the quantum tends to stop the electron; if the electron were moving in the opposite direction the frequency of the quantum would be diminished by the collision. If V is the tangential velocity of the electron in the first case, the increase in frequency is

$$\frac{mv}{h}\left(\mathrm{V}-\frac{v}{2}\right),$$

where v is given by equation (4).

In the second case the diminution in the frequency is

$$mv\left(\mathrm{V}+\frac{v}{2}\right),$$

where V is the tangential velocity.

Since the lines of electric force in the quantum are circular, when a quantum passes over an electron moving tangentially the electric force in one half of the quantum ring will tend to stop the electron and therefore increase the frequency of the quantum, while in the other half it will tend to accelerate the electron and diminish the frequency. As the chance of the electron being at the collision in one half of the ring is the same as its being in the other, the quantum in its journey will sometimes have its frequency increased by collisions and sometimes diminished. Thus, if a number of quanta started with the same frequency, they would, after collision with many electrons, possess frequencies distributed according to the law of probability, the chance of a departure between x and $x+\delta x$ from the original frequency being proportional to $\epsilon^{-\frac{x^2}{n\alpha^2}}dx$, where n is the number of collisions and α the change of frequency due to one collision. Thus the spectrum, which before the collisions was a sharp line, will be drawn out into a band, the breadth of the band being proportional to $\sqrt{n}$. Since the increase of frequency due to a tangential velocity V is proportional to $\mathrm{V}-v$, while the diminution is proportional to $\mathrm{V}+v$, the diminution is greater than the increase; so that besides drawing the line out into a band, the collisions will produce a continuous diminution in the frequency proportional to n.

In the preceding calculations we have supposed, in accordance with the usual classical theory, that the communication of momentum to an electron in an electric field is continuous and that infinitesimally small increments of momentum can occur. Results of the same general character would ensue if the action of the electric field consisted in the discontinuous additions of finite increments of momentum. For suppose that δt is the time taken by the quantum to pass over the electron, and that $\omega\delta t$ is the chance that the electron will receive an increment of momentum in this time, let k be the increment it receives, if it receives any, then $k\omega\,\delta t$ will be the probable increment of momentum in time δt. Thus the constant force F, which on the average will produce the same effect as the discontinuous addition of momentum, is equal to $k\omega$.

Let us consider what the difference between the continuous and discontinuous actions would be; suppose that k, the increment of momentum on the discontinuous view, is greater than $F\,\delta t$. Thus, when a quantum did lose momentum by collision with an electron it would on the discontinuous view lose more than on the continuous one and would be deflected through a greater angle; on the other hand, it might pass over an electron without suffering any deflexion. If the angle of deflexion for an effective collision on the discontinuous view were twice that on the continuous one, the probability that the quantum suffered any deflexion at all would be only 1/2 Thus when a large number of deflexions were combined, in one case the result would be that of combining N/2 deflexions each equal to 2ψ, and in the other N deflexions each equal to ψ.

The characteristic quantity occurring in the distribution law, $\frac{2}{p\alpha^2}e^{-\frac{\theta^2}{p\alpha^2}}.\theta.d\theta$, is $p\alpha^2$, where p is the number of collisions and α the deflexion at each collision; when p is halved and α doubled, $p\alpha^2$ is doubled; this means that the scattering is more evenly distributed and is not so concentrated along the smaller angles.

If a quantum gives no momentum at one collision but a double quantity at the next, since double momentum corresponds to four-fold energy, the quantum after passing over the same number of electrons would lose twice as much energy and suffer twice the change in frequency as one losing one unit of momentum at each collision.

If the magnitude of the momentum imparted to the electron were such that the moment of momentum is quantized into multiples of h, then

$$kr = nh, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where r is a length proportional to λ, the wave-length of the light.

Since the momentum of the quantum is h/λ, as λ is greater than r, k, the transverse momentum communicated to the electron, will be greater than h/λ, the forward momentum of the quantum; so that the quantum will, if deflected at all, be deflected through a large angle. From equation (9) k varies inversely as λ. Since $k\omega$ is equal to F and F varies as $1/\lambda^2$, ω will vary as $1/\lambda$; the chance that an electron struck by a quantum receives momentum is $\omega a/c$, now a varies as λ, and since ω varies as $1/\lambda$; the chance that an electron when struck by a quantum should receive some momentum is independent of the wave-length of the light corresponding to the quantum.

Summary.

This paper develops further the theory of light sketched by the author in a paper published in the Philosophical Magazine, Oct. 1924. The effects which on this theory would follow from collisions between quanta and electrons are discussed; it is shown that the quanta would be deflected and lose frequency by colliding with electrons. This deflexion would produce a scattering of the light made up of a great number of small deflexions combined according to the Theory of Probability; this scattering is in addition to that which is proportional to $(1+\cos^2\theta)$, which is accounted for by the wave theory. It is shown, too, that when the electrons are in rapid motion the theory indicates that collisions between the quanta and the electrons will produce an *increase* in the frequency of some of the quanta.