

THE FORM OF THE ETHER

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1. THE MEANING OF THE WORD ETHER. In the past many attempts have been made to define the properties of an ether—a supposed active agent in all physical phenomena which exists even in interstellar space and is responsible for the transmission of light from the stars. In the introduction of this concept of an ether the chief aim was to establish the unity of science, an aim which was shown to be worth while by the subsequent discovery of relations between electricity, magnetism and light. This success has led men to expect more and more from the ether and occasionally, when men have been disappointed in their expectations, the idea of an ether has lost its popularity and has even been ridiculed. Thus today many scientists assert that there is no ether, but presumably they do not intend to abandon hope of unity in science. If the ether were regarded simply as a means of unification, the contention of these scientists could be expressed by saying that the ether has been supposed to be endowed with properties which it cannot actually possess. This simple view of the ether is not, however, satisfying and most men associate with it a number of hypotheses, the choice of which is largely a matter of personal preference. Sometimes these hypotheses turn out to be conflicting, as we have just remarked.

If a scientist denies altogether the existence of an ether he should either state precisely what he means by such an assertion or else propose a substitute for the ether. The truth may be that men are growing tired of the word ether and that unless a new word is happily coined we shall simply speak in the future of radiation. It is quite common now for scientists to maintain that every body has its own ether and in simple cases it appears to be radiated directly from the body.¹ The theory of Faraday tubes leads one naturally to such a point of view, especially when

¹In 1600 Dr. William Gilbert thought that electric force was produced by a kind of effluvium or atmosphere which was emitted by the electrified body during the process of electrification by friction and was supposed to be carried about by the body just as the Earth's atmosphere is retained by the Earth. If we interpret the word atmosphere to mean lines of force we have here an idea that differs very little from our modern notions.

we consider the lines of force of an electric pole as physical entities. This is clearly emphasized in the work of Sir J. J. Thomson² and Leigh Page.³ With some writers an ether has come to mean very nearly the same thing as an electromagnetic field. Thus Leonard⁴ speaks of the Faraday tubes or ether of a body as analogous to vortex filaments in a continuous fluid or "uraether." This is practically the ether sponge theory of Lord Kelvin and G. F. Fitz Gerald.

The idea of vortices in an ether has been a familiar one since the time of Descartes and has been favored by many of the British physicists. The point to be noticed is that there is a continual tendency to change the significance of the vortices. While Descartes regarded a vortex as being produced by matter, Lord Kelvin regarded a vortex as representing matter. Some modern writers, on the other hand, regard a vortex as electricity and we finally have the view just mentioned that an ether is a vortex. It is quite evident that at the present time the word ether is used in several different senses and a review of some of the meanings that have been attached to it may be useful.

I suppose that some people still think of the ether as a convenient medium which enables one to give a definite meaning to absolute motion. It is this idea of the ether that is so firmly rejected by the supporters of the modern theory of relativity, attack being specially directed against the supplementary hypothesis that an ether, in the sense just mentioned, is unique.

The most popular idea of the ether is that which has developed from the attempts of Hooke and Huygens to develop a wave theory of light.⁵ Hooke's idea of transverse vibrations produced in the ether by the small vibration of minute particles in a luminous body has been made more definite by the electromagnetic theory of Maxwell, Hertz, Rayleigh and Lorentz, which now includes the modern theory of electrons.

Although it is generally agreed that light is produced by the vibration of electrons, it is not understood why in certain types of vibration apparently no light is emitted. Many other difficulties relating to the transformation of energy arise in this theory,

²Recent Researches (1893).

³*American J. of Sci.*, **38**, 169, 1914.

⁴*Jahrb d. Radioakt. u. Elektronik*, Bd. 17 (1921), Heft. 4.

especially when an attempt is made to explain the laws of the quantum theory of radiation. In such a situation it may be wise to examine carefully the various hypotheses that have been made regarding the properties of the ether and, in particular, to pay some attention to the ideas of the original inventor. We shall mention at this point just one hypothesis that may not be quite justifiable. While an electromagnetic field acts on an electron, it is generally assumed that the electron does not act directly on the field. This does not seem quite reasonable because it is more natural to assume that an electron creates its field at the expense of previously existing fields. In general, however, the ether particles which give rise to a particular field may be supposed to be infinitely small in number compared with the ether particles which are associated with other superposed fields that balance one another. The electron's field, then, may be regarded as being produced almost entirely at the expense of the latter fields and there is usually no noticeable effect. In a very strong electric field things may be different.⁶

The idea of a fiery ether was introduced by Heraclitus, a poet and philosopher of Ephesus, who lived in the fifth century before Christ. The ideas of fire and burning were used to express vividly and poetically the opinion that a continual process of change is the real secret of the world. According to Heraclitus, the ether is the substance of all things and the Universal Intelligence. There is no rest in Nature, only perpetual change; the generation of one thing means the destruction of another.⁷ "All things move and nothing stays," says Heraclitus, "all things flow."⁸

According to Plato, the word ether itself is meant to convey the idea of perpetual motion, although it was probably derived from the Greek word *αἴθω*, I burn.⁹

In some respects the teaching of Heraclitus is similar to that of his great predecessor Thales, the father of science. Thales,

⁶For a history of the subject we may refer to E. T. Whittaker's "History of the Theories of Aether and Electricity," Longmans Green & Co. (1910). References will be given here chiefly to recent work.

⁷This point is discussed in *Bull. Amer. Math. Soc.* (Feb., 1921).

⁸G. P. Macdonell, "Article on Heraclitus, The New Calendar of Great Men" (1920), p. 94.

⁹Gilbert Murray, "A History of Ancient Greek Literature" (1915), p. 155.

¹⁰J. Larmor, "Article on Aether," *Encyclopædia Britannica*, 11th Ed. Larmor quotes a passage from Plato's "Cratylus." Cratylus was the chief exponent of the teaching of Heraclitus.

however, thought that water was the active agent in all changes. The chief contribution of Heraclitus was the choice of language and the invention of the ether. The symbols used by Heraclitus are commonly employed in every-day speech; thus we speak of a spark of intelligence, "firing a person's imagination," "a spark of the divine fire."

We have dealt at some length with the ideas of Heraclitus because they suggest a theory of the ether which has not received as much attention as it deserves. This theory may be called a chemical theory of the ether because electromagnetic fields are supposed to be produced by a process analogous to chemical action.

2. IS THE ETHER A SUBSTANCE? It may be thought that it is unnecessary to ask this question, since the ether is defined in many encyclopædias and dictionaries as a material substance, the word material being used in the sense given to it by Descartes, extension being the sole essential property of matter and matter a necessary condition of extension. Nowadays matter has a more definite meaning and it is sometimes said that a substance can be material without consisting of matter.¹⁰ The ether, then, is generally regarded as a substance, but there is the possibility that the unifying agent in physical science may be a type of motion that can occur in more than one type of substance. The theory of Faraday tubes indicates that such a view is not unreasonable. Again, certain philosophers believe that there is an identity between ether and electricity,¹¹ but since we generally have to distinguish between positive and negative electricity the possibility of two different types of fundamental substance must be considered. Unfortunately, the nature of electricity is not understood. We may be wrong in regarding electricity as a substance; indeed, a common point of view is that we should try to imagine a single type of fundamental substance in terms of which both positive and negative electricity can be explained.

At first the ether was regarded as analogous to a fluid, but unfortunately Hooke's idea of transverse vibrations was forgotten and scientists imagined waves of light to be analogous to

¹⁰Sir Oliver Lodge, Presidential Address, Brit. Assoc., Birmingham (1913).

¹¹Cf. C. Flammarion, *Dictionnaire encyclopédique illustré*.
Maxwell's view was nearly equivalent to the above.

waves of sound, with the result that the wave theory of light lost its popularity and was superseded for a time by the corpuscular theory of Newton, which seemed then to account for more of the facts.

When the idea of transverse vibrations was revived by Fresnel and Young it was thought that the ether was more like an elastic solid and the elastic solid theory of light was developed by Fresnel, Cauchy, Green, F. E. Neumann, Stokes, Lord Kelvin, Boussinesq and many other writers. The theory of reflection proved a stumbling block and different types of elastic solid were invented to overcome the difficulties. It was realized eventually that the ether could not be an elastic solid of the ordinary type, and the ether of McCullagh, which leads to equations analogous to Maxwell's electromagnetic equations, has been described as a rotationally elastic medium, *i. e.*, a medium every particle of which resists absolute rotation. In Sir Joseph Larmor's theory of the ether the lines of electric force radiating from an electric pole are regarded as lines of torsional strain in the ether.

The theories of Larmor and Boussinesq are, perhaps, the most satisfactory of the quasi-elastic theories, as they are able to account for many physical phenomena. It is the custom nowadays, however, to try to account for elasticity by means of electric and magnetic forces, and so scientists do not generally speak of an elastic ether. The idea of strain in the ether is, however, a very general one¹² and is retained in the most recent works on electricity.¹³ Now that a comparison between the ether and an ordinary elastic solid has proved unsatisfactory, there seems to be a temptation to go back to the fluid analogy. The Kelvin-Bjerknes theory of pulsating spheres in a fluid, which has been developed further by Hicks, Leahy and Korn, is still regarded by the last named as a possible basis of electromagnetism. Korn has, in fact, derived from his form of the theory a set of equations which differ only slightly from the equations of Maxwell's electromagnetic theory.¹⁴

¹²A theory in which electricity is supposed to consist of a strain in the ether has been developed by C. V. Burton, *Phil. Mag.* 1 (1907).

¹³See, for instance, Poynting and Thomson's *Electricity and Magnetism*. The idea of strain is also used by Leigh Page in his theory.

¹⁴*Phys. Zeitschr.* Bd. 19 (1918).

The opinion that the ether is a gas is expressed in Le Sage's theory of gravitation, a theory which has been discussed by Maxwell, Darwin, Drude and others.¹⁵ Maxwell objects to the theory on the ground that a body may be expected to continually gain heat on account of the impacts, but Drude sees in this a possible explanation of the permanence of the high temperature of the Sun and stars.¹⁶ A curious relation between intrinsic temperature and intensity of gravitation has been noticed more than once,¹⁷ but no good theoretical explanation of it has been given.

The idea that the ether is a gas has been revived, too, by Mendeleef¹⁸ and Zehnder,¹⁹ the former having endeavored to develop a chemical theory of the ether.

Nearly all scientists are agreed that if the ether is not absolutely continuous it must be exceedingly fine grained.²⁰ The idea of a continuous ether is favored by many and is strongly supported by Sir Oliver Lodge.²¹ If, however, we wish to compare the ether with a solid or fluid we must, at least temporarily, adopt the other view, and then the question which of the two is most suitable may require for its answer a knowledge of the length of the mean free path of an ether particle.

If we go back to the ideas of Heraclitus and adopt a chemical theory of the ether it may be useful to compare the ether with a mixture of gases. In language similar to that used by Heraclitus such a theory can be described as follows:

Electricity is the flame which burns the ether. An idea somewhat similar to this was used by Riemann as a basis of a theory of gravitation, matter being regarded as an ether sink which sucks up the ether. Riemann's theory has recently been developed by S. B. McLaren²² with the additional hypothesis that the ether is a polarized fluid.²³ The fundamental idea that matter may not always consist of the same ether particles is illustrated

¹⁵A somewhat similar theory, in which radiant energy and light pressure act like the corpuscles of Le Sage, has been developed by C. F. Brush. *Science*, **33**, 381 (1911).

¹⁶*Ann. d. Phys.*, Bd. **62** (1897).

¹⁷Leray, *Comptes Rendus*, **69**, 6:15; H. Fricke, *Phys. Zeitschr.*, **22**, 636 (1921).

¹⁸"An Attempt Towards a Chemical Conception of the Ether." Trans. by G. Kamensky. Longmans Green & Co. (1904).

¹⁹"Der ewige Kreislauf des Weltalls" (1914), p. 314.

²⁰The fineness of the structure is discussed by C. V. Burton [*Phil. Mag.*, **18**, 872, 1909] in connection with an alleged discovery of dispersion in the ether by Nordmann and Tikhoff.

²¹Loc. cit.

by FitzGerald²⁴ in an interesting manner when he says that a particle of matter may move in much the same way as a drop of water moves through a block of ice. C. V. Burton expresses the same idea as follows:

“A given portion of matter consists, not of any individual portion of æthereal or other substance, but of modifications in the structure or energy or other qualities of the æther, and when matter moves it is merely these modifications of structure or of energy or of other qualities which are transferred from one portion of the æther to another.”

The idea which will be developed here is that a change is produced in an ether particle when it encounters a particle of electricity. This change may be simply one of orientation, but it seems better, in accordance with the ideas which have just been mentioned, to assume that it is more radical in character. Furthermore, if we adopt the view that everything changes, we must consider the possibility that a collision between two ether particles may produce a radical change in both and that some change may occur in an ether particle in the interval between two collisions.

3. AN ELECTRICAL THEORY OF THE ETHER. To develop a theory on the lines just indicated we shall make the following assumptions:

(1) An ether particle is normally an electric doublet whose constituents carry the elementary electric charge ϵ and travel along straight lines with constant velocity c . According to Maxwell's equations, an electric charge does not produce a field while it is traveling with velocity c along a rectilinear path; any field that is associated with the charge is produced either when the charge is created or when its direction of motion is changed by a collision.

²²*Phil. Mag.*, **26**, 636, 1913.

²³This is practically a revival of Maxwell's view. The idea of polarization in the ether occurs implicitly also in Einstein's theory of gravitation and is discussed explicitly by W. Alexandrow, *Phys. Zeitschr.*, **22**, 593, 1921. The equivalent idea of a specific inductive capacity of the ether is discussed by Sir Oliver Lodge, *Phil. Mag.*, **37**, 465, 1919, and used by E. Wiechert, *Ann. d. Phys.*, **63**, 1920, and by H. A. Wilson, *Phys. Review*, **17**, 54, 1921, in their theories of gravitation, in which gravitation is attributed to local variations in the specific inductive capacity of the ether. A somewhat similar theory in which local variations in density and specific inductive capacity of the ether are supposed to be the cause of gravitation was proposed by R. A. Fessenden, *Science*, **12**, 325, 740, 1900. This was partly a revival of an old idea due to Newton.

²⁴See a note at the end of a paper by C. V. Burton on strain figures in the ether, *Phil. Mag.*, **1**, 191, 1892.

There is a possibility that the velocity c may not be quite the same for all doublets, that it may vary between two limits c_1 and c_2 . This will produce a complication in electromagnetic theory, but the possibility must be given some consideration on account of Einstein's theory of gravitation. The moment of a doublet which travels undisturbed has an approximately constant longitudinal component in the mean direction of motion of the two electric charges $-\epsilon$ and $+\epsilon$. The transverse component increases at a constant rate.

The distance between the charges is always small but is generally much greater than the linear dimensions of the geometrical entities carrying the electricity. These entities are supposed to be smaller even than the positive nucleus of an atom.

(2) An elementary electrostatic field is produced by the collisions and transformations resulting from the rapid oscillation within a small closed region of a free electric charge of the same nature as one constituent of an ether doublet. The volume of this region is probably of the order of magnitude of that of an electron when the charge is negative, and of the order of magnitude of that of a positive nucleus when the charge is positive.

As a result of the collisions ether-doublets are supposed to be broken up and new ones formed, the free electric charge taking the place of one of the constituents of a dissolved doublet and leaving the other one free to act as the free charge.

On an average doublets are emitted equally in all directions, but the direction of motion of a doublet is not exactly along the radius from the center of the field, consequently doublets are continually colliding with one another and the field loses doublets on this account, the transformation of the doublets giving rise to a secondary field. If it were not for these collisions there would be a continual flow of electric doublets to infinity.

(3) If the electric force and electric displacement are proportional to the total electric moment per unit volume, the inverse square law can be easily accounted for, when there are no collisions, provided we assume that the transverse components either cancel out or are non-existent. Let us consider in fact the ether doublets within a small region bounded by a cone of solid angle $d\omega$ and two spheres of radii ct and $c(t + dt)$ respectively,

the vertex of the cone and the centers of the two spheres being at O , the center of the field. As t varies this region will move away from O in a radial direction and its volume will increase so as to be proportional to t^2 or r^2 . The total amount of the doublets will, however, remain constant. This will still be true when there are collisions if, in a collision between two doublets AB and CD , two charges B and C of opposite signs become practically coincident and leave the field; for the moment of the residual doublet AD is then equal to the sum of the moments of the two original doublets AB and CD .

Let n denote the number of ether-doublets within the region at time t , then, on account of the loss by collisions, n decreases in accordance with an equation of type

$$\frac{dn}{dt} = -an^2$$

where a is a constant. The rate of decrease is proportional to n^2 because there are n positive and n negative charges within the region.

Integrating this equation and assuming that n is infinite when $t = 0$, we have

$$n = \frac{1}{at} = \frac{c}{ar}$$

where $r = ct$. The value $r = 0$ is really excluded on account of the finiteness of size of the region within which the free charge oscillates.

The number of doublets per unit volume of the region is now Kr^{-2} , where K is a constant which is assumed to be the same for all directions. The rate at which doublets are transformed per unit volume is clearly proportional to n^2/r^2 or r^{-4} and so is proportional to the density of the energy in the field. Since all the doublets which enter the field at the center eventually leave the field, it appears that the number of doublets transformed by the free charge per unit time is proportional to the total energy of the field, *i. e.*, to the mass associated with the free electric charge.

The result that doublets leave the field at a rate proportional to the density of energy is of some interest in connection with Bohr's theory of the atom in which an electron is supposed to move steadily in a circular or elliptic orbit round a positive

nucleus. The rate at which such an electron radiates energy, according to the classical theory, is roughly proportional to the density of energy in the field of the nucleus. To retain the classical theory we must assume that the electron is supplied with energy in some way and the secondary field of the nucleus seems a likely source. It may be mentioned that we may avoid a radiation of energy to infinity if, instead of the usual expressions for the electromagnetic field vectors, we adopt the mean values of these expressions with respect to c over an interval from c_1 to c_2 . The acceleration terms, which ordinarily give the radiation, then become multiplied by a factor of type $(\sin kr)/kr$ where k is a very small constant. This factor is approximately unity except for exceedingly large values of r , when it is very small. If we imagine radiations of similar nature, but traveling with slightly different velocities, to come from the same source there will be marked interference only at a very great distance from the source.

4. REMARKS ON THE PRECEDING THEORY. The theory just sketched is in many respects incomplete because as yet no values have been assigned to the various constants. The theory needs to be extended also so as to be applicable to the case in which a number of electromagnetic fields are superposed. We may have to consider the possibility of collisions between ether doublets belonging to different fields and the developments of a complete theory will be necessarily complex.

There is one point, however, that may be considered here. We have assumed that collisions actually occur, *i. e.*, that events do happen. This implies that the ether has a definite geometrical form; in other words, the aggregate of ether-particles must move according to definite geometrical laws, some of the lines of motion must intersect and the particles must be timed right. A particle traveling along a straight line with velocity c has five coördinates and the ether-complex may be represented by a number of equations between these coördinates. The form of these equations is entirely unknown, but it is easy to imagine simple systems of particles in which collisions are bound to occur.

Let us suppose, for instance, that a set of particles are situated initially on the line $x + y = 0$ and that some move with

velocity c parallel to the axis of x , while the others move with velocity c parallel to the axis of y . If a particle continues its motion after a collision but with altered properties we can say that all the particles moving in one direction are bound to collide sooner or later with all the particles moving in the other direction. The simple system just described is a particular case of a more general system in which particles move along the two sets of generators of a hyperboloid of revolution and are initially in a plane perpendicular to the axis of symmetry of the surface. More complex systems can be built up from a number of simple systems of the type just described and an ether particle may be supposed to leave one simple system after a collision and enter another.

It is noteworthy that the importance of the idea of incidence or collision is emphasized in Einstein's general theory of relativity and gravitation. The physical equations are supposed to be covariant with respect to transformations of the space and time coördinates in which the new coördinates (x', y', z', t') are functions of the old coördinates (x, y, z, t) . A fundamental property of such a transformation, when it is biuniform, is that what may be described geometrically as a collision, when one system of coördinates is used, may also be described as a collision when the other system of coördinates is used.

The conditions for incidence are lost sight of in the ordinary development of the continuous ether theory because disturbances in the ether are supposed to radiate in all directions from an electric pole. The conditions must be supposed to exist all the same and it is noteworthy that in the case of the fields of two moving electric poles the condition for the incidence of lines of force can be expressed in a particularly simple form.²⁵ It is probable that the lines of force should be considered as discrete entities just as Sir J. J. Thomson has so often taught. The ether-particles forming the different lines of force may in fact be produced one at a time instead of a large number at a time, and consequently a wave front should be treated as a thin shell instead of a surface, there being very slight differences in phase between the different particles in the shell.

²⁵*Phil. Mag.*, **34**, 405, 1917.

The present theory cannot be regarded as final because no attempt has been made to describe the properties of electricity. The ultimate aim should be that of Pythagoras. We should endeavor to describe all physical quantities in terms of numbers. The representation of mass and energy by means of a number of collisions is a step in the right direction. Let us see now whether any progress in the same direction can be made with electricity itself.²⁶

5. SCALAR QUANTITIES CONNECTED WITH THE ETHER. An electromagnetic theory of physical phenomena is not sufficiently fundamental because it is based on the idea of a vector field. From a mathematical point of view a theory which deals primarily with scalar quantities may be regarded as more elementary.

Let us consider an entity which is endowed with certain properties at an instant of time τ and which for some time retains a few of these properties unchanged while it moves along a straight line with constant velocity c . Our object, in the first place, is to obtain a number of simple quantities in terms of which these properties may be expressed.

Like a human being, the entity may be supposed to remember the date of its birthday. If, at time t , the entity has traveled a distance r from its birthplace, the date of its birthday is given by the relation $c\tau = ct - r$. If we express $d\tau$ in terms of dt and the increments dx , dy , dz of the rectangular coördinates of the entity at time t , the coefficients of dx , dy , dz , dt are quantities which remain constant during the entity's career. These quantities may all be expressed in terms of the real and imaginary parts of a complex quantity σ , the ratio of $X + iY$ to $Z + r$, where X , Y and Z are the rectangular coördinates of the entity relative to its birthplace. If α and β are two functions of σ and τ , the real and imaginary parts of α and β are quantities which remain constant during the entity's career and may consequently be regarded as expressing permanent properties of the entity.

Now consider three consecutive entities whose birthplaces and birthdays are consecutive. Let $d\alpha$, $d\beta$ denote increments of the quantities α , β when we pass from the first entity to the

²⁶A theory in which electricity is explained in terms of ideas relating to space and time has already been given by Hermann Weyl, Berlin, *Sitzungsberichte*, May 30, 1918; Raum, *Zeit. Materie*, 1918; *Nature*, Feb. 17, 1921. See also A. S. Eddington, *Proc. Roy. Soc.*, London, Ser. A., 99, 104.

second, $\delta\alpha$, $\delta\beta$ the increments of α and β when we pass from the second entity to the third. We can express the quantity $d\alpha\delta\beta - d\beta\delta\alpha$ in terms of quantities of types $dy\delta z - dz\delta y$, $dx\delta t - dt\delta x$, and the ratios of the coefficients of these 6 quantities will be found to depend only on σ , the ratios in three cases being simply equal to $-ic$. The coefficients of $dy\delta z - dz\delta y$, $dz\delta x - dx\delta z$ and $dx\delta y - dy\delta x$ may be regarded as the components of a complex vector $Q = H + iE$ and it is easily verified that the vectors H and E satisfy Maxwell's electromagnetic equations and can be regarded as the magnetic and electric forces in an electromagnetic field.²⁷ The vector field specified by the vectors E and H is a particularly elementary type of electromagnetic field, the flow of energy is along the radius from the birthplace of one of the entities at the point under consideration. Both E and H are perpendicular to this radius and to one another, and are equal in magnitude. It should be remarked that the different possible birthplaces of an entity are supposed in general to be the consecutive positions of a point which moves with a velocity less than c .

It is found that a vector field of this type always has singular points which travel along particular radii with the velocity c . To arrive at the ordinary electromagnetic theory we must get rid of these singularities and this may be done as follows:

Let the field just obtained be called a field of the first order. To obtain a field of the second order we form four different fields of the first order by taking different pairs of functions of σ and τ as the functions α and β . Differentiating the corresponding vectors Q with respect to x , y , z and t respectively and adding the results we get the complex vector Q of a field of the second order. A field of the third order may be derived from a field of the second order by repeating this process, and so on. It is sufficient for our purpose, however, to remark that by suitably combining a field of the first order with a field of the second order it is possible, in one important case at least, to get rid of all the singular points which travel with velocity c and be left with a total field with just one isolated point singularity which moves along the curve of birthplaces with the velocity, mentioned

²⁷For a more complete development, with full mathematical analysis, see *Proc. London Math. Soc.*, **18**, 95, 1919.

before, which is less than c . This resultant field is the electromagnetic field of a moving electric pole. The particular type of first order field needed for the derivation of an electrostatic field of an electric pole was first obtained by O. Heaviside²⁸ for a different purpose. The birthplaces of the entities all coincide at a point O and the singular points which move with velocity c all move along a line ZOZ' , the two directions OZ , OZ' being singular rays. The magnetic force H at a point P is perpendicular to the plane POZ and is inversely proportional to the distance from OZ ; the electric force E is in the plane POZ and is perpendicular to OP , it has moreover the same magnitude as H . If now we take a point O' on OZ consecutive to O and construct a similar field with singular rays $O'Z$, $O'Z'$ it is clear that we can arrange matters so that the magnetic forces cancel when the two fields are superposed and we are left with the electrostatic field of an electric pole at O .

We can infer from these examples that the classical electromagnetic theory can be derived from considerations in which the primary quantities are scalar quantities. The new point of view is of some interest because it indicates that all the solutions of Maxwell's equations may have a physical meaning. Following up this line of thought we may surmise that some of the solutions with singular lines which have not yet been interpreted²⁹ may be just the ones needed for the specification of the secondary field of 3. It seems likely in fact that the lines of force in a primary field must be supposed to produce a secondary field. On account of the r^{-4} law of collision the intensity of this field may be negligible except in the immediate neighborhood of the free electric charge. It is possible that the total field may have the properties of the type of electrostatic field which occurs in Einstein's theory of gravitation.

²⁸"The Electrician," Nov. 29, 1901; "Electromagnetic Theory," 3, 122.

²⁹Electrical and Optical Wave Motion, Cambr. Univ. Press, 1915.