PART IV

THE TROUTON-NOBLE EXPERIMENT

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SYMBOLS

c = velocity of light.
E = electric intensity in electrostatic units.
f = force per unit charge (stationary or moving).
G = electromagnetic momentum, g = same per unit volume.
H = magnetic intensity in electromagnetic units.
K = dielectric constant.
L = torque.
N = number of doublets per unit volume.
P = polarization (displacement of electricity in dielectric matter).
U = electric energy in ether $= \iiint E^2/8\pi \, d\tau$.
u = velocity of convection.
V = electrostatic potential.
v = velocity of motion of electricity.
$\beta = u/c$.
$\gamma = 1/\sqrt{1-\beta^2}$.
θ = angle of rotation of condenser.
ρ = volume density of electricity in electrostatic units.
σ = surface density of electricity in electrostatic units.
φ = retarded scalar potential.
Ψ = convection potential.
$\cdot \times$ denotes scalar and vector products, resp.
∇ denotes gradient.
Vectors are in black-face type.

I. GENESIS OF THE EXPERIMENT

1. *The Trouton-Noble experiment* originated in a suggestion of G. F. FitzGerald's. We quote F. T. Trouton (1902)[1]: "The fundamental idea of the experiment is that a charged condenser, when moving through the ether, with its plates edgeways to the direction of motion, possesses a magnetic field between the plates in consequence of its motion" · · · "The question then naturally arises as to the source supplying the energy required to produce this magnetic field." "FitzGerald's view" · · · "was that it would be found to be supplied through there being a mechanical drag on the condenser itself at the moment of charging, very similar to that which would occur were the mass of any body situated on the surface of the Earth to suddenly become greater."

An impulse due to such a drag was looked for by Trouton, using a condenser mounted on one end of a cross arm, but with a negative

[1] Scientific writings of G. F. FitzGerald, p. 557.

result; the experiment is described in the paper just cited. Unfortunately the method employed to test the sensitiveness of the apparatus was dynamically fallacious and the effect was really far too small to be detected[2] (see also below, Sec. 5).

Having thus convinced himself that the FitzGerald drag did not exist, Trouton came to the conclusion that a charged condenser moving through the ether with its plates inclined to the direction of motion ought to experience a torque tending to turn it so as to diminish its magnetic energy and therefore so as to set its plates perpendicular to the motion. Modern theory confirms this conclusion except that the direction of the torque is just the opposite of that described.

This effect was sought in the Trouton-Noble experiment.

II. The Experiment

2. *Description.*[1] A mica-plate condenser 7.7 cm. in diameter and having a capacity of 0.0037 microfarads was suspended with its plates vertical from a phosphor-bronze strip 37 cm. long. The condenser was charged to 2100 volts by way of the suspension and a separate wire on the bottom dipping into dilute sulphuric acid. Deflections about the vertical axis were observed upon charging the condenser, by means of a mirror and telescope.

The sensitiveness of the apparatus was determined by finding the elastic constant of the suspension in a separate vibration experiment. The torque that ought to be exerted upon the condenser by the field was then calculated upon the assumption that the torque is equal to the angular rate of decrease of the magnetic energy.

The principal observations were taken during ten days in March, at which time the horizontal drift through the ether was shown to be near a maximum if one included the supposed velocity of the solar system through the ether; three of the observations were taken at noon, when the effects of the earth's orbital motion considered by itself would be a maximum, the others at 3 or 6 P.M. The plane of the plates lay N.E.–S.W.

Eleven observations are reported. The calculated deflections as given by the authors ranged from +.8 to −2.6 cm. at 1 M. distance for the orbital motion alone, and from 0 to −6.8 cm. for the combined motion. The observed deflections ranged from −.12 to −.35 cm., without obvious correlation with other circumstances.

3. *Critique.* A study of the report leaves one with the conviction that the experiment was ably performed and that the data given are reliable. But two points in the interpretation are, at least nowadays, open to serious criticism.

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[2] H. A. Lorentz, K. Akad. Amst. Proc. 6, p. 830, 1904.
[1] Trouton and Noble, Phil. Trans. A, 202, p. 165, 1903.

It is regrettable that good observations were not made and reported for several different periods distributed throughout the year. It is certainly possible that the earth might be almost stationary in the ether during March in consequence of the sun's proper motion through the ether, which we have no really good means of estimating. Or, again, the earth's speed might have been considerable during the period of the observations and yet, as an inspection of the geometrical relationships shows, its component of velocity in a suitable horizontal direction, which would alone be effective in producing a twist about the vertical, might have been always comparatively small. Probably our conviction as to the correctness of the result will not be greatly shaken by this circumstance, especially since the final results were obtained only "after many months of experience" with the apparatus; nevertheless, in contrast, the Michelson-Morley experiment certainly gains considerably in conclusiveness through the avoidance of just this sort of doubt.

A more serious matter is that the authors omitted the "Roentgen current," hardly known in their day, in calculating the magnetic field in the condenser. For they put $H=4\pi\sigma w$ where w = velocity parallel to the plates and σ = surface density of charge on the conducting plates (in electromagnetic units). But the convection of the apparent charges on the mica dielectric also contributes to the magnetic field, and in the opposite sense, making the resultant field $4\pi\sigma w/K$. This illustrates the fact that in a moving electrostatic system the magnetic intensity due to convection is proportional at every point to the electric intensity.

In consequence the authors overestimate the torque to be expected in the ratio K^2. But later they make a slip by substituting for μK in the numerator v^2 in the denominator, where v = "velocity of electric propagation" in the dielectric, and then taking $v=3.10^{10}$ in their numerical calculation. This wrongly divides the deflection by K and leaves it only K times too great. Thus, taking $K=6$, the deflection to be expected becomes, for the orbital motion, from $+.13$ to $-.43$ cm., or almost within the range of the errors of observation.

III. Theory of a Moving Condenser

The theory of the experiment is presented in detail in Laue's "Relativitätstheorie," but he does not consider the effect of a material dielectric and he supposes the moving condenser to undergo the relativity contraction. The introduction of this contraction does not appreciably alter the result but it seems preferable to omit it in the present discussion since the object of our inquiry is the result predicted by the fixed-ether theory. Accordingly we shall omit the contraction and give an outline of the argument.

4. According to the *Maxwell-Lorentz* theory the fundamental equation for the calculation of all ponderomotive forces of electromagnetic origin is

$$\mathbf{f}=\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H} \tag{1}$$

where $\mathbf{v}$ denotes velocity of the electricity relative to the ether. The field vectors are connected in turn with the electricity by the equations (the units being ordinary Gaussian)

$$\left.\begin{aligned} &curl\ \mathbf{H}=\frac{1}{c}\left(4\pi\rho\mathbf{v}+\frac{\partial\mathbf{E}}{\partial t}\right). && div\ \mathbf{H}=0. \\ &curl\ \mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}. && div\ \mathbf{E}=4\pi\rho. \end{aligned}\right\} \tag{2}$$

These equations lead, by a purely mathematical calculation, to the following well-known principle:

(*A*) The moment about any fixed axis of all the forces exerted by the field upon electricity equals the rate of decrease in the total moment about that axis of the electromagnetic momentum in the field, the electromagnetic momentum per unit volume being defined as

$$\mathbf{g}=\frac{1}{4\pi c}\mathbf{E}\times\mathbf{H}. \tag{3}$$

A special case (axis at infinity) is this:

(*A'*) The vector sum of the forces exerted by the field upon electricity equals the rate of decrease of the total vector electromagnetic momentum in the field.

For *convection of an electrostatic system* we easily find further that

$$\mathbf{H}=\frac{1}{c}\mathbf{u}\times\mathbf{E} \tag{4}$$

where $\mathbf{u}$ = velocity of convection: while the problem of finding $\mathbf{E}$ can be reduced to an electrostatic problem by the following device:

(*B*) Imagine the moving system S, including its distribution of electricity, stretched uniformly in the direction of motion in the ratio γ and then brought to rest, forming what we shall call the "associated stationary" system S_1. Then the plot of the lines of $\mathbf{E}$ simply stretches into the plot of lines in S_1, and, letting $\mathbf{E}_1$ and V denote electrostatic intensity and potential, respectively, in S_1 and taking the x-axis parallel to the direction of motion, we have the following relationships between magnitudes at corresponding points in the two systems:

$$\varphi=\gamma V,\ E_x=E_{1x},\ E_{y,z}=\gamma E_{1y,z} \tag{5}$$

where φ = retarded scalar potential in S.

From this we find easily that

$$\mathbf{f} = -\nabla\psi,\ \psi = (1-\beta^2)\varphi = V/\gamma, \tag{6}$$

$$f_x = E_x = E_{1x},\ f_{y,z} = (1-\beta^2)E_{y,z} = 1/\gamma\ E_{1y,z} \tag{7}$$

where ψ is the "convection potential" and **f**, the force per unit charge.

Clearly the condition for electric equilibrium of a conductor is that in S ψ must be uniform, and in S_1, V; but (6) shows that both conditions are of necessity met simultaneously. Accordingly:

(*C*) To find the distribution of given charges which will be in equilibrium on a set of conductors in S, we have only to find the distribution of the same charges in electrostatic equilibrium in S_1 and then contract everything in the ratio $1/\gamma$ parallel to the motion.

Proofs of these principles and formulas may be found in Abraham's *Theorie der Elektrizität,* Vol. II, or in Richardson's *Electron Theory of Matter.*

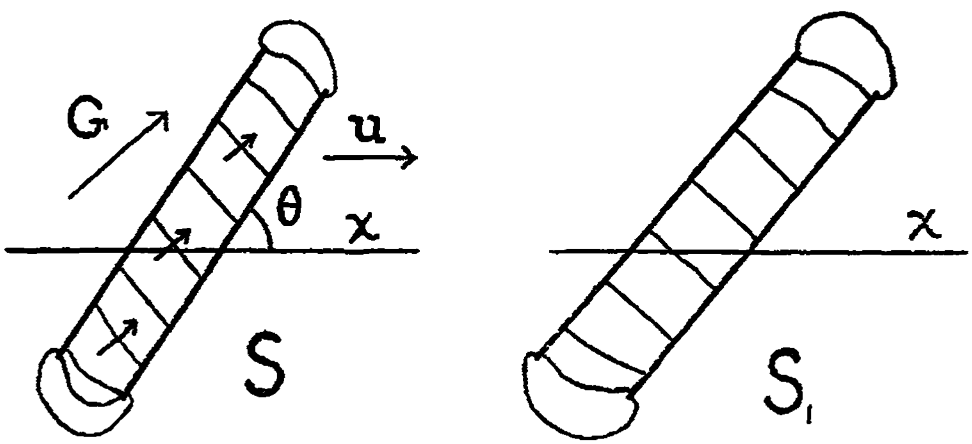

FIG. 1.

5. *Condenser with Ether as Dielectric*: β *small.*—Applying (B), we have as S_1 a stationary condenser that is slightly warped but with the plates still parallel. The warping slightly distorts the field in S_1; and then when we contract it to obtain the electric field in S, the lines become slightly inclined to the plates as shown. Accordingly, (3) and (4) show that **g** is inclined to the plates at a small angle (as shown by the arrows in the figure). But since the contraction is of order β^2 we shall incur relative errors only of the same order if we assume that the electric intensity is the same in the moving condenser as if there were no motion. **g** is then practically parallel to the plates and (3) and (4) give for its magnitude

$$g = \frac{\beta}{4\pi c} E^2 \cos\theta \tag{8}$$

where θ = angle between the plates and the direction of motion. The total momentum is therefore

$$G=\frac{\beta \cos \theta}{4\pi c}\int\int\int E^2 d\tau \text{ or } G=\frac{2\beta}{c}U \cos \theta, \tag{9}$$

where U = electric energy in the ether, the direction of **G** being practically parallel to the plates as shown in the figure.

A further slight error is introduced by the edge-field but this can be reduced indefinitely in the usual way by approximating the plates. The calculation given in Laue differs by a further small quantity of order β^2 because there the system S is supposed to undergo the Relativity contraction.

The *mechanical reactions* of the field upon the condenser are now readily obtained.

When the condenser is charged, (A′) shows that it will experience a backward impulse (the FitzGerald drag) nearly parallel to the plates and equal to $(2\beta/c)\ U \cos \theta$. For the earth's orbital motion $\beta=10^{-4}$, so that the maximum impulse would be less than $U\cdot 10^{-14}$. With modern apparatus a linear impulsive velocity of 10^{-3} cm. per sec. ought to be measurable to 10 per cent; but then $10^{-3}m=U\cdot 10^{-14}$, $U/m=10^{11}$, or we should have to load the condenser with 10^4 joules of electrical energy per gram of weight. At present this is probably impossible.

A more important conclusion for our present purpose is that the condenser also experiences a torque. For, taking a fixed axis at right angles to **G** and to the direction of motion (normal to the paper in the figure), we see that, since **G** is being carried along with a transverse component of velocity $u \sin \theta$, the moment of **G** about this axis is steadily increasing at the rate $Gu \sin \theta$ or, by (9), $2\beta^2 U \sin \theta \cos \theta$; hence by principle (A) there is a torque on the condenser equal to

$$L=2\beta^2 U \sin \theta \cos \theta; \tag{10}$$

and its direction will clearly be such (clockwise in fig.) as to tend to set the plates parallel to the motion.

This result confirms Trouton and Noble's calculation for the case $K=1$ except that the direction of the torque is reversed. The torque is small but ought to be readily detectable.

As a check, we shall attempt to calculate the torque directly from (1) and (4). At any point between the plates $H=\beta E \cos \theta$; as we pass through the charged layer on one plate the magnetic intensity decreases in proportion to the amount of charge passed over and becomes zero at a point outside of the charged layer. Hence one easily finds for the force per unit area on the plate $(H/2)(\sigma u/c)$ or, since $\sigma=E/4\pi$, $(\beta^2 E^2$

$\cos\theta)/8\pi$, the force being perpendicular to the direction of motion. Pairing off opposite elements of the two plates, we find a torque upon them equal per unit area to this force times the distance between the plates times $\sin\theta$, and summing for the whole condenser we arrive at just half of L as given by (10).

The discrepancy is not caused by the electric intensity, for this can be seen to exert no torque if we resolve the field into the elementary fields due to the separate elements of charge. In each of the latter fields the electric intensity points away from the element producing it and the law of action and reaction consequently holds for the interaction due to this cause between the members of any pair of elements.

As a matter of fact, the discrepancy is due to an excess force on the edges of the plate whose moment increases with the size of the plates and hence cannot be made negligible by increasing indefinitely the ratio of area to plate distance; details of this, slightly modified by the relativity contraction, are given in Laue's *Relativitätstheorie.*

6. *Effect of a Material Dielectric of Unit Permeability.*—This deserves careful consideration because mica condensers were used in the experiment.

Part of the theory can be extended to this case simply by including the polarization electricity in our distribution of charge. For then principles (A), (A′), (B), and equations (1)–(2), (4)–(7) will still hold and not only for the values at a point but also for the observable values of the vectors and the density, which represent mean values taken throughout a "physically small" volume. But (3), which is not linear, and (C) require elaboration.

The polarization $\mathbf{P}$ in S, being measurable as electricity displaced across unit area, stretches into a polarization $\mathbf{P}_1$ in S_1 given by

$$P_x = P_{1x}, \quad P_{y,z} = \gamma P_{1y,z} \tag{11}$$

Now if the dielectric were isotropic in both systems $\mathbf{P}$ would be parallel to $\mathbf{f}$ and $\mathbf{P}_1$ to $\mathbf{E}_1$; but then (11) would be inconsistent with (7). Hence we shall suppose an isotropic dielectric in S to become anisotropic in S_1, the polarization constant, $(K\text{-}1)/4\pi$, being decreased in a direction transverse to the motion in the ratio $1/\gamma^2$, so that

$$K_{1x} = K, \quad K_{1y,z} = K - \beta^2(K-1) \tag{12}$$

The polarization will then be in equilibrium in both systems simultaneously, and it follows that principle (C) can be extended to this case. The slight distortion of the field produced by the anisotropy in S_1 is easily seen to be of no appreciable consequence in the present connection.

Finally, in (3) let us split $\mathbf{E}$ and $\mathbf{H}$ into two parts and write $\mathbf{E}=\mathbf{E}'+\mathbf{E}''$, $\mathbf{H}=\mathbf{H}'+\mathbf{H}''$, where the plain letters denote values at an actual point and the single-primed letters, the observed mean values; then on the average $\mathbf{E}'\times\mathbf{H}''=\mathbf{E}''\times\mathbf{H}'=0$, and the mean momentum per unit volume is

$$\mathbf{g}'=\frac{1}{4\pi c}\left[\mathbf{E}'\times\mathbf{H}'+\mathbf{E}''\times\mathbf{H}''\right]. \tag{13}$$

The first term on the right leads at once to equations (8)–(10), that is, this term gives, for a fixed potential difference between the plates, the same value for the torque as if the material dielectric were removed. This is the result to which we are led if we follow the usual rule of simply ignoring fine-grained irregularities of field.

7. *Local Torque in a Dielectric.* The second term on the right in (13) requires special consideration. The general result appears to be that it yields an additional torque which may greatly exceed the one given by (10) but which can have either sign and which has nothing to do with the ordinary electrical properties of the substance but depends directly upon its atomic structure. Indeed, one would expect a torque to exist in many crystals even in the absence of an electric field.

We shall illustrate the possibilities of the case by considering a simple group of electric doublets whose dimensions are small relative to their distance apart.

The mechanical forces upon a given doublet arise in part from the field of its neighbors and of distant charges. This part will depend only upon the strengths and positions of the doublets and of other charges.

A second part arises from the magnetic interaction between the constituent charges of each doublet. If the latter are $+e$ and $-e$ and are separated by a displacement l making an angle ζ with the x-axis, which we suppose to be the direction of motion, the doublet experiences a torque tending to set its axis at right-angles to the direction of motion and of magnitude (the magnetic intensity being $\beta e \sin\zeta\ l^2$).

$$L_1=\frac{\beta^2e^2}{l}\sin\zeta\cos\zeta. \tag{14}$$

Let there be N doublets per unit volume with parallel axes. Then the torque per unit volume is $L=NL_1$, while the resulting polarization is $P=Nel$, so that the torque per unit volume can also be written

$$L=\frac{\beta^2P^2}{Nl^3}\sin\zeta\cos\zeta. \tag{15}$$

This torque can attain any magnitude independently of P through variation in l. Hence a counterbalancing of it by other effects which depend upon P can occur only as a matter of accident. The denominator Nl^3 is, according to our assumptions, much less than unity.

From this result one seems justified in drawing the broad conclusion that *crystals* ought, in general, to experience an appreciable torque when moving through the ether. Definite calculations cannot be made without the adoption of some definite atomic theory, but all modern theories suppose electrical separations to exist in the atom which by themselves would produce enormous values of P, and in general one would expect these to give rise to an outstanding torque of the magnitude of L as given by (15) with P a large number and Nl^3 at least less than unity. For instance, if 10^{23} electrons per unit volume were displaced relative to the atoms a distance of only 10^{-10} cm. the resulting polarization would be $10^{23} \cdot 10^{-10} \times 4.77 \cdot 10^{-10} = 4{,}770$ electrostatic units. This value of P gives, even with Nl^3 replaced by unity, a value of L which is hundreds of times bigger than the torque per unit volume on Trouton and Noble's condenser; for in the latter case the electric intensity appears not to have exceeded 700 electrostatic units, so that U in (10) divided by the volume would be about 20,000.

Since the occurrence of such a torque would constitute an effect of uniform motion through the ether, Relativity requires that any torque due to this cause must be compensated within the crystal by an equal and opposite torque of different origin (presumably of the same nature as that described below in Sec. 10). Thus the fact that no such torque has ever been observed lends a certain amount of support to Relativity.

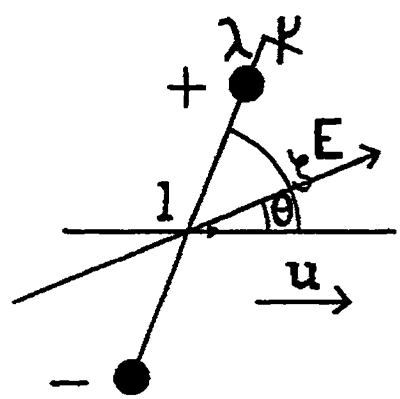

FIG. 2.

To illustrate now the possibilities in an *isotropic* medium, let us consider the effect of applying an electric field in a direction inclined at an angle θ to the direction of motion (Fig. 2), and let us suppose the doublets to be oriented irregularly except that their axes are all parallel to the plane defined by these two directions. Let the field displace the positive charge a small distance λ in the direction of the axis and a small distance μ at right angles to it where

$$\lambda = aE \cos (\zeta - \theta), \quad \mu = bE \sin (\zeta - \theta). \tag{16}$$

In this case there is an outstanding second-order torque for which, using (14), I find the value

$$L=\frac{N\beta^2e^2}{16l^3}(2a^2-8ab+5b^2)E^2\ sin\ 2\theta,$$

or, introducing $P=Ne(a+b)E/2=(K-1)E/4\pi$,

$$L=\left[\frac{(K-1)^2}{8\pi Nl^3}\ \frac{2a^2-8ab+5b^2}{(a+b)^2}\right]\frac{\beta^2E^2}{8\pi}\ sin\ 2\theta, \tag{17}$$

where L = torque per unit volume.

This expression is the same as (10) divided throughout by the volume of the condenser, except for the factor in brackets. It is hard to assign a plausible value to the latter factor without assuming some definite theory of atomic structure; but according to our assumptions Nl^3 is small compared with unity, hence the whole bracket ought for a solid dielectric to be comparable with unity and might greatly exceed this value, and it might have either sign. For instance, for $Nl^3=1$, $K=6$ (mica) and $a=0$ (approximately pure rotation of the doublets) the bracket equals $+5$, while for $Nl^3=1/10$, $K=6$ and $a=1.5\ b$ the bracket equals -4.

In an actual dielectric, circumstances will no doubt be very different from those of the simple case here treated. Yet we seem justified in concluding that, according to the electromagnetics of Lorentz, a local torque of appreciable magnitude is very likely to act upon a moving polarized dielectric and that this torque might conceivably mask completely the effects of the torques upon the true and apparent charges.

8. *The Effect of the Mica in Trouton and Noble's Condenser* remains therefore in doubt. It may have caused the null result; more likely, however, being crystalline, it should have greatly increased the effect, perhaps with a reversal of sign.

9. *The Fine-Structure of the Charge on the Plates* might conceivably have a similar effect. The removal or addition of electrons on the surface might form doublets with axes more or less parallel to the surface, and these, by urging the plates toward a position at right angles to the motion, might mask the main effect. Such an effect could be distinguished in a repetition of the experiment through the circumstance that it would depend only on the charge and not, like the main torque, also upon the difference of potential.

10. *The Explanation by Relativity* of the null result is a dynamical one and is fully given in Laue's *Relativitätstheorie*. The torque occurs as a secondary effect superposed upon the far larger electrostatic attraction between the plates: according to Relativity, the intermolecular stresses which balance the latter attraction do not obey Newton's

Third Law but themselves produce a torque which just balances the electromagnetic one. The null result is thus explained, not exactly by the Lorentz-FitzGerald contraction itself, but rather as a consequence of the same cause that produces this contraction.

11. *Other Ways of Excape* are hard to find. So far as the writer is aware, there is no rival to the Maxwell-Lorentz theory which explains all ordinary phenomena (including Hertzian waves) and also removes the torque on the condenser. Of course, drag of the ether by the earth would do it, but this assumption leads to well-known difficulties. Instead of speculating upon possible modifications of electromagnetic theory it seems more profitable to pass in review the experimental facts upon which the prediction of the torque rests. They are (for vacuum as dielectric):

(1). Moving charged bodies generate a magnetic field. The charged body used in experiments like Rowland's is very similar to either plate of our condenser, the only important difference being that in those experiments the average convection current was closed.

(2). Magnetic fields (of certain kinds, at least) act on moving charged bodies (probably verified only for charged molecules or electrons).

(3). No difference has yet been detected between magnetic fields arising from different causes.

These basic facts lead by very simple reasoning to the predicted torque. As a matter of logic one might attack the sufficiency of the present experimental basis for (2) and (3), but the prospect of discovering any error at this point seems very small. Perhaps the most promising thing to try out would be whether a moving charged *conductor* really is acted upon by a magnetic field.

12. *In Conclusion,* the situation may be summed up as follows:

Trouton and Noble's negative result might have been due to either (1) insufficient sensitiveness of their apparatus (Sec. 5), or (2) insufficient distribution of their observations over different times of year (Sec. 5), or (3) a special effect of the mica dielectric (Sec. 7, 8), or (4) a similar special effect in the charged surface of the plates (Sec. 7, 9).

If, however, it be accepted as an experimental fact that an air condenser never experiences a torque due to the earth's motion, then this fact speaks forcibly in favor of Relativity and can hardly be explained on any other basis.

Probably few physicists will refuse today to accept this as an experimental fact, nevertheless the importance of the problem seems to justify a repetition of the experiment, with observations taken on an air condenser, at different times both of day and of year, and with various distances between the plates.