suggested, but the balance of internal evidence in the paper points to faulty pumping as the cause. In filling lamps on a Toepler pump, I always leave the lamp evacuated at the pump in connexion with a large P_2O_5 tube for at least 24 hours before running in the metal in order to be sure of the removal of water vapour. I also usually cast the cadmium into sticks by suction into a heated glass tube fitted with a tap in order to be sure of the absence of blow-holes. The process of filtration, if properly carried out, is quite sufficient to free the molten metal from solid extraneous matter such as oxide, and I should certainly have considered the operation faultily performed if I had ever noticed traces of oxide in the lamp such as Mr. Bates reports he occasionally observes. The peeling off of "thin sections of the quartz from the walls," reported by Mr. Bates, certainly points to the formation of a cadmium-glass due to oxide as the result of water vapour, and not to the action of adhering metal.

I am, Gentlemen,
Yours faithfully,
HENRY J. S. SAND.

The Sir John Cass Technical
Institute, London.
March 20th, 1920.

LXX. *Mass, Energy, and Radiation.*

By J. J. THOMSON, *O.M.*, *P.R.S.*[*]

THE object of this paper is to endeavour to supply a method of representing in terms of physical conceptions the processes occurring in physical phenomena. It is an attempt to help those who like to supplement a purely analytical treatment of physical problems by one which enables them to visualize physical processes as the working of a model; who like in short to reason by means of images as well as by symbols.

The ideas on which the method is based were suggested by the consideration, from the electrical point of view, of the origin of the mass of an electron. From this point of view this mass is distributed throughout the region surrounding the electron, and for an electron at rest the mass per unit volume at any point in this region is proportional to the square of the electric force at the point. The electrostatic potential energy per unit volume at this point is also proportional to the square of the electric force and is thus proportional to the mass. In fact (see 'Electricity and Matter,' J. J. Thomson, Chap. 2) the electrostatic potential

[*] Communicated by the Author.

energy is equal to the kinetic energy which the mass would possess if it moved with the velocity of light.

This result suggests that the potential energy in the electrostatic field is really the kinetic energy possessed by the mass which is distributed throughout the field, the mass being regarded as an aggregate of equal particles each one of which moves with the velocity of light. In a stationary electric field we may suppose that these particles revolve with this velocity round the lines of electric force, much as the electrons from a hot wire can be made to revolve, though at a slower speed, round lines of magnetic force.

It seems natural to generalize this result and to suppose that all mass, that of atoms as well as that of electrons, is distributed through space with a density determined by the electric field at the place where the mass is supposed to exist; and that energy of every kind, kinetic, potential, thermal, chemical or radiant, is of one and the same type, being the kinetic energy possessed by the particles which are supposed to constitute mass, these it is assumed always move with the velocity of light.

On this view there is no such thing as the *transformation* of energy, if by that we mean a discontinuous change from something of one kind into something of another; on our view the transformation of energy is merely the flow of the mass particles from one place to another. Thus for example, on this view when a body gains kinetic energy, it is not because any of its mass particles are moving faster; it is because the mass of the body has been increased and the increase in the mass implies a proportional increase in the energy.

It will perhaps make it clearer if we follow out in detail this process in a special case—we will take that of a moving electron. When an electron is moving relatively to the bodies around it, the lines of electric force which start from it are no longer uniformly distributed in all directions, those running in directions at right angles to the direction of motion of the electron get more concentrated, and those running parallel to this direction more diffuse. The total number of lines starting from the electron is unaltered by the motion and depends only upon the charge on the electron. Since the mass per unit volume at any place in the neighbourhood of the electron is proportional to the square of the number of lines of force passing through unit area at that place, the amount of mass between two spheres with their centres at the electron and whose radii differ by unity, will be proportional to $\int N^2 dS$, where dS is an element of the area of surface of one of the spheres, and N the number of lines of force passing through unit area of the sphere. Since the charge is given, $\int N\,dS$ is fixed, and when this is so, $\int N^2\,dS$ will be least when N is uniformly distributed over S, and for other cases the excess over the minimum value will increase with the amount by which the lines of force are concentrated in definite directions. The greater the velocity of the electron the greater is this concentration and therefore the greater the value of $\int N^2\,dS$, *i. e.* the greater the value of the mass in the region close to the electron. Thus the moving electron has more mass in its immediate neighbourhood than an electron at rest, and as each unit of mass possesses, since the mass is moving with the velocity of light, a definite amount of energy, the energy of the moving electron will be greater than that of an electron at rest. This increase in energy is what is usually called the Kinetic Energy of the moving electron. It is necessary to say a few words about the definition and measurement of kinetic energy. When, as in ordinary dynamics, the kinetic energy of a body is defined by the expression $\frac{1}{2}mv^2$, it depends essentially upon the axes with respect to which the velocity is measured, the kinetic energy of the same body may be increasing when measured with reference to one set of axes and decreasing when measured with reference to another. The *changes*, however, of the total kinetic energy in a self-contained system, *i. e.* one which is not acted upon by any external forces, will, if action and reaction are equal and opposite, be independent of the axes used. What may be called the localization of energy, *i. e.* the assignment of a certain amount of energy to each member of a dynamical system, is a problem which, as far as rigid dynamics goes, has an unlimited number of solutions; any one of these solutions will give the same changes in the configuration of the system as any other, so that the localization of energy could not be deduced without ambiguity from observations of the configuration of the system.

On the method considered in this paper, the energy associated with an electron, for example, could be determined independently of any axes of reference if we had the power of counting the individual mass particles in its vicinity. We know, however, of no physical phenomenon which will enable us to do this, all that with our present knowledge of physics we are able to do is to compare the number of mass particles in one region with that in another, and this will make the measurement of the mass of an electron, for example, depend upon the position of our measuring instruments. We may illustrate this point in the following way. Suppose we have a region A in which all the atoms and electrons were initially at rest relatively to each other. Now suppose that under electrostatic attraction an electron gets set in motion. From our point of view this means that some of the mass particles which initially were remote from the electron have come much closer to it; this will produce an increase in its mass, and from the equations of electrodynamics we can calculate the ratio of the increased mass to the mass of the electron when it started from rest; we can also, even if every constituent atom or electron of the system gets set in motion under the electrostatic attraction and the mass of each is in consequence increased, calculate the ratio of the increased mass of each constituent to its original mass.

Suppose, however, that the whole region A gets set in motion as a rigid body by the action of an external system B; while the velocity of A is increasing the mass particles will be streaming into it, and while this is going on it is possible that the relative masses of the constituents of A may be affected. But when the velocity of A has become steady and there is no longer any influx of mass particles into it from the outside, the particles which have come into it while this state was being reached will distribute themselves so that the number of new particles in any region is proportional to the number that were present before the influx. Thus the relative masses of two constituents of A, say an electron and an atom, will be unaltered. Thus an observer in A will be unable to detect any effect due to a uniform motion of translation of this region, for though the mass of one of the constituents, as measured by the number of mass particles associated with it, may be altered, the mass of the unit by which that of the constituents is measured will be altered in the same proportion, so that the alteration will not be detected. The argument is the same as that which applies to any changes which the motion may produce in the shape or size of the constituents of the region A; these escape detection by an observer in A because his units are altered in the same proportion as the quantities measured. If, however, we had any method of counting the mass particles within the region A, an observer in this region ought to be able to detect an effect due to changes in the velocity of translation.

Again, if an observer in a region C which did not participate in the motion of A had the means of comparing the mass of an electron in his region with that of one in A, he would find that the ratio depended on the velocity of translation of A.

Following the ideas suggested by these illustrations we get what I think is a consistent scheme for visualizing physical processes, if we assume the existence :—

1. Of particles all of the same kind and with the same mass. These particles all move with the velocity of light. Since the mass particles are moving with the velocity of light they would on the Lorenztian transformation have this velocity whatever might be the axes to which their motion was referred. Any force on a particle due either to other particles or to the electric field is always at right angles to the direction of motion of the particle. Thus, though a particle may be deflected its velocity remains unaltered.

The mass of one of these particles must, as we shall see, be exceedingly small compared with that of an electron.

All mass, whether of electrons or atoms or radiant energy, arises from the presence of these particles, and inasmuch as each particle possesses an invariable amount of energy, wherever there is mass there is an amount of energy proportional to it.

The distribution of these particles and their movement from one place to another is determined by the distribution of the lines of electric force. For we assume that in addition to the mass particles we have in the universe :

2. Lines of electric force spreading through space. These lines may be closed or they may begin or end at definite points. These points are the seats of what we call electrical charges, the electron being at one end of a line of force and a unit of positive electricity at the other. Each electron and each unit of positive electricity forms the end of an invariable number of lines of electric force. The connexion between the distribution of the mass particles and the lines of force is given by the rule that the mass per unit volume at any point P is proportional to

$$\left\{ f^2 + g^2 + h^2 + \frac{1}{c^2}(\alpha^2 + \beta^2 + \gamma^2) \right\},$$

which is also proportional to the energy per unit volume. f, g, h are the number of lines of force passing through a unit area at P at right angles to the axes of x, y, z respectively, α, β, γ are the components of the magnetic force. c is the velocity of light through a vacuum.

We regard magnetic force as due to the motion of the lines of electric force past the observer who is measuring the magnetic force. The relation between the electric and magnetic force when all the lines of electric force at P are moving with the same velocity is given by the equations

$$\alpha=4\pi(gw-hv)\ ;\ \beta=4\pi(hu-fw)\ ;\ \gamma=4\pi(fv-gu)\ ;$$

when u, v, w are the components of the velocity of the lines of electric force relative to axes fixed with reference to the observer of the magnetic force.

From this equation combined with the expressions for the energy per unit volume, we see that P, Q, R, the components of the momentum per unit volume at P are given by the equations

$$\mathrm{P}=\frac{1}{4\pi}(\mathrm{Z}\beta-\mathrm{Y}\gamma)\ ;\ \mathrm{Q}=\frac{1}{4\pi}(\mathrm{X}\gamma-\mathrm{Z}\alpha)\ ;\ \mathrm{R}=\frac{1}{4\pi}(\mathrm{Y}\alpha-\mathrm{X}\beta)\ ;$$

where X, Y, Z are the components of the electric force.

We can also, by the principle of varying action, deduce from the expression for the value of the energy the Maxwellian expressions for the stresses in the electric and magnetic fields which reproduce the mechanical forces existing in those fields.

From the expression for the energy in the electric field, we see that the mass particles are concentrated in the places where the electric field is strongest. Thus when the electric charges are electrons or positively charged units of exceedingly small dimensions—when, in consequence, the electric force is exceedingly strong close to the charge—by far the greater part of the mass will be quite close to the charge. Thus, for example, if the radius of an electron is 10^{-13} cm., only one thousandth part of its mass will be at a distance from the electron greater than 10^{-10} cm. Thus, though the mass particles are present wherever there is an electric field an enormous majority of them cluster close round the electrons and positive charges.

The mass particles perform the functions both of æther and matter. They perform the function of matter by endowing the electrons and positive charges found in the atoms of the chemical elements with mass, and when they are moving through space and carrying energy with them with the velocity of light they are performing functions usually ascribed to the æther.

By themselves the particles are not the whole, either of matter or of æther, for lines of electric force are an integral part both of æther and matter. We only get matter when we have lines of electric force anchored on to electrons or the units of positive electricity; we only get radiation when we have along with the mass particles, closed lines of electric force. The distribution and movement of the lines of electric force determine the distribution and movement of the mass particles.

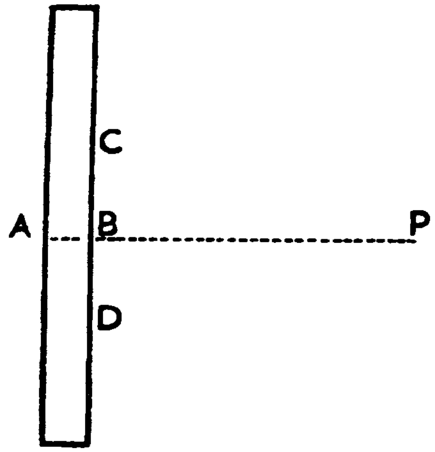
Comparing the physical universe with a living organism we may regard the mass particles as the flesh, the lines of electric force as the nervous system. Mass and energy are contributed by the mass particles, but the distribution, localization, and movement of both mass and energy are determined by the lines of electric force.

The mass particles in a steady electrostatic field, though moving with the velocity of light, are constrained to follow closed paths round the lines of electric force. This produces a tension along the lines of electric force, and these are only prevented from breaking away by being anchored to the electrons and positive charges, and so being obliged to drag about with them wherever they may go the masses condensed about these charges.

In a steady electrostatic field all the lines of electric force have their ends either on electrons or positive charges, none of these lines form closed curves. When, however, the electric field is changing, either by the motion of the positive and negative charges or otherwise, the lines of electric force may get looped, and some of them may form closed curves. These closed curves are not anchored to electrical charges, there is nothing to prevent the mass particles from dragging them away; thus the mass particles will travel out through space with the velocity of light through a vacuum, dragging after them the closed lines of electric force. This, on the view we are considering, is the way in which radiation is supposed to originate. Since both the energy and mass are due to the mass particles, we see that, on this view, radiation involves a transference of mass proportional to the transference of energy. The speed with which the radiation travels is the speed of the mass particles, this speed is invariable and equal to the velocity of light through a vacuum: it is independent of the medium through which the particles are travelling; the velocity of light, however, depends upon the medium, and we have to show that an invariable velocity of the mass particles carrying the energy is consistent with the variation in the velocity of light with the medium through which it is travelling. When a wave of light passes through a refracting medium the electrons in the medium are set in vibration and give out secondary waves; the effect of these secondary waves is to make the apparent velocity of the light through the medium depend upon the number of electrons in that medium, though all the constituents which make up the resultant wave travel with the velocity of light through a vacuum.

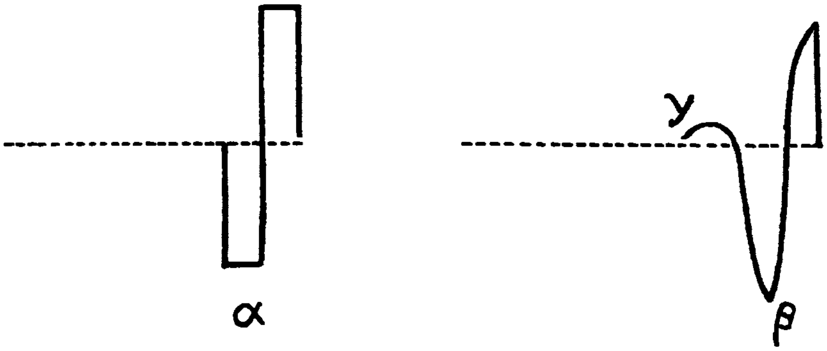
A detailed analytical investigation of this effect will be given in another paper, but the general principles on which the results depend may be illustrated by considering the special case of a pulse of electric force travelling through a slab of refracting matter bounded by planes at right angles to the direction of propagation of the pulse. Let us suppose that the electric force in the pulse is parallel to the axis of x and that the pulse is travelling parallel to z and bounded by two parallel planes at right angles to z. Let the thickness of the pulse be $2d$, let the electric force in the pulse before it strikes the slab be constant in the front half d and equal to X, while in the rear half it is also constant but equal to $-$X. Let us consider the effect produced by this pulse when it strikes the slab of electrons. When the force X strikes the electrons it will accelerate them, and in consequence they will emit secondary waves in which the electric force is in the opposite direction to X.

Fig. 1.



Let AB be the slab containing the electrons and suppose the front of the pulse has reached P; the only secondary radiation which has had time to reach P is that coming from the electrons at B, the part of the slab nearest to P. When a little later a part of the pulse a little in the rear of the front reaches P, the secondary radiation carrying a negative X will have had time to come up from outlying places like C and D, and will diminish the electric field in this part of the pulse. Now consider what will happen when the positive half of the pulse has just passed P and the negative half is just arriving. At first the secondaries which arrive at P will be those excited by the front and positive half of the pulse, and the force on them will be in the negative direction, *i. e.* in the same direction as that in the part of the primary pulse which is now arriving at P, and thus the secondaries will increase the magnitude of the electric force in the pulse. After a time the secondary radiation excited by the negative part of the pulse will begin to come up; the force on this will be in the positive direction, and will diminish the intensity of the force in the primary pulse. The secondary radiation from outlying regions will continue to arrive at P after the primary pulse has passed, so that the primary pulse will have developed a tail. Before passing through the slab the distribution of electric force in the pulse would be represented by a graph like α (fig. 2), while after passing through the slab it will be represented by β.

Fig. 2.



We see that the result of passing through the slab has been to diminish the energy in the front half of the pulse, to increase it in the rear half, and to develop a positive tail γ. Now let this modified pulse go through a second slab; the energy in the front half will be still further diminished, the energy in the rear half and in the tail γ will be increased, and another tail of negative force δ will be developed. This process will go on as the pulse passes through other slabs until the energy in the front part is reduced to insignificance and the second half of the pulse will be the active front; this will in its turn be worn down by the same process and the tail γ will take its place, this will be succeeded by the tail δ, and so on. In this way the virtual front of the pulse is continually falling behind the place which the pulse would have reached if it had not been passing through the slabs of electrons, and the amount by which it lags behind will depend on the density of the electrons in the slab. Thus the velocity of the pulse through the medium containing the electrons will differ from that through empty space and will depend upon the nature of the medium, in spite of the fact that all the radiations which make up the pulse travel with the velocity of light through a vacuum. Hence we see that the constancy of the velocity of the mass particles which carry the energy and mass of light is consistent with light travelling with quite a different velocity when passing through a refracting medium.

On the view we are discussing the radiation as it were carries its æther along with it. The medium which carries the radiation is not something uniformly distributed through space but fragments torn from matter, carrying along with them lines of electric force as an integral part of the radiation.

Though this theory of radiation may be described as an emission one, yet since the velocity of the mass particles is invariable the velocity of light will not be affected by the motion of the source, or when the light is reflected, by the speed of rotation of the mirror. Experiments recently made by Majorana are in accordance with this result.

Since on the view we are discussing energy is made up of a number of equal units, the transference of energy from one body to another must take place by definite steps, and no transference is possible unless the amount to be transferred exceeds a finite amount. This involves that the dynamics of processes involving very small transferences of energy must differ fundamentally from ordinary dynamics.

We are not yet in the position to calculate the mass of one of these mass particles, but it is certain that it must be an exceedingly small fraction of that of an electron. For the energy of an electron is about 10^{-7} erg, which can be represented by the fall of the atomic charge of electricity through about 6×10^4 volts. Now the average energy of a molecule of a gas at 0° C. corresponds to the fall of the atomic charge through a potential difference of about $\frac{1}{30}$ of a volt. Hence if the mass of a mass particle is ω times the mass of an electron, the smallest amount of energy which could be transferred from one body to another would be about $1{\cdot}8 \times 10^6 \times \omega$ times the mean energy of a gas molecule at the temperature of 0° C. Now suppose a gas is raised from absolute zero to a higher temperature, if each molecule of the gas receives the minimum amount of energy possible, the temperature of the gas would be raised to

$$1{\cdot}8 \times 10^6 \times \omega \times 273 \text{ absolute};$$

when the temperature is less than this only a fraction of the molecules will acquire any additional energy from the rise in temperature. When a large number of molecules have acquired no additional energy at all, it would seem improbable that any large number should have acquired the extra energy corresponding to additional degrees of freedom, for example, for a diatomic molecule to have acquired the energy due to its rotation round the centre of mass as well as that corresponding to energy of translation, but unless it did this the specific heats of diatomic gases at temperatures less than $1\cdot8\times10^6\times\omega\times273$ absolute, would approximate to those of monatomic gases; this consideration shows that ω must be less than 10^{-8}. Again, we know from Michelson's experiments on the green line of mercury that the source of this line can give out more than 400,000 vibrations without abrupt change of phase; from Planck's rule, the energy in this radiation is that due to the fall of the atomic charge through a potential difference of 2·5 volts, *i. e.* is about $1/(2\cdot4\times10^4)$ of the energy of an electron. If there is only one mass particle per wave-length of the radiation, there will be more than 4×10^5 mass particles in this amount of energy, so that the energy of one of these particles will be less than $1/(2\cdot4\times10^4\times4\times10^5)$ of that of any electron. Since the ratio of the masses is the same as that of the energies we conclude that at least 10^{10} and probably many more mass particles are required to supply the mass of an electron.

If energy is indivisible beyond a certain limit, then the inverse square law of electrical attraction cannot hold at all distances. For when this law holds, the energy outside a sphere of radius r with its centre at an electron, bears to the energy of the electron the ratio a/r, where a is the radius of the electron; hence if a/r is less than ω the energy outside the sphere will be less than the energy possessed by one mass particle. Thus since the particles are indivisible there would be no particles and no force when r is greater than a/ω, so that the law of electric force cannot be the inverse square law over more than a certain finite distance.