ON DOPPLER'S PRINCIPLE.

By W. MICHELSON.

IF n be the number of vibrations per second produced by the source, a, the velocity of the source along a straight line uniting it to the observer, N, the number of vibrations perceived by the observer, and b, the rate at which he moves along the same line, then, according to Doppler's assertion,

$$N = n\frac{v \pm a}{v \pm b}\,, \tag{1}$$

where v is the velocity of propagation of the wave-motion in the given medium.

Apart from the assumptions by which it is supported, Doppler's principle has by itself a purely kinematic meaning, and therefore cannot be called in question.

But some of the assumptions on which its application is based are in great measure arbitrary, and can hardly be proved except *a posteriori*, by experimental verification. I shall mention but two of these assumptions: (1) that the period of vibration of the source is not influenced by its motion along the line of sight; (2) that the medium carrying on the waves is at rest as a whole, and that its properties are not changing.

When we have to deal with sound, the source of the waves as well as the medium in all its parts are within our reach. Therefore it is generally easy to test the above mentioned assumptions.

It is quite different when we are observing the displacement of lines in the spectra of celestial bodies. In this case we can neither verify immediately nor prove indirectly either of the assumptions referred to. It is very likely that these displacements are actually due to those motions by which they are usually explained in astrophysics, but, from a strictly logical point of view, it cannot be asserted as yet that no other explanation is possible.

Nor do I mean to remove from Doppler's principle its hypothetical part, which probably belongs to it by the very nature of the question.

All I want is to give it a somewhat different expression in order to comprise under one law also those cases where a change of the frequency is caused not only by the motion of the source or that of the observer, but also by a rapid alteration in the density of the medium crossed by the ray.

It is hardly possible to produce or observe, on a large scale, such rapid changes of density on the Earth, but they not only are conceivable, but very probably take place in the Sun's atmosphere.

Let us return to Doppler's formula (1). As the velocity a and b are generally small compared to the velocity of light, the ratios $\frac{a}{v}$ and $\frac{b}{v}$ are small fractions, whose squares and higher powers can be neglected. Hence the equation (1) can be represented as

$$N = n\frac{1+\frac{a}{v}}{1+\frac{b}{v}} = n\left(1+\frac{a-b}{v}+\ldots\right), \tag{2}$$

reckoning a and b as positive in the direction from the source of vibrations to the observer.

If l be the variable distance of the source from the observer it is evident that

$$b - a = \frac{dl}{dt}$$

is the derivative of the distance to time.

Doppler's formula can then be written as follows:

$$N = n\left(1 - \frac{1}{v}\cdot\frac{dl}{dt}\right). \tag{3}$$

This equation holds also for rays which do not travel in a straight line from the source to the observer, but undergo any number of reflections or refractions on their way. In this case, however, the distance l should be replaced by the optical length of the path of the ray L from the source to the observer.

If the geometrical length of the single parts of the ray's path through different consecutive media be l_1, l_2, l_3, . . . l_n, and the corresponding refractive indices of the media referred to one of them (ether) be μ_1, μ_2, μ_3, ... μ_n, it is obvious that

$$L = l_1\mu_1 + l_2\mu_2 + l_3\mu_3 + \ldots . l_n\mu_n = \sum l\mu \ ,$$

and Doppler's principle will be expressed by the equation

$$N = n\left[1 - \frac{1}{v}\sum\left(l\frac{d\mu}{dt} + \mu\frac{dl}{dt}\right)\right] , \qquad (4)$$

where v represents the velocity of light in the medium to which the indices of refraction are referred.

In this equation the additional members of the type

$$\frac{1}{v}\sum\mu\frac{dl}{dt}$$

represent the change of frequency which alone is usually considered in Doppler's principle. This involves also the cases where the length of the ray's path is altered by a rapid displacement of a mirror reflecting it. Mr. W. Wien[1] has made a successful application of a similar change in the period of vibrations produced by reflection from a moving mirror, to the thermodynamics of radiant energy.

These additional members may represent as well the cases where several media of unvarying properties are moving in the path of the rays so as to change rapidly the distance crossed in each of them.

Let us examine two special examples illustrating the case. Suppose that the monochromatic light issuing from the source S passes first through the ether ($\mu_1 = 1$) over the distance l_1, then in a liquid with the index μ_2 over the distance l_2 and then reaches the observer.

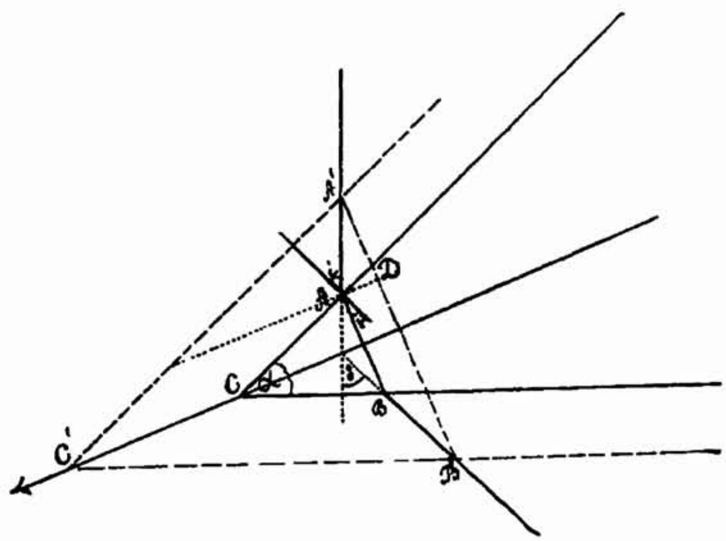
Let l_1 and l_2 change in such a manner that the limiting surface of the ether and the liquid remaining parallel to itself should be displaced rapidly (at the rate c) in the direction of the source.

[1] WILLY WIEN, *Sitz-ber. d. Berl. Acad.*, 6, 1893, and *Wiedemann's Annalen*, **52**, p. 156, 1894.

In other words, the thickness of the liquid l_2 increases at the rate c. Then

$$\frac{dl_1}{dt} = -c\ ;\ \frac{dl_2}{dt} = c\ ;\ N = n\left[1 - \frac{c}{v}(\mu_2 - 1)\right]. \tag{5}$$

If c is commensurable with the velocity of light and μ_2 differs considerably from 1, the displacement of the line can be of the same order as in the case of a rapid motion of the source or the observer. As a second example let us examine the case of a prism moving rapidly across the path of the ray in such a way that the length of the path inside the prism continually increases.



Suppose that the ray passes through the prism at the angle of minimum deviation, which we shall denote by δ. The prism is moving in the direction of CC_1 at the rate c, and its side faces are supposed to be unlimited in their length.

Then let the refracting angle of the prism be $SCB = a$
the angle of incidence of the ray, i
the angle of refraction, r
the refractive index of the prism, μ
the length of the ray's path inside the prism, l_2
the length of the ray's path outside the prism, l_1;

then, according to the previous notation,

$$N = n\left[1 - \frac{1}{v}\left(\frac{dl_1}{dt} + \mu\frac{dl_2}{dt}\right)\right]. \tag{6}$$

If the diagram shows the displacement of the prism in unit of time, then evidently

$$\frac{dl_1}{dt} = -2AA_1 = -2c\frac{\sin\frac{a}{2}}{\cos i};$$

$$\frac{dl_2}{dt} = +2A'D = 2c\frac{\sin\frac{a}{2}\cos\frac{\delta}{2}}{\cos i}.$$

Putting these formulæ into equation (6) and remembering that

$$r=\frac{\alpha}{2}\,;\quad i=\frac{\alpha+\delta}{2}\,;\quad \mu=\frac{\sin i}{\sin r}\,;$$

we get

$$N=n\left[1-\frac{2c}{v}\sin\frac{\delta}{2}\right]. \tag{7}$$

The displacement of the spectral line depends only upon the movement of the prism and the angle of deviation of ray δ. If this angle equals 60°, the displacement is equal to that produced by the receding of the source of light at the rate c. If the prism moved in the opposite direction the displacement of the lines should take place in the direction of the violet end of the spectrum.

In spite of the artificiality of these suppositions, some more or less distant analogies must occur in nature. Immediately before a total eclipse of the Moon the Fraunhofer lines in the Moon's spectrum must be displaced toward the red in consequence of the rapid drift of the Earth's atmosphere across the path of the rays. A reverse displacement ought to be observed at the end of totality. A far more considerable displacement must occur immediately before some star is covered by Jupiter, in whose atmosphere the optical path of the rays is considerably lengthened. But one must confess that it will hardly be possible to witness these phenomena by means of observation on account of the strong light of Jupiter himself, as well as on account of the refraction of the rays passing through his atmosphere.

It is not so with the Sun. There phenomena of this kind not only are continually going on, but they have probably been the object of manifold observation, although, to my knowledge, no attempts have as yet been made to explain them in the direction I have indicated. If a velocity of 500 kilometers per second is attributed to the luminous gases of the photosphere and the chromosphere in order to account for the displacement and the distortion of the spectral lines, there is no reason to suppose that higher and non-luminous gases should be less movable. The thinness and sharpness of the Fraunhofer lines show that the gases of the "reversing layer" are already in a considerably rarefied condition, and are therefore capable of a very swift motion. On the other hand, it can hardly be doubted at present that Schmidt's theory concerning the bearing of the refraction in the Sun's atmosphere is true, at least to a certain extent. Accordingly the rays of the Sun describe probably a very long curvilinear path in the atmosphere before reaching us. This especially concerns the rays issuing from the edges of the solar disk. In this case even comparatively small fluctuations of the solar atmosphere may bring layers of different density into the path of the ray emerging from a certain point of the photosphere or the chromosphere. As "the optical length" of the ray's path may change thereby very rapidly and irregularly, these motions of the non-luminous gases may at least in part account for those extremely rapid and irregular distortions of the spectral lines belonging to Sun-spots and protuberances, which have been observed by Sir N. Lockyer, Professor C. A. Young, and others.

The difficulty of explaining these rapid deviations of the spectral lines in the ordinary way lies less in the assumption of enormous velocities for luminous elements than in the necessity of admitting the existence of inconceivable forces and accelerations which are hardly compatible with the rarefied condition of matter in the Sun's atmosphere. Whereas, according to the explanation I am proposing, a given displacement may be accounted for, in certain cases, also by much smaller velocities; here are acting not only the velocities in the direction of the ray, but also those perpendicular to it. More than this, constant (stationary) velocities may be the cause of most irregular and even opposite distortions of the lines in accordance with the changes of density which take place in the matter carried before the luminous point.

In the case of increasing density in the layers traversed by the ray, the lines are displaced towards the red end of the spectrum, whilst a decrease of density produces a shifting towards the violet end.

Until now it has apparently been considered as an incontrovertible statement that the displacement or distortion of single spectral lines indicates a motion precisely of that matter to which those lines belong. Whereas it becomes evident from the above statements that the motion of hydrogen or helium may have an influence on the displacement of the lines belonging to calcium, iron, etc. It is generally said that if the displacement of the lines depends upon any processes taking place in the path of the rays, and not in the source itself, it would affect not only some of the lines, but all the spectral lines equally. That is true if by "the source of light" one means the whole Sun with the whole of its atmosphere. But since it must be admitted that different elements of this atmosphere may be endowed with velocities of different magnitude and directions — in other words, that they are not mixed, — one must necessarily admit also that the optical paths of the rays issuing from different elements may be different and may be altered almost independently of one another. This may be sufficient to explain in certain cases how the lines belonging to one element may be displaced, while other lines show no disturbances at all.

A strictly scientific solution of the questions, considered from an elementary point of view in this brief account, is hardly possible at present, since it involves the difficult problems of the connection between the ether and ponderable matter.

Moscow,
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