If B is zero then $p=t$ and D must also be zero. In this case we have

$$\begin{cases} d=\dfrac{2A}{t^2} \\ p=t, \\ a=\dfrac{C}{t}+\dfrac{A}{t^2}. \end{cases}$$

Example: Find the sum to n terms of the series whose n^{th} term is $n(n+1)(2n+1)$

Here $A=2$, $B=1$, $C=1$, $D=0$, and condition (5) is satisfied.

Taking $t=2$, we have $d=1$, $p=3$, $a=1$, and

$$\frac{n}{2}\{2p+(n-1)t\}=\frac{n}{2}(2n+4)=n(n+2).$$

The sum to n terms of our series is thus equivalent to the sum of $n(n+2)$ terms of the $A.P.$ 1, 2, 3, ..., and is therefore equal to $\frac{1}{2}n(n+2)(n+1)^2$.

A. A. KRISHNASWAMI AYYANGAR.

Kepler's Law of Refraction.

The correct law of refraction, $\sin i=\mu \sin r$, is usually assumed to have been first discovered by Snell in 1621, although he did not express it in this form. The astronomer Kepler laboured hard to discover it, but in vain, and Whewell in the *History of the Inductive Sciences* says that it is strange that he should have failed, when the law is so simple. There is, however, a good reason for his want of success.

Kepler attacks the question in chap. IV. of the *Paralipomena ad Vitellionem* which was printed at Frankfort in 1604. The problem is to get a mathematical expression which will fit Vitellio's table of the refraction from air to water. After a vain attempt to connect it with various properties of the conic sections he gives a practical rule which is equivalent to the formula

$$i=\frac{\mu r}{\mu-(\mu-1)\sec r},$$

where i is the angle of incidence and r the angle of refraction. He does not give the formula explicitly, partly because he is more interested in $i-r$ than r, regarding the deviation as more important than the refraction; but it is easy to write the formula down from his procedure.

This formula like the modern one is a one-constant formula, and, like the latter, it reduces to $i=\mu r$, when i and r are small, for then $\sec r=1$. The first three columns of the following table give Kepler's verification of the formula, the third column being calculated on the basis of $\mu=1.317$:—

i	r Vitellio's Observations.	Excess of r calculated by Kepler's Formula over Vitellio's Observations.	Excess of r calculated by Modern Formula over Vitellio's Observations.
10	7° 45′	− 11′	− 16′
20	15° 30′	− 29′	− 39′
30	22° 30′	− 19′	− 28′
40	29° 0′	+ 2′	− 10′
50	35° 10′	+ 14′	+ 4′
60	40° 30′	+ 22′	+ 1
70	45° 30′	+ 19′	− 41′
80	50° 0′	+ 0′	− 2° 22′

In the fourth column I have calculated the excess of r over the observations as given by $\sin i=\mu \sin r$ on the basis of $\mu=1.333$, the correct value for water.

Kepler's formula agrees much better with the observations than the modern one owing to the last experimental value being very far out. He states emphatically that his formula agrees with the observations within the error of observation, which is, of course, right. Thus the failure is not a failure, as he states the problem; if he had checked the observations, I think that there is little doubt he would have discovered the true formula.

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