EXPERIMENTAL TEST OF DOPPLER'S PRINCIPLE FOR LIGHT-RAYS[1]

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The first attempt to test Doppler's principle for light-rays in the laboratory was made by A. Bélopolsky.[2] For this purpose he constructed a special apparatus, consisting of two systems of light wheels coupled in pairs. Each pair carried eight mirrors mounted near the periphery of the wheels. Special electric motors set the two systems of eight mirrors in very rapid rotation in opposite directions. These wheels with mirrors were so arranged that a light-ray falling upon them would undergo several reflections from the silvered glass surfaces. By inclination of the direction of the incident beam the number of reflections could be varied at will.

If λ represents the wave-length of the incident ray, v_1 the linear velocity of the center of the mirror, V the velocity of light, and n the number of reflections, then, according to Doppler's principle, the wave-length of the incident beam after the nth reflection will have undergone a change $\delta\lambda$, which is represented with sufficiently close approximation by

$$\delta\lambda = \pm 2n\frac{v_1}{V}\lambda. \qquad (1)$$

If the systems of mirrors on the upper side of the wheels, where the reflection takes place, turn toward each other, the wave-length is shortened and the negative sign is employed; if the direction is opposite to this, the positive sign is used.

A detailed description of the apparatus and the method of its use in testing Doppler's principle is given in the communication by Bélopolsky cited above. Consequently we shall content ourselves with referring to that paper.

Bélopolsky used sunlight as the source of light in his investigation. For the dispersion in the spectrograph two compound prisms were

[1] Translated from *Bulletin de l'Academie impériale des Sciences de St. Pétersbourg*, (6) 1907, No. 8, May 1.

[2] *Bulletin de l'Academie des Sciences de St. Pétersbourg*, XIII, No. 5, 461, 1900; *Astrophysical Journal*, 13, 15, 1901.

used. The photographic exposures were made in the region of the spectrum from λ 4380 to λ 4500. The displacement of several lines on each plate was measured; from these the mean displacement was deduced and then the corresponding velocity in the direction of the beam was computed.

Since the apparatus used by Bélopolsky possessed no very great dispersion, and the displacements were extremely small, even after the sixfold reflection which he employed, these measurements could make claim to no great accuracy. In fact, from the measurement of some lines the displacement was opposite to that expected from Doppler's principle, but nevertheless the mean value in every case gave a displacement which represented correctly the direction of rotation of the mirror.

Bélopolsky secured six different series of observations and compared the velocities derived with those obtained directly from the number of rotations of the wheels.

Considering the comparatively crude means with which the investigation was conducted, the agreement of the values may be considered quite satisfactory. With such dispersion only a skilful observer like Bélopolsky could obtain such good results.

Since Bélopolsky considered the investigation only as a first trial, it seemed to us very desirable to repeat the same experiment with more powerful means, employing the large Michelson echelon spectroscope of the Physical Laboratory of the Academy of Sciences, which possesses such a powerful dispersion. Bélopolsky kindly loaned to us the apparatus with the rotating mirrors, and with this we conducted a number of experiments which we now describe.

The theory of the echelon and the various methods of application of this valuable instrument have been worked out and examined by one of us,[1] and at that time its eventual application to the examination of Doppler's principle was spoken of. Consequently we refer to that paper in the discussion which follows.

We used an Arons' mercury arc lamp as the light-source. This was fed with a nine-ampere, but later thirteen-ampere, current from

[1] See Fürst B. Galitzin, "Zur Theorie des Stufenspectroscops," *Bulletin de l'Academie Impériale des Sciences de St. Pétersbourg*, V[e] Série, T. XXIII, Nos. 1 and 2, p. 67, 1905.

the distance of the middle of the small mirrors, each 2 cm wide, from the axis of rotation, and v_1 the linear velocity of the center of the mirrors, then

$$v_1 = 2\pi N r \ .$$

At the *n*th reflection

$$v = 2 n v_1 , \tag{4}$$

or

$$v = 4 n \pi N r . \tag{5}$$

The test of Doppler's principle consists in comparing the values of v computed from the formulae (3) and (5).

The distance r was obtained by direct measurement. It is

$$r = 0.112 \text{ meter.}$$

A special speed-counter from a mercury interrupter was coupled with the rotating-mirror apparatus to determine the number of rotations N of the wheels corresponding to various exposures. The conversion factor was previously obtained by a series of experiments with a common revolution-counter and a Löbner second-counter which permitted hundredths of a second to be read off.

In all cases we sought to give the wheels the highest possible velocity. This was achieved with a current of about 7.3 amperes. The mean number of rotations per second varied for various series between $N = 41.1$ and $N = 46.2$, which represent linear velocities of the centers of the mirrors from 28.9 to 32.5 meters per second. During single series, for the same direction of rotation of the mirrors, N remained very constant.

Before the investigation was begun, the outer surfaces of the mirrors were carefully silvered by a special method.

The photographic exposures were made in part on Edwards' isochromatic plates and partly on Seed's extra-rapid plates.

At first we wished to photograph, together with the green and indigo-blue lines, the yellow line at λ 5791. But trial exposures showed that the exposure-time necessary to obtain sharp and well-measurable lines with the mirrors rotating was too long; and, since for long intervals of time we could not be sure of maintaining the echelon at sufficiently constant temperature—an indispensable condition in these experiments, as we shall presently see—the yellow line the electric mains of the Academy of Sciences. After several successive reflections the beam was concentrated by the lenses upon the slit of the echelon spectroscope. Two mercury lines, that in the green at λ 5461 and that in the indigo at λ 4358, were photographed after passage through the echelon. For these exposures, the lower half of the collimator slit was first occulted by means of a screen mounted independently of the spectroscope, and an exposure was made with the mirrors rotating. Then the upper half was covered and a second exposure was made with the mirrors rotating in the opposite direction. The movement of the screen was so regulated that there was only a very small space between the ends of the two halves of the lines. After developing and drying the plates the displacement, $2\,\delta m$, of the two halves of the lines with reference to each other was measured under a microscope. $2\,\delta m$ is given in divisions of the head of the screw of the ocular-micrometer, a division of which is equal to $\frac{1}{400}$ mm. This displacement represented a double velocity in the direction of the light-ray.[1]

The exposures were always secured on the side of the echelon of the greater dispersion and in spectra of various orders. It is known that for one and the same line in neighboring orders of the echelon the difference in wave-length, $\Delta\lambda$, is independent of the order of the spectrum. If we represent the separation of the same lines on the photographic plate by Δm, the change in wave-length, $\delta\lambda$, corresponding to the displacement, δm, of the lines for the rotating mirrors relative to the stationary is expressed by

$$\delta\lambda = \delta m \frac{\Delta\lambda}{\Delta m}. \tag{2}$$

When $\delta\lambda$ is determined, the desired velocity can at once be easily computed. It is, regardless of sign,

$$v = \frac{\delta\lambda}{\lambda} V. \tag{3}$$

In this manner the velocity is expressed in terms of displacement of the lines.

The same quantity may be determined from the number of rotations per second of the wheels carrying the mirrors. Let r represent

[1] The ends of the half-lines lay so near to each other, that in the measurement of the displacement the influence of the curvature of the lines could be entirely neglected.

was given up. Indeed the line turned out to be superfluous, for the green and indigo-blue lines belong to regions of the spectrum sufficiently widely separated to give a quite extended test of Doppler's principle.

The exposure-time for the two lines employed was varied for different photographs. The longer the exposure-time, just so much sharper are the lines, and just so much easier is it to measure the relative displacement. On the other hand, too long exposures are dangerous on account of the possible variation of temperature.

After successive exposures with rotating mirrors, exposures were always made with stationary mirrors, on another portion of the plate, in order to determine the dispersion, or the value of Δm, for the given position of the echelon.

The value of Δm need not be measured with great accuracy; nevertheless we give Δm as the mean of six or more measurements, three always being made by each of us.

The chief emphasis of these tests lay in the determination of $2\delta m$. Each value of δm given below is the mean of 20 single measurements, 10 by each of us. We may mention that the agreement of the single values in general is entirely satisfactory and in no case did we get a negative result, that is, a deplacement which is not in agreement with Doppler's principle in respect to the direction of rotation of the mirrors. On the contrary, the measured displacements, as we shall see further on, correspond very well with the values to be expected from Doppler's principle, in view of the admissible errors of observation. Most of the exposures were obtained with a fourfold reflection of the rays, but exposures were also made with six reflections.

Let us now consider a little more closely the influence of a possible oscillation of temperature on the results of these measurements.

It is evident in advance that a change of temperature can be very disturbing, for the echelon in a certain sense may be regarded as a very sensitive interference refractometer, and consequently each variation of temperature, which changes the height of the steps and the index of refraction of the glass, produces a wandering of the fringes. Let us see how great an error a change in temperature of $0^\circ.01$ C. will produce in the velocity v derived from the displacement of the lines.

In the paper cited earlier, "Zur Theorie des Stufenspectroscops" (p. 117) is found the formula

$$\delta\psi\frac{n_2}{r}\left\{\delta\mu+(\mu-1)a\delta\tau\right\}, \tag{6}$$

which gives the angular displacement of a spectrum line for a change of temperature $\delta\tau^\circ$ C.

μ is the index of refraction of the glass for a given spectrum line; $\delta\mu$, the change of μ, when the temperature increases $\delta\tau$ degrees; a is the linear coefficient of expansion of the glass, $=0.0000085$; n_2 and r are two quantities which are defined by the formulae (26) and (29) (*loc. cit.*). Let m be the linear distance in divisions of the head of the ocular micrometer of the microscope corresponding to the angle ψ, then we may make

$$m=A\psi,$$

where A is a constant, dependent on the properties of the optical train.

If $\Delta\psi$ is the angular distance between two fringes of adjacent orders, then

$$\Delta m=A\Delta\psi.$$

According to the formula (36) (*loc. cit.*)

$$\Delta\psi=\frac{1}{r}.$$

If for brevity we set

$$\delta\mu+(\mu-1)a\delta\tau=s, \tag{7}$$

then

$$\delta m=n_2\Delta ms.$$

δm is also the error in the measured displacement $2\,\delta m$, in consequence of a change of temperature $\delta\tau$.

We can therefore place

$$\delta(2\delta m)=n_2\Delta ms.$$

It follows from formula (2) that

$$\delta(\delta\lambda)=\tfrac{1}{2}n_2\Delta\lambda s,$$

or, from equation (3),

$$\delta v=\tfrac{1}{2}n_2\frac{\Delta\lambda}{\lambda}Vs. \tag{8}$$

This very simple formula permits the error in v to be computed directly.

From the numerical data which are given in the paper to which we have referred, and the values of $\delta\mu$ for flint glass for various spectrum lines (from the tables of Landolt and Börnstein), we compute the following values for the constants contained in formula (8), calculating δv for a temperature-change of 0°.01 C.

	Green Line	Indigo-blue Line
λ	5461	4358
$\Delta\lambda$	0.4766	0.2859
n_2	18277	22901
μ	1.5781	1.5918
$\frac{\delta\mu}{\delta\tau}$	0.00000396	0.00000556
s	0.0000000887	0.0000001059 (for 0°.01 C.)
δv	0.021 km	0.024 km

We see that a change of temperature of only 0°.01 C. affects the velocity by 21 to 24 meters per second. Hence if an echelon is to be used in actually testing Doppler's principle, the observer must exercise the utmost care to keep the temperature constant during both exposures with rotating mirrors.

In practice this is indeed quite a difficult matter and at first gave us much trouble, but finally we overcame the difficulties and obtained a quite constant temperature during the consecutive exposures. For this purpose the echelon-spectroscope with all the accessories was inclosed in a glass case, and the interior, where a change of temperature was most to be feared, was filled with cotton. A layer of cotton was also put on the cover over the echelon. In addition to this the whole was mounted in the basement of the main building of the Academy of Sciences where the daily oscillation of temperature is very small, and here the windows were covered. A very sensitive thermometer, divided in fiftieths of degrees, whose bulb was near the echelon, gave extremely small variations. In spite of this the observations commonly had to be confined to the morning hours only, when the sun had not yet shone around the corner of the building; and even then only one line (two consecutive exposures) could be investigated on one and the same day because the air of the laboratory became disturbed by the rotation of the mirrors, as was indicated on the thermometer after a time. A small change of temperature at the beginning of the observations is not so dangerous, because the poor conductivity of the glass makes it probable that the echelon assumes this temperature much later. But if the investigation is pushed farther one cannot be sure of the conditions of temperature in the echelon. In no case was the measured temperature-change greater than from 0°.01 to 0°.02 C., with a single exception where the change amounted to 3½ hundredths.

By observing all these precautions, the results obtained were very satisfactory, as will be recognized in the summary of the investigation given below.

The numerical results are given in the following tables, I and II. The first gives the cases of fourfold reflection, the second of sixfold.

The first column gives the date of the observation; the second, the emission line employed; the third, the rotation number N. It should be remarked that N is the mean of four readings made at the beginning and end of the two consecutive exposures. The fourth column gives the exposure-time for each plate; the fifth the displacement sought (moving mirror relative to stationary mirror) in divisions of the head of the ocular micrometer ($2\delta m$, the sum of the two displacements, was measured directly[1]). The sixth column gives the value of Δm, that is, the distance of the two bands in neighboring orders, likewise in divisions of the head. The values of $\frac{\Delta\lambda}{\Delta m}$ are collected in the seventh column. These quantities give a measure of the dispersion of the apparatus; that is, the number of Ångström units corresponding to a division of the micrometer-head. In the eighth column are the velocities derived from the observations. The ninth gives the velocities computed by the number of rotations per second, N. Finally, the last column gives the difference Δv of these values $\{v$ (from number of rotations) $- v$ (from displacements)$\}$.

With reference to the determination of v from the number of rotations per second N, it should be stated that we undertook to mount the mirrors so that the rays reflected from the center of the mirrors, while the reflecting surface was parallel to the slit, should coincide as closely as possible with the middle of the slit, where the

[1] It is to be remarked that, in order to vary as much as possible the conditions of the experiments, the investigations were begun at one time with one direction of rotation, the next time with the opposite direction.

TABLE I

Date	Line	*N*	Exposure	δm	Δm	$\frac{\Delta\lambda}{\Delta m}$	*v* from Displacement	*v* from No. of Rotations	Δv
			mins.	divs.	divs.		km. per sec.	km. per sec.	km. per sec.
April 10......	Green	45.1	15	4.75	524.9	0.000908 Å. U.	0.237	0.254	+0.017
" 11......	Green	45.4	15	5.28	559.6	852	0.247	0.256	+0.009
" 12......	Indigo-blue	46.2	30	6.24	399.3	716	0.308	0.260	−0.048
" 15......	Indigo-blue	45.9	60	4.80	403.1	709	0.234	0.258	+0.024
" 16......	Green	45.3	30	5.11	564.4	845	0.237	0.255	+0.018
" 17......	Green	45.4	30	5.16	567.3	840	0.238	0.256	+0.018
" 18......	Indigo-blue	45.5	50	6.02	429.3	666	0.276	0.256	−0.020
						Means	0.254	0.256	

TABLE II

Date	Line	*N*	Exposure	δm	Δm	$\frac{\Delta\lambda}{\Delta m}$	*v* from Displacement	*v* from No. of Rotations	Δv
			min.	divs.	divs.		km. per sec.	km. per sec.	km. per sec.
April 20......	Green	45.0	60	7.60	491.0	0.000971 Å.U.	0.405	0.379	−0.026
" 21......	Green	44.0	60	6.68	490.1	973	0.357	0.372	+0.015
" 22......	Green	41.1	60	6.27	495.4	962	0.331	0.346	+0.015
						Means	0.364	0.366	

displacements are measured. In the computation of v, from formula (5), we have taken for r the distance of the center of the mirrors from the axis of rotation. If the reflection had taken place on one of the edges of the mirrors, the computed velocity would have been in error about 10 per cent.

When we consider the results in these two tables we see that the difference between the velocity v computed from the displacement and from the number of revolutions amounts in the mean to only about twenty meters per second.

Considering the difficulty of these measurements and the influence of variation of temperature mentioned above, the agreement can be considered as entirely satisfactory. Therefore, Doppler's principle for light-rays is, within the permissible range of error of observation, fully confirmed.