Further:

$$\varepsilon''' - \varepsilon'' = \tfrac{10}{21}\,\varepsilon^2\,;$$

$$\gamma_2 = \tfrac{10}{21}\,\alpha_2{}^2\varepsilon_s'' \quad ; \quad \gamma_1 = \tfrac{10}{21}\,\alpha_1{}^2\varepsilon_s'' \quad ; \quad \gamma_s = \tfrac{9}{21}\,\varepsilon_s''.$$

A rather rough computation gives an approximate value of P. It was then found that in each of the four cases there is only one value of P satisfying the conditions:

$$p_2 < p_1 < 1$$

and $$\Delta_1 > 0 \quad ; \quad \Delta_2 > 0.$$

The result is:

	I	II	III	IV
δ_s	·500	·500	·600	·600
p_2	·63	·55	·63	·55
P	1·560	1·747	1·557	1·718
p_1	·9828	·9608	·9806	·9449
Δ_1	·290	·375	·174	·267
Δ_2	1·014	1·178	1·047	1·206

Assuming for the mean density of the earth 5·52, and expressing the radii in kilometers, we have:

	I	II	III	IV
radius of inner surface of discontinuity	3941 *km*	3441 *km*	3941 *km*	3441 *km*
depth „ „ „ „ „	2430 *km*	2930 *km*	2430 *km*	2930 *km*
radius „ upper „ „ „	6149 *km*	6011 *km*	6134 *km*	5911 *km*
depth „ „ „ „ „	222 *km*	360 *km*	237 *km*	460 *km*
density above upper surface of discontinuity	2·76	2·76	3·31	3·31
„ between the two surfaces of discontinuity	4·36	4·83	4·27	4·79
„ at centre	9·96	11·33	10·05	11·44

The depth of the upper surface of discontinuity comes out rather small. Still it corresponds to a comparatively small change of density. Modern seismological results seem to point to an important discontinuity of the elastic properties (not necessarily of density) at a great depth, corresponding to our inner surface, and a number of smaller discontinuities above that. The theory of CLAIRAUT, on which the above computations are based, of course takes account of the density only, and not of the other properties of matter. It is very difficult to predict what the result of a computation introducing more surfaces of discontinuity would be.

Doppler's principle and the ballistic theory of light, by *W. de Sitter.*

In *A. N.* 5319 (**222**, p. 249, Sept. 1924) Prof. LA ROSA argues that the assertion made by me in *B. A. N.* **57**, p. 121 and in *Proc. Acad. of Sciences Amsterdam*, **27**, p. 291 (May 1924), viz: that the effect of a radial motion of the source on the apparent intensity*) and on the frequency of the observed light depend on the same factor $1 + q = dt'/dt$, though correct for the wave theory of light, is not true for the ballistic theory. As I am not acquainted with the details of Prof. LA ROSA's theory, and the freedom cannot be denied him to construct whatever theories he likes, it may be as well to restate briefly my objections, so that he may know exactly which difficulties his theory has to meet. My assertion in *B. A. N.* **57** was based on the assumption that the frequency could be defined by

$$\nu = \frac{n}{\Delta t},$$

and the intensity by

$$i = \frac{I}{\Delta t},$$

if n represents the number of waves, or impulses, and I the total energy, during the interval of time Δt. To be entirely correct, we should say that ν and i are the limiting values of these ratios for an infinitely small interval of time. Consequently, if every impulse carries the energy I_0, we have

$$I = n.I_0,$$

and therefore

$$i = I_0.\nu,$$

and it is evident that i and ν must be proportional, and if one of them changes the other must change in the same ratio, whatever the cause of this change may be, so long as the amount of energy carried by one impulse remains constant.

*) In *B. A. N.* 57 I supposed that the effect of relative motion on intensity had never been noticed before Prof. LA ROSA pointed it out in the beginning of this year. I have since found that it was already mentioned by EINSTEIN in 1905 [*Annalen der Physik*, 17: Zur Electrodynamik bewegter Körper, § 7].

If, however, this definition, either of the frequency or of the intensity, is abandoned, and in that case only, this proportionality may cease to exist, whatever theory of propagation of light may be adopted. The question whether my assertion is correct thus depends not on the theory of *propagation* of light, but on that of *radiation*. Now the above definition of the frequency, however evident it appears to be at first sight, may not be tenable if modern theories of radiation are adopted. It may be that in BOHR's theory the frequency is *not* the total number of vibrations, or impulses, per unit of time, but an inherent property imparted *instantaneously* to the light in the act of radiation, and only interpreted by us as having the character of a number divided by a time*). I am not competent to decide whether this is so or not, nor to say what in that case the effect of a relative motion of the source and the observer on the observed frequency will be. Prof. LA ROSA states (*l. c.* p. 251) that the ordinary formula of Doppler's principle holds good whether the velocity of the source is added to the velocity of light or not. Assuming for the moment this to be correct, then of course the objection to RITZ's theory first raised by ZURHELLEN (*A. N.* 4729, **198**, p. 1, 1914), and repeated by me in *B. A. N.* **57**, loses its force. My original objections (*Proc. Acad. of sciences Amsterdam*, **15**, pag. 1297, Feb. 1913 and **16**, p. 395, Oct. 1913), however, are not affected, and I still maintain that these objections are absolutely decisive against that theory.

The observed radial velocities of spectroscopic double stars are in the great majority of cases completely explained by orbital motion under the action of gravitation, and the orbital motion is in some cases confirmed by visual observations and in others by the star being an eclipse variable. This explanation of the observed shifts of the spectral lines becomes impossible if the time needed for the light to reach us differs appreciably for different parts of the orbit.

*) Prof. LA ROSA (*l. c.*, footnote p. 251/252) seems to think that we may simply neglect the change of velocity of the source during the time Δt of emission, on account of its smallness. This change may however easily amount to some meters per second per second, which for a star at a distance of the order of 10 lightyears would on the ballistic theory cause a retardation or acceleration of the beginning of a train of waves emitted during one second relatively to the end of the order of one second, so that the beginning and end would reach the observer either simultaneously or with an interval of two seconds, according to the sign of the acceleration of the source.

Thus, in order to have no effect of a change of velocity of the source on the observed frequency, the act of emission must be time-less: the frequency, which we interpret as a number of impulses per unit of time, must be imparted to the emitted light instantaneously.

If we take as an example a star whose apparent radial velocities are represented by a simple sine curve

$$v = K \sin \frac{2\pi\,(t' - t_0')}{T} + v_0,$$

t' being the time of observation, then, if these values of v are interpreted as real velocities, and if we adopt RITZ's theory, we have

$$t' = t + \frac{\Delta}{c - v},$$

t being the time of emission, and the true law of velocities would be very approximately of the form:

(1) $$y = \sin 2\pi\,(x + \alpha y),$$

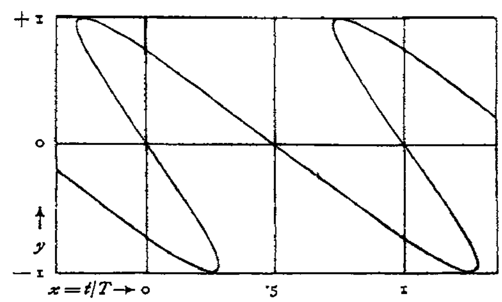
where we have put

$$y = \frac{v - v_0}{K}, \qquad x = \frac{t - t_0}{T},$$

and we have*)

$$\alpha = 0{\cdot}0039\,\frac{\Delta . K}{T},$$

Δ being the distance in parsecs, K the semi-amplitude in km/sec, and T the period in days. If the curve of observed velocities differs from a simple sine curve, an entirely analogous distortion takes place. To show the absurdity of this I give in the accompanying diagram the curve (1) for $\alpha = \frac{1}{2}$.**) It appears that the star would in a part of its orbit have three different velocities at the same time.



There are many stars showing periodic displacements of their spectral lines, for which the value of α is of the same order or much larger than this. If the ballistic theory is to be adopted, a satisfactory explanation of these spectral shifts, i. e. variations of frequency of the observed light, must be given, since that theory makes the evident explanation by orbital motion under the action of gravity impossible. We look forward to Prof. LA ROSA for this explanation, but until it is forthcoming, I maintain my conclusion of 1913, that the ballistic theory is unacceptable.

*) This α is the quantity called Kb by LA ROSA in *Zeitschrift für Physik*, 21, 6, p. 333, 1924. The sign of v is here taken in accordance with the usual custom for radial velocities, positive for recession.

**) Thus, *e.g.*: $T = 4^d$, $K = 100$ km/sec, $\Delta = 5$ parsecs.