Amherst in 1902 and from Johns Hopkins in 1915. According to letters from Professor Barrois, of the University of Lille, a further honor was soon to have been his through election to fellowship in the French Academy.

A study of John M. Clarke's works shows clearly that he was one of the greatest paleontologists of his time and one of the geniuses of science, "standing on the mountain-top and catching the first rays of the rising sun," pregnant with new views of nature. But an intimate knowledge of his life also reveals that his path to eminence was hewn out with much labor among his beloved fossils, taxing to the full the many-sided equipment that was his from home, college and environment.

Charles Schuchert,
Rudolf Ruedemann

SOME MATHEMATICAL ASPECTS OF COSMOLOGY

(*Continued from page 99*)

There are many more postulates that are worthy of discussion, but let us suppose that they have been read by title, and that our system of postulates is complete. Everything else that happens in our cosmology must be in harmony with them, for they are esthetic propositions and are not to be profaned with evidence. Evidence and experience are dealt with by *hypotheses*, which include all those statements which we usually call the laws of nature. Perhaps the most fundamental and the best verified of all hypotheses is Newton's law of gravitation, and yet the Neumann-Seeliger proposition, which we have already mentioned, shows that our mathematical formulation of it can not be rigorously true, since it conflicts with our system of postulates. The statement that the effects of a displacement of a body are perceived at distances, however remote, instantaneously is quite likely to be in conflict with any serious system of postulates. Newton's formulation is delightfully simple, and its predictions are almost perfect, but I should very much prefer to think that at distances sufficiently great the attraction of any body whatever is rigorously zero, rather than merely very small. However that may be, we must not push Newton's law "to the limit"; nor, indeed, are we justified by evidence in pushing any physical law "to the limit."

Similarly, the inverse square law enables us to compute in an entirely satisfactory manner the attraction of an electrically charged surface for an oppositely charged particle, provided the particle is not in the surface. If the particle is in the surface the situation is mathematically indeterminate. We escape this evil consequence by a hypothesis of fine structure, so that what is a mathematical surface for some purposes is not at all a mathematical surface for others. Again we must not push the law of attraction to the limit. Perhaps a theory of fine structure could be made to account for the complete disappearance of gravitation at distances sufficiently great. However fine the structure may be, eventually it becomes too coarse for gravitation to act.

A second conflict with our postulates is found in the law of radiation, which, again, is an inverse square law. We have already seen that if this law were rigorously true the entire sky would be as bright and as hot as the disk of the sun. The evidence is squarely against it. Relative to such a situation the sky is very dark and cold, and we must admit that the law is not rigorously formulated. But radiation is energy, and energy can not disappear into empty nothingness. It was this difficulty which led me some ten years ago to make the hypothesis[38] that radiant energy can and does disappear into the fine structure of space, and that sooner or later this energy reappears as the internal energy of an atom; the birth of an atom with its strange property of mass being a strictly astronomical affair. Indeed, with an infinite sequence of physical units, no smallest one and no largest one, each an organized system of smaller units, and none eternal, one can hardly escape the hypothesis that energy runs up and down the entire sequence, and that on the whole as much energy is ascending as is descending.

The rate at which radiant energy is being absorbed in space, and consequently the rate at which atoms are being formed, must be very small relative to the standards of a physical laboratory. Trigonometric parallaxes show that there are only six or seven thousand stars within 100 light years of the sun, while estimates for the entire galaxy run from one to two billion. The distance of most of the stars must be great as compared with 100 light years. Assuming the rate of loss of energy to be proportional to the distance travelled, we find that the radiant energy decreases according to an exponential law, and since the reliable distances are certainly very great the rate of loss must, with equal certainty, be very low. But if this loss is only one per cent. in one hundred light years, the Andromeda nebula is at a distance of less than 50,000 light years instead of 1,000,000 light years as at present estimated.

There is nothing particularly strange about the idea that atoms, or electrons, are formed from

[38] *Astrophysical Journal*, July, 1918. See, also, *Scientia*, January-February, 1923.

smaller units by the addition of a suitable quantum of radiant energy. We all agree that the periodically recurring beauties of the springtime are due to a similar process and that the organic molecules, with their host of marvelous properties, are somehow built up by radiant energy from inorganic molecules. The properties of the organic molecules are not less marvelous than the property of mass, but the rate at which these systems come and go enables us to observe many cycles, while the lives of the atoms are in general very long. Possibly a scientifically inclined mosquito might wonder why the process of vegetable growth has not exhausted the carbon dioxide of the atmosphere long ago.

The hypothesis that atoms are generated by the radiant energy of space does much more than merely account for the blackness of the night sky, which suggested it. It accounts for the existence of that nebulosity with which cosmogonists have always started, and which is so striking a feature of the astronomer's photographs. Even in the apparently dense Orion Nebula it is extremely attenuated, the wonder being that it is visible at all. There is nothing, however, to suggest that these nebulae contract into stars, as was taught during the nineteenth century, and is still largely believed to-day. The twenty million years assigned for the life history of the sun by the contraction theory of Helmholtz is absurdly small even for the requirements of the geologists, perhaps not over one or two per cent. of the required amount; and it vanishes almost completely in comparison with the vast stretches of time which are fundamental in the dynamics of the galaxy.

For example, the close approach of two stars is a primary event in the evolution of a cluster of stars, corresponding to collisions in the kinetic theory of gases. The expectancy of any one star for an approach as close as the earth to the sun, that is, one astronomical unit, is of the order of a million billion (10^{15}) years. If we call such an interval of time an *eon*, then the *eon* is a convenient unit of time in describing the history of the galaxy. The statistical studies of Charlier and of Jeans have shown that the galaxy has made observable progress towards the steady state which we can regard as the state of maturity.[39] The phenomena of star clusters and star clouds, groups of stars possessing common motion, shows that the galaxy is still a youthful aggregate of stars. Quite likely its present age is to be measured by hundreds of eons, and its state of maturity is still distant by thousands of eons, if it ever arrives. Our information is quite inadequate to probe the possibilities of such vast stretches of time.

[39] See Jean's "Problems of Cosmogony," p. 236.

It should be said, however, that smaller aggregates, the globular star clusters, seem to have arrived at the steady state.

Such considerations force the problem of the source of stellar energies vividly upon our attention. But if the atoms are systems containing energy, as we have supposed, then here is a source that, at least, is worthy of investigation. Possibly in the sun these energies are released just as the stored radiant energy of a cord of wood is set free in a fire. The mechanism of that release is to be found in the intense gravitational stresses which exist in the interior of a star. The earth is a small body astronomically, but the pressure at its center is 22,000 tons per square inch, or a hundred times the greatest pressure attainable in our physical laboratories. For bodies of the same density the pressure varies as the square of the radius. For bodies of the same material in the same physical state, increase of pressure results in increased density, and therefore the pressure increases faster than the square of the radius. A body similar to the earth, but of twice its radius, has a central pressure of 100,000 tons per square inch; double it again, and the pressure rises to 500,000 tons, and we have only reached the size of Uranus and Neptune which are still small bodies astronomically.

If we appeal to the postulate that ***no organised system can withstand an unlimited amount of violence,*** it is evident that there is an upper limit to the mass of a solid body. The atoms break down and give up their energy. Imagine the earth to be growing by the addition of meteoric material and nebulosity picked up from space, and imagine this material similar to that which the earth already has. The mass begins to get hot. Permanent gases escape from the interior and enlarge the atmosphere. Eventually, even the surface becomes too hot, and the ocean rises in a cloud of steam. The more volatile substances pass over into the atmosphere, and there is a gradual change from the solid state to a gaseous state accompanied by a marked decline in the mean density. The gaseous state having been reached, a further increase in mass results at first in an increase in density due to compression, just as it does in the solid state. Increase in density can not go on indefinitely in the gaseous state, however, any more than it can in the solid state. The expansive effect of the heat which is liberated by the increasing mass gradually overtakes the compressive effects of gravitation, and there is a second maximum in the density mass curve. For still greater masses the density continues ever afterwards to decline, owing to the excessive generation of heat; the curve becoming asymptotic to the axis of zero density. The mass begins to glow with a dull red heat, becoming brighter as the mass increases until the entire mass is white hot.

These are consequences which follow from the hypothesis that the atoms are destroyed by sufficiently great gravitational stresses. How does it fit the evidence? Experiment, of course, is out of the question, but we can examine at least some of the astronomical bodies. Commencing with the satellites and planets of our own system, we find that all bodies smaller than the earth are solid and that on the whole the density rises as the mass increases. The next bodies more massive than the earth are Uranus and Neptune, 14 and 16 times the mass of the earth, respectively. Their density is approximately the same, and about one fourth of the density of the earth. The maximum solid body is apparently slightly more massive than the earth, and Uranus and Neptune are in the transitional stage from solids to gases. Passing next to Saturn, which is 95 times as massive as the earth, we find a density only .6 that of water. Saturn is near the beginning of the dark gaseous state. Jupiter is more than three times as massive as Saturn and its density is nearly twice as great. Jupiter is the largest dark body in our planetary system. There are not enough bodies in our system to locate exactly the second density maximum. There is also a value at which the mass becomes red hot, and is therefore a dull, feeble star. This point is perhaps 100 times the mass of Jupiter, as there is no star whose mass is known to be less than one tenth of the mass of the sun.

One of the fundamental modern contributions to our knowledge of the stars was made by Russell in 1911 in establishing, by statistical methods, the existence of the dwarf and giant series, a classification due originally to Hertzsprung, on the basis of absolute luminosities. Stars of all spectral classes occur in both series. The dull-red, dwarf stars were found to be dense, and to average one half the mass of the sun. As the stars of the dwarf series brightened and became yellow and then white, the average mass increased and the density decreased until for the very white stars the mass was five and one half times the mass of the sun. Passing then to the giant series, as the star's colors passed from the white to the yellow to the red, the mass still further increased to about fifteen times the mass of the sun, while the luminosity increased but slightly, and the density fell to very low figures.

Russell's interpretation[40] of these facts was very different from that which I am suggesting, but it can not be doubted that these facts are precisely those which I should anticipate. In the case of the giant red stars with a diameter of two or three hundred millions of miles, the furious radiation near the center must be blue white, but this type of radiation can not penetrate its enormous atmospheric envelope, which is of course relatively much cooler; and the star is red, partially for the same reason that the sunset is red, partially because the radiations from the relatively cool atmosphere also are red.

The energy which a star can draw from its own mass is limited, just as the energy which it can draw from the contraction theory is limited. But as a star moves through space it picks up atoms and molecules or stray meteors or a comet and adds to its mass. Occasionally it enters a distinctly nebulous region, and its mass grows with relative rapidity. We have only to suppose that, on the whole, it picks up as much mass as it loses by radiation to provide for an indefinite duration to its period of luminescence. Its brightness will fluctuate with its mass. At times it will decline to the point of extinction; at other times it will pass over into the giant stage.

Let us see what we might anticipate for the future of our solar system during the next few eons. The mass of the sun will fluctuate, but the planets can scarcely do anything but grow. When the sun declines in mass the planets will recede, the distances of the planets being inversely proportional to the sun's mass. Under these circumstances, the planets become more sensitive to the perturbations of passing stars, and there is greater possibility of the eccentricities being increased. When the mass of the sun is growing, however, as it will when in a densely nebulous region, the planets are growing too. Assuming that the ratios of the masses are maintained, the planets draw closer to the sun, the distances being inversely proportional to the cube of the masses,[41] and the eccentricities tending towards zero. If the material is gathered in at random from all directions the planets will grow without substantially altering their distances, or eccentricities. In this manner we see the planets gradually growing towards starhood. Let us suppose that Jupiter has grown to be a dwarf red star, while the sun has just held its own. The distance between them will be reduced, but how much will depend upon the circumstances of growth. Let us suppose it is one half their present distance. Suppose finally they enter a nebulous region, and their masses slowly grow to four times their initial masses. Their distances will be reduced to four million miles and their period to about a day and a half. Jupiter and the sun will form a typical spectroscopic binary star. The earth and the inferior planets will have been

40 "The Observatory," 1913, 1914.

41 See MacMillan, "The growth of the solar system," *American Mathematical Monthly*, October, 1919.

swallowed up by the sun. The fate of Saturn, Uranus and Neptune is not clear, but the probabilities seem to favor their extinction also. If the masses of Jupiter and the sun were increased to five times the initial mass, their distances would be reduced to two million miles and their period to about twelve hours. They would be inside of Roche's limit and there would be some kind of a cataclysm, possibly of a type that would account for the existence of cepheid variables. If a star were once started into pulsations, which is Moulton's hypothesis for cepheids,[42] there would be an excessive release of energy at the time of compression, and this extra energy would keep the pulsations going.

We see then that the matured state of a planetary system is a binary or perhaps even under favorable circumstances a multiple star. From the developments of the planetesimal hypothesis, it is to be expected that planetary systems are normal to all stars; that 40 per cent. of the stars, which is the percentage estimated to be binaries, should have matured families is not surprising. If four eons is the expectancy of any one star for the generation of a family, then four eons should measure the normal existence of a planetary system, including the binary star stage, though of course there would be wide variations from the mean.

It is evident, too, that if the rate of radiation of a star is proportional to some power of the mass higher than the first power, which is the case, according to Eddington's and Jeans's figures,[43] then the masses of a binary star tend towards equality, which, as we have already observed, is strikingly the case. It should be remarked, however, that if the disparity of masses is too great, say, ten to one, the chances of discovery that a star is binary is much diminished. If a binary star belongs to the dwarf series we should expect the less massive star to be redder, but if the star belongs to the giant class we should expect the more massive star to be the redder. Shajn[44] has recently stated that this is the case.

Let us imagine that a large volume of extra galactic space has become nebulous in the course of eons by the passage of radiant energy through it. It is penetrated by wandering stars which we recognize within the galaxy as the runaway stars, that is to say, stars within the galaxy, but, on account of their high velocities, not permanent members of it. As the star gathers in the nebulosity and adds to its own mass, its velocity relative to the nebula is reduced, so that it is unable to escape the gravitative control of the nebula. In the course of time many stars will be caught in the same way, and we have the beginnings of a star cloud. Many star clouds in the same neighborhood, if their total moment of momentum was not zero, would begin to move about one another and form such a system as our own galaxy. Or, if the star cloud was single and isolated, it would develop into a globular star cluster, of which there are some eighty examples. As Shapley's researches indicate, these clusters are very remote.

The mode of disintegration of such systems also is clear. Occasionally two stars will approach in such a way that one of the pair is given a velocity sufficiently great for it to escape from the system altogether. Even though such events are extremely rare, a few stars must be lost in this way; but a general disintegration of the system is due to an exhaustion of nebulous material. In a sufficiently prolonged period of famine the masses of the stars decline, the cluster expands, and one by one the stars escape from the group control and resume their primitive state of solitude. So far as I can see, a star can lose its identity only by colliding and uniting with another star, but a star cluster, which includes even the galaxy, loses its identity by a process which is similar to evaporation.

Such, in a hasty way, is the astronomical evidence which justifies a consideration of the hypothesis that the energies of the stars are derived from the consumption of their own masses and that new atoms are generated in the depths of space through the agency of radiant energy. Let us turn now to the domain of physics and see what justification we can find from modern physical concepts. Every one knows that the modern physicist regards all atoms as being built up of positive and negative electrons, which are very small as compared with an atom. Each positive electron carries a positive charge of electricity, and each negative electron carries a negative charge of electricity. These charges are all sensibly equal numerically. Unlike charges attract each other according to the inverse square law, and similar charges repel each other. Matter is electrically neutral, because the atoms are composed of equal numbers of positive and negative electrons. The hydrogen atom, which is the simplest atom, is a binary star, while other atoms are multiple stars of more or less complexity; and the physicists are busy working out the electronic orbits. As for their concepts of the nature of mass, I can not do better than the following quotation from Millikan:[45]

> But though we have thus justified the statement that electricity is material, have we any evidence as yet that all matter is electrical—that is, that all inertia is of the same origin as that of an electrical charge? The answer is that we have *evidence* but as yet no *proof*. The theory

[42] *Astrophysical Journal*, May, 1909.

[43] Monthly Notices, January, 1925, p. 209.

[44] Monthly Notices, January, 1925, p. 248.

[45] R. A. Millikan, "The Electron" (1917), p. 183.

that this is the case is still a speculation, but one which rests upon very significant facts. These facts are as follows:

If a pith ball is spherical and of radius a, then the mass m due to a charge E spread uniformly over its surface is given, as is shown in appendix D, by

$$m = 2/3\ E^2/a \qquad (32).$$

The point of especial interest in this result is that the mass is inversely proportional to the radius, so that the smaller the sphere upon which we can condense a given charge E, the larger the mass of that charge. If then we had many means of measuring the minute increase in mass of a pith ball when we charge it electrically with a known quantity of electricity, we could compute from equation (32) the size of this pith ball, even if we could not see it or measure it in any other way. This is much the position in which we find ourselves with respect to the negative electron. We can measure its mass, and it is found to be accurately 1/1,845 of that of the hydrogen atom. We have measured accurately its charge and hence can compute the radius a of the equivalent sphere, that is, the sphere over which e would have to be uniformly distributed to have the observed mass, provided we assume that the observed mass of the electron is all due to its charge.

The justification for such an assumption is of two kinds. First, since we have found that electrons are constituents of all atoms and that mass is a property of an electrical charge, it is of course in the interests of simplicity to assume that all the mass of an atom is due to its contained electrical charges, rather than that there are two wholly different kinds of mass, one of electrical origin and the other of some other sort of an origin. Secondly, if the mass of a negative electron is all of electrical origin, then we can show from electro-magnetic theory that this mass ought to be independent of the speed with which the electron may chance to be moving unless that speed approaches close to the speed of light. But from one tenth the speed of high up to that speed the mass ought to vary with speed in a definitely predictable way.

Now it is a piece of rare good fortune for the testing of this theory that radium actually does eject negative electrons with speeds which can be accurately measured up to ninety-eight hundredths of that light. *It is further one of the capital discoveries of the twentieth century that within these limits the observed rate of variation of the mass of the negative electron with speed agrees accurately with the rate of variation computed on the assumption that this mass is all of electrical origin.* This leaves no room for a mass of any other kind to be associated with the free negative electron. Such is the experimental argument for the electrical origin of mass. . . .

In the case of the positive electron there is no direct experimental justification for the assumption that the mass is also wholly of electrical origin, for we can not impart to the positive electrons speeds which approach the speed of light, nor have we as yet found in nature any of them which are endowed with speeds greater than about one tenth that of light. But in view of the experimental results obtained with the negative electron, the carrying over of the same assumption to the positive electron is at least natural. Further if this step be taken, it is clear from equation (32), since m for the positive is nearly two thousand times larger than m for the negative, that a for the positive can be only 1/2,000 of what it is for the negative. In other words, the size of the positive electron would be to the size of the negative as a sphere having a two mile radius would be to the size of the earth. From the standpoint then of the electromagnetic theory of the origin of mass, the dimensions of the negative and positive constituents of atoms in comparison with the dimensions of the atoms themselves are like the dimensions of the planets and asteroids in comparison with the size of the solar system. All these computations, whatever their value, are rendered possible by the fact that e is now known.

Now we know from methods which have nothing to do with the electromagnetic theory of the origin of mass that the excessive minuteness predicted by that theory for both the positive and negative constituents of atoms is in fact correct, though we have no evidence as to whether the foregoing ratio is right.

Without concerning ourselves as to the ultimate nature of electricity we can write down the mutual electrostatic potential energy of a positive and a negative electron, *viz.*, $V = \frac{e^2}{r}$, where e is the charge of an electron,[46] 4.774×10^{-10}, and r is the distance between them. In the atom the electrons are in orbital motion and they do not fall together for the same reason that the planets do not fall into the sun. In the interior of a star, however, the integrity of an atom can not be preserved on account of the violence of the gravitational stresses. There must be a vast quantity of free electrons moving at extraordinary speeds. If a positive and a negative electron collide and unite, so that their electrical fields are exactly superposed, the two opposite charges of electricity neutralize each other, and the property of mass disappears for the combined unit. The energy released could be computed if we knew the value of r at which the energy changes from the potential to the radiant form. If it is assumed to be the radius of the negative electron, 2×10^{-13} as given by Millikan, it is found that one gram of matter (equal to 6.06×10^{23} hydrogen atoms) is equivalent to 1.2×10^{10} calories; while if we take the radius of the positive electron it is 2.4×10^{13} calories.[47]

[46] Millikan, "The Electron," p. 119.

[47] The idea that the energies of the stars and of the sun are derived from the consumption of their own masses was suggested to me some ten years ago by the blackness of the night skies. It was frequently discussed with my colleagues and my classes and was published in the *Astrophysical Journal* in July, 1918. The idea that the

The theory of relativity has an advantage here in that it gives a perfectly definite relationship between mass and energy.[48] According to this theory one gram of matter is equivalent to 9×10^{20} ergs, or 2.17×10^{13} calories; and this gives a value of the radius at which the energy is transformed about 10 per cent. larger than Millikan's value for the positive electron. As the sun radiates approximately 1.5 calories per year per gram of its mass, the sun's present mass would supply its radiation for about fifteen thousand billion (15×10^{12}) years, or 1/70 of an eon, if one adopts the larger figures, as I am inclined to do. On this basis the sun radiates 1.2×10^{20} grams per year. Taking the sun's effective radius for sweeping up the materials of space at 14,000,000 miles,[49] and its present speed of about twelve miles per second, the mean density of space necessary to maintain the sun's mass is of the order of 10^{-19}, a density perhaps not impossible. This would be the density if one cubic foot of normal atmospheric air were expanded so as to fill a cube the edge of which was thirty miles.

I do not insist upon these figures, however, as they depend upon hypotheses which can not be verified directly. It is overly optimistic, perhaps, for us to expect any direct experimental evidence which will guide us with certainty over those vast stretches of time for which the eon is a convenient unit, and which certainly are necessary in a consideration of the dynamics of the galaxy and of super-galaxies. The main point is that modern physics furnishes a model already made in the theory of electrons, for our hypothesis, which was based originally upon astronomical evidence, that the energies of the stars are derived from the consumption of their own masses and that the atoms are generated by the radiant energy in what we ordinarily call empty space, although, according to our postulates, space is nowhere empty; furthermore, the energy which is furnished by this model is sufficiently great to meet the immediate requirements of astronomy. I do not think that it tells the whole story, nor do I think that the whole story will ever be told, however long the human race may live or however wise it may become; but it does relieve us of our pressing embarrassments.

It permits us to see that in our physical laboratories and in our observations of nature we are merely watching the courses of the atoms as they are tossed about by the various forces which they encounter on their journey from their birthplace in the depths of space to the place of their extinction in the interior of some star. We are studying only one aspect of the transformations of energy, and hence we derive our second law of thermodynamics. The water which we see is all on its way down the hill. We have ignored the existence of the radiant energy of space, and the question as to what becomes of it. It is inaccessible and out of sight. It is only with the imagination that we can follow it, just as it is only with the imagination that we can follow the water as it changes to vapor at the surface of the sea and condenses back to water again high up among the clouds. We should expect some such doctrine as that of entropy in the world of matter, but it is not a valid doctrine for all possible transformations of energy. According to postulate 13 the universe does not tend constantly in any one direction.

There is a corollary to such a universe as we have postulated that has a strong human appeal. Life is not a phenomenon peculiar to the earth. It exists upon the earth because the conditions upon the earth have been favorable for a sufficiently long period of time. In the past million years or so, it has developed a certain small degree of intelligence, and the race of man is beginning to pry into the secrets of nature with a real curiosity. Elsewhere in an infinite universe there are other suitable abodes, infinitely many, with races of living beings upon them. Some of these races are young, some of them are vastly older than ours, more highly developed, much wiser. Such races existed before the earth was formed or even before the sun started upon its career as a star; after the earth and even after the sun has passed out of existence, other races of living beings elsewhere will

property of mass was lost by the exact superposition of the electrical fields of the electrons was suggested to me early in 1919 by Millikan's book, "The Electron," and this idea fitted perfectly into the gap which I had left in my previous paper. It was stated in a lecture before the Sigma Xi on March 11, 1920, and published in SCIENCE July 23, 1920.

I have learned recently that in a letter to *Nature*, Vol. 99, p. 445, Aug. 2, 1917, Eddington mentioned as a conceivable idea "a gradual annihilation of matter by positive and negative electrons occasionally neutralizing one another" and ascribed the idea to Jeans. Jeans did not regard the idea as worthy of discussion in his book, "Problems of Cosmogony," and definitely tied his cosmology to the contraction theory of Helmholtz."

[48] A. Einstein, "Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?" Annalen der Physik 18, 639 (1905). In this paper Einstein states that from the point of view of relativity the mass of the sun is diminishing on account of its radiation, and he gives the numerical relationship mentioned in the text. He does not suggest, however, that the number of its atoms is diminished thereby, nor that its gravitational field is weakened.

[49] See MacMillan, "The Growth of the Solar System," *Am. Math. Monthly*, October, 1919, p. 328.

be repeating with infinite variations the experiences which we are having upon the earth at the present time.

Atoms, living beings, stars and galaxies are permanent forms in the universe. It is the individuals only that come and go.

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THE CENTENARY OF WILHELM HOFMEISTER

Doubtless many can recall certain books which have greatly influenced their lives, and in my own case one stands out especially—a translation of Hofmeister's epoch-making treatise on the comparative morphology of the archegoniate plants. This book, studied while an undergraduate at the University of Michigan, was undoubtedly the most important factor in determining the trend of my botanical investigations for many years.

It was, therefore, particularly interesting for me to find myself a few years later a student in the botanical institute at Tübingen, where Hofmeister spent the last years of his life.

This picturesque old Suabian town, not far from the Black Forest, lies in the beautiful valley of the Neckar, surrounded by an extremely attractive country. Tübingen will always be famous in botanical annals as the domicile of a line of great botanists, among whom three may be especially mentioned—Mohl, Hofmeister and Pfeffer—surely a sufficiently notable trio for one small university.

Mohl, one of the greatest botanists of his time, founded the botanical institute at Tübingen, in its earlier days the best equipped in Germany.

During my sojourn for the summer semester of 1887, Pfeffer was director, but in the autumn of that year he removed to Leipzig, where his brilliant record is familiar to all botanists, and where many American students studied under his direction.

These memories of Tübingen were recalled through a recent address[1] by one of Hofmeister's most distinguished students, Professor Goebel of Munich. This was delivered at Tübingen, at the celebration held on May 18, 1924, the hundredth anniversary of Hofmeister's birth.

In these days, when the study of comparative morphology is looked at more or less askance by many of our younger botanists, the immense significance of Hofmeister's early work is scarcely understood. These remarkable investigations, necessarily lacking some of the precision made possible by modern technical methods, nevertheless form the solid foundation upon which has been raised the great edifice of comparative morphology, and there is no question that Hofmeister's work will remain as probably the most brilliant contribution ever made to this fundamental department of botany.

Hofmeister's activity began in the period which Goebel has called the "renaissance of botany," when botanists began to break away from the Linnean tradition which for the first third of the nineteenth century was still dominant, and made taxonomy the all-important subject of botanical activity. The brilliant beginnings of anatomy and physiology, made in the seventeenth and eighteenth centuries, had almost sunk into oblivion.

Among the great names of this renaissance is Hugo von Mohl, whose name will always be associated with the study of protoplasm, to which he gave the name still in use. As we have already stated, Mohl was the first director of the Tübingen botanical institute, and was succeeded by Hofmeister.

Hofmeister's first paper was published when he was twenty-three. *A propos* of this Goebel writes:[2] "This was especially remarkable as he was entirely self-taught. It is true that at this time there were no botanical institutes where one could receive instruction in botanical investigation. The technical methods were not so developed and mechanical as is the case to-day, when often the technique of a botanical investigation has a greater specific weight than its 'Gedankennihalt'!"

In 1851, when he was twenty-seven years old, he published his remarkable studies on the structure and development of the archegoniate plants—mosses and ferns; and somewhat later his investigations were extended to include the seed-bearing plants as well. It is these "Vergleichende Untersuchungen" which are Hofmeister's greatest contribution to science and which rank with the most important that have ever been made.

These investigations covered a wide range of forms, and demonstrated beyond question the essential similarity between the archegoniates and the lower seed-plants, and effectively broke down the supposed barrier between "Cryptogams" and "Phanerogams." They showed the essential likeness in the life-histories of all these plants, the regular alternation of sexual and non-sexual generations; and eight years before the appearance of the "Origin of Species," gave a concrete demonstration of the derivation of the higher types of plants from lower ones.

The importance of these investigations as bearing

[1] Goebel, K. Wilhelm Hofmeister. Tübinger Naturwissenschaftliche Abhandlungen. 8. Heft. Tübingen, 1924.

[2] *Loc. cit.*, p. 2.