signed by both Sir William and Lady Huggins. Lady Huggins has been a modern Caroline Herschel and in her bereavement she will have the satisfaction of knowing that her name will always be bracketed with that of her illustrious husband.

The writer had the honor of visiting Sir William and Lady Huggins at their home at Tulse Hill three years ago. The astronomer and his wife were kindness itself and at the age of eighty-three Sir Willlam was as keen and alert mentally and physically as a man many years his junior. The great astronomer had a peaceful and vigorous old age in posession of all his faculties until the end came. He has passed away full of years and honors leaving behind him a noble record of untiring devotion to the cause of Science.

SPECTROSCOPIC AND VISUAL BINARIES.

FRANK SCHLESINGER AND ROBERT H. BAKER.

A barrier of difficulties that seems at the present time all but insurmountable, confronts any direct inquiry as to the course of stellar evolution. What are the facts bearing on this question, that our instruments are capable of revealing? The spectroscope will determine the types of the stars under investigation, and the photometer will tell us their relative brightness both in the visual and in the photographic regions of the spectrum. We may also ascertain their proper motions and in rare cases their distances. But beyond these there are other data that seem indispensable to an investigation of stellar development. The size of a star, its density, the brilliancy for each unit of surface, its age, the local conditions which may have affected its progress—these in general must for the present remain unknown.

The case is far more hopeful if we turn from the general question of stellar development and confine our attention to the relative development of stars forming binary systems; for many of the difficulties we have just mentioned at once disappear. First, we may be certain that the two stars in each pair have had a common origin and are therefore of the same age. It is true that we can imagine circumstances, such as the operation of a resisting medium or the temporary intervention of a third star, that will leave together two stars which were originally remote and which were brought into proximity by reason of their proper motions. But if such circumstances arise at all, they can be only very rare and we may leave them entirely out of the reckoning. Questions as to local differences in space, such as for example the abundance or scarcity of hydrogen that was to be found in the neighborhood of a star during its early stages, may be put aside at once in the case of a binary system, since these conditions must have been the same for both components. Again, the orbits of the two stars in a binary are exceedingly small as compared with their distances from us and we may therefore infer the ratio of their intrinsic brightness without knowing anything of their parallax.

For reasons of this character it would seem highly probable that when we finally come to have something like definite ideas concerning stellar evolution, the key to this knowledge will be found in the study of binary systems. Visual doubles, that is those in which the two components are far enough apart to be separately seen, have been the subject of careful study during the past century, but the accumulation of data concerning them is necessarily a slow business. Few of these objects have periods less than a century, and until a pair has gone through a circuit, or at least the greater part of a circuit, the elements of the relative orbit can be determined only very roughly. In 1905 Professor Aitken of the Lick Observatory compiled a list of all the visual binary orbits that had been determined with even tolerable accuracy. His catalogue contains only 53 objects, and of these the periods of 17 range from 105 to 347 years.

The detection of the first spectroscopic binaries by Pickering and by Vogel in 1889, followed by the surprisingly numerous discoveries of Campbell, Frost and others, has opened to us a rich mine of information bearing upon questions of binary evolution. So vigorously has this mine been worked that it is not too much to say that we have learned more in the past ten years concerning spectroscopic binaries than a whole century has taught us with regard to visual systems. This is due principally to the short periods of the former; very few of those thus far detected have periods as great as one year, and most of them complete a circuit in a few days. But it is proper here to remark that the distinction between visual and spectroscopic systems is in all probability not a real one, being merely a matter of what instrument has been employed to observe them.

Thus to cite only one example: Sirius, long known to be a visual binary with a period of 49 years, has now been observed with the spectrograph and variations in its radial velocity have been detected; it may therefore be very properly included in catalogues of both classes of binaries. In a few of the spectroscopic binaries the linear separations between their components are of the same order as the distance between the Earth and the Sun, and it is therefore natural to expect that some of these will be resolved into visual doubles. Up to the present time however this interesting observation has not been made, and no system first detected with the spectroscope has yielded to the high resolving power of the almost perfect refractors of our day.*

There are now available the orbits of more than sixty spectroscopic binaries. In volume I, No. 21, of the Publications of the Allegheny Observatory we have collected these orbits and we have found them sufficient to bring out some interesting characteristics. At the request of the Editor of POPULAR ASTRONOMY a short account of these results is given here.

It should first be explained that among these 63 orbits are those of eleven variable stars of the δ Cepheid type, that is, short period variables in which the duration of *decreasing* light is less than that of *increasing* light. It is an interesting fact that the variable stars of this class thus far examined from this point of view, beginning with Belopolsky's investigation of δ Cephei, have all been found to be spectroscopic binaries as well; and the persistent researches of the Lick astronomers upon these objects have proven beyond doubt that their variations in light are caused by, or are at least intimately associated with, their orbital motions. Furthermore our list contains the orbits of six Algol variables; that is, variables (always of very short periods) that fluctuate in light in such a way as can be accounted for by an eclipse. Beginning with Vogel's investigation of Algol itself every variable of this class whose radial velocity has been measured has proven to be a spectroscopic binary, with just the kind of velocity curve as would be demanded by the eclipse explanation of its variation in light.

* Capella has been suspected as a visual double by Lewis and others at the Greenwich Observatory, but other observers in better climates and provided with more powerful telescopes have been unable to confirm this.

1. Distribution of Eccentricities.

If we count the number of spectroscopic binaries whose eccentricities fall within successive limits of 0.10, we obtain the following table in which the δ Cepheid variables have been considered apart from the other spectroscopic binaries, and in which are included the 23 visual binaries from Aitken's catalogue.

e	<0.10	.10 to .19	.20 to .29	.30 to .39	.40 to 49
Spectroscopic Binaries	24	9	1	3	4
Visual Binaries	0	3	3	9	14
δ Cepheid Variables	0	2	2	4	3
e	.50 to .59	.60 to .69	.70 to .79	.80 to .89	>0.90
Spectroscopic Binaries	4	2	2	1	0
Visual Binaries	10	5	4	4	1
δ Cepheid Variables	0	0	0	0	0

We see that among visual binaries there is a decided tendency* toward eccentricities between 0.40 and 0.50; and that more nearly circular orbits than this are quite as rare as more elongated orbits. Among spectroscopic binaries on the other hand, the great majority of orbits are nearly round, the number falling off rapidly as the eccentricity increases. It is a curious fact that in this respect the δ Cepheid variables conform more closely with visual binaries than with the other spectroscopic binaries, while all the orbits of the Algol variables have eccentricities under 0.10.

2. Relation Between Periods and Eccentricities

If we form a list of spectroscopic binaries (excluding the δ Cepheid variables) in the order of increasing periods and set down the corresponding eccentricities, we find a remarkably close relation between these two elements: of the 21 of these objects having periods not exceeding six days, only one has an eccentricity over 0.10. This exception is Polaris with a period of four days and an eccentricity of 0.20, about the same as in the case of the planet Mercury. Again, of the sixteen spectroscopic binaries having periods greater than 21 days, Capella (period 104 days) is the only one with a very small eccentricity (0.02), the next smallest (0.15) being that of η Pegasi with a period of 818.0 days. The eleven δ Cepheid variables are again distinct from other spectroscopic binaries; their average eccentricity is 0.31, while their average period is only 7.3 days.

* From a discussion in 1896 of the elements of 40 orbits Dr. See has already called attention to this characteristic of visual doubles.

If we arrange the 53 orbits of visual systems in the order of increasing periods and take the means in sets of ten, we obtain the following:

	Mean Eccentricity.	Mean Period.
First set of ten	0.44	20 years
Second set of ten	.43	42 years
Third set of ten	.51	64 years
Fourth set of ten	.53	104 years
Fifth set of ten	.56	201 years
Last three	[.60]	[327] years

The progressive increase in these mean eccentricities may be accidental but we are inclined to believe that it is real, and that visual binaries, like the spectroscopic, have a tendency to show greater eccentricity as the period increases.

Let us divide the spectroscopic and the visual binaries each into two classes of equal number, according to the length of period, excluding the δ Cepheid variables and the three visual binaries of longest period, for which the data are uncertain. Each group will, as it happens, contain twenty-five stars, and for them we obtain the following means:

	Mean Eccentricity	Mean Period
Spectroscopic binaries of short period	0.07	4 days
Spectroscopic binaries of long period	.35	129 days
Visual binaries of short period	.45	36 years
Visual binaries of long period	.54	136 years

Beginning thus with binaries that revolve in a few days and ending with those that require several centuries for a complete circuit, there seems to be a steady increase in the ellipticity of the orbits.

We must not fail to mention that as early as 1877 Doberck* suspected that visual binaries of long period have more eccentric orbits than those of short period, and that in 1898† he was able to adduce additional evidence in the same direction.

3. Distribution of the Longitudes of Periastra.

About two years ago Mr. J. Miller Barr‡ called attention to the curious fact that among the thirty spectroscopic binaries whose orbits had then been determined, in only three cases did the longitude of periastron exceed 180°. Mr. Barr adduced some reasons for believing that this preponderance might have

* *Astronomische Nachrichten*, 91, 119.

† *Astronomische Nachrichten*, 147, 251.

‡ *Journal of the Royal Astronomical Society of Canada*, 2, 70, 1908.

a physical basis, and it is of interest to see whether more recently computed orbits show the same tendency. Collecting the data from this point of view we find

36 cases in which the longitude is less than 180°,
19 cases in which the longitude is greater than 180°,
1 case in which the longitude is equal to 180°.

Mr. Barr's rule has been violated more frequently than it has been observed in the orbits that have been computed since the publication of his paper, and we must conclude that the one-sided distribution of periastra noted by him was nothing more than a somewhat extraordinary coincidence.

4. Presence and Character of the Secondary Spectrum.

In the first spectroscopic binary ζ Ursæ Majoris, discovered by Pickering in 1889, the spectra of both components are visible, but in the spectrograms of most of the binaries found soon afterwards by Vogel, Campbell, Frost and others, only the brighter component appeared. Some surprise was expressed among astronomers regarding the large number of systems thus revealed in which one of the bodies is comparatively dark and yet is of considerable mass. Very recently however it has been found that the presence of the fainter spectrum is not rare, and there is now reason to believe that it would appear in perhaps half the cases, if photographed under proper conditions with present-day instruments, when the difference in velocity of the two components is sufficient to produce an effective separation of the two sets of lines.

The large number of double spectra that have recently come to light is partly due to the use of fine-grained plates. In this kind of work Seed 27 plates have hitherto been used more than any other. While these are of very satisfactory grain for so rapid an emulsion they are much inferior in this respect to the Seed 23 plates, which however require exposures from two to two and a half times as long. We have found that what appears as a fairly conspicuous secondary spectrum upon a Seed 23 plate may entirely escape notice in the coarser grain of a more rapid plate, and in several instances where we have been led by certain characteristics of the published orbits to suspect the presence of the secondary spectrum, we have verified it upon the first fine-grained plate secured.

Our list of orbits shows fifteen binaries in which the fainter spectrum has been measured, one (κ Cancri) in which it was seen but not measured, and two others (α Carinæ and β Cephei) in which its presence is suspected. Doubtless in many of the remaining cases the fainter component is bright enough to be seen, for in very few of these have fine-grained plates been used, and in others the range of velocity is not sufficient to produce an effective separation of the lines. In still other cases the fainter spectrum does not seem to have been sought, as several instances might be cited in which the observer has made no mention of duplicity of the lines, which further examination proved to be conspicuous on the original plates.

Of the fifteen binaries showing two spectra for which orbits have been computed, there are thirteen in which the brighter spectrum is of the first type. In each of these thirteen cases the fainter spectrum is in all probability an exact duplicate of the brighter. This is explicitly stated by Mr. Ichinohe to be the case with η Virginis.* We have had occasion to examine spectrograms of the remaining twelve of these stars and have failed to find even one line in a fainter spectrum that is not present in the brighter, or a sufficiently strong line in the latter that is not duplicated in the former.

In addition to those cases in which orbits have been computed there are many binaries in which the second spectrum has been seen or suspected. Among them we find fourteen binaries of the first type in which the fainter spectrum is certainly present, and in which something is known as to its character. In each of these fourteen cases the observers remarks indicate that the two spectra are probably the same. There appears then to be no contradiction to the rule that *in spectroscopic binaries of the first type the secondary spectrum whenever seen is a duplicate of the brighter.*

It would be interesting to examine spectroscopic binaries of *later* types from this point of view. Unfortunately the material at hand for this purpose is very scant, and all that can be said at the present time is that while the two spectra in each of these binaries usually bear a close resemblance to each other there seem to be cases in which there are noticeable differences. We shall have occasion to discuss this matter in more detail later.

Besides these binaries that have been studied with slit spectrographs we should not neglect to mention the "composite spectra" discovered at Harvard College Observatory by Miss Maury and Miss Cannon, upon objective-prism plates.† These

* *Astrophysical Journal*, 26, 283, 1902.

† *Annals* of the Harvard College Observatory, 28, 93, 1897; 28, 229, 1897; and 56, 113, 1908.

objects show two spectra in close proximity and some of them exhibit considerable difference in type. Fourteen of them are known visual doubles too close to be separated upon these plates; ten have since been found to be spectroscopic binaries, generally of long periods. Of the remaining twelve, it is not impossible that some are (1) optical doubles, (2) close visual binaries with small relative velocities or (3) single stars with peculiar spectra. These objects are especially worthy of investigation with both the telescope and the slit spectrograph.

Frost* has recently called attention to four spectroscopic binaries in which the H and K lines of calcium oscillate neither in accord with the other lines nor in such a way as to permit us to ascribe them to the other star of the binary. It should also be mentioned that Hartmann has found the K line to be stationary in the spectroscopic binary δ Orionis and Mr. Jordan of the Allegheny Observatory has recently found another case in which this is true.

The star ε Aurigæ seems to be quite unique. Ludendorff has shown† that its light varies in much the same fashion as that of an Algol variable, but with a period one thousand times as great, 27 years. The researches of Vogel, Eberhard and Ludendorff indicate that the spectrum is normally like that of α Cygni, with many metallic lines; but that during light minimum a second spectrum of the F type is superimposed. Ludendorff concluded that there can hardly be any doubt as to the reality of a relation between the light changes and the velocity oscillations, but that "If we wish to explain the observed line displacements in ε Aurigæ as being due solely to changes in velocity the assumption of two bodies is not sufficient."‡

5. The Relative Masses of the Two Components.

The two sets of lines in a double spectrum are displaced from the position in which they are coincident by amounts inversely proportionate to the masses of the two components. Consequently we may compute the ratio of these masses from the two amplitudes. In the table, column (2) contains for each binary the mass of the fainter component in terms of the brighter. Eleven of these masses have been derived from data secured at Allegheny; and for these is added, in column (3), an estimate of the relative intensities of the two spectra. In addi-

* *Astrophysical Journal* 29, 235, 1909.

† *Astronomische Nachrichten*, 164, 81, 1903.

‡ *Astronomische Nachrichten*, 171, 49, 1906.

tion it is known that the two stars of ζ Ursæ Majoris are nearly equal in brightness. These estimates are not intended to give the absolute brightness of the secondary but merely to indicate the proper sequence in which these twelve binaries should be arranged from this point of view.

The numbers in column (2) are all less than unity, or in other words, *among spectroscopic binaries that show both spectra the brighter component is invariably the more massive.*

(1)	$\frac{m_s}{m}$ (2)	$\frac{I_s}{I}$ (3)
ν Andromedæ	0.72	0.6
o Persei	0.81	0.4
α Aurigæ	0.81	
η Orionis	0.95	0.7
ψ Orionis	0.76	0.4
β Aurigæ	0.99	1.0
o Leonis	0.86	
η Virginis	0.70	
ζ Ursæ Majoris	0.99	1.0
α Virginis	0.61	0.5
ε Herculis	0.68	0.4
u Herculis	0.40	0.4
θ Aquilæ	0.89	0.5
57 Cygni	0.96	0.8
2 Lacertæ	0.81	0.7

There appears to be a close correspondence between relative mass and relative brightness: where the two spectra are equally conspicuous, as in ζ Ursæ Majoris or β Aurigæ, the two masses are also equal; but where one spectrum is barely discernible, as in *u* Herculis or ε Herculis, the corresponding mass is small. We may perhaps infer that in those binaries in which the fainter component does not show at all, the mass of the brighter star is all the more preponderant.

6. Masses of Spectroscopic Binaries Compared with that of the Sun.

There are very few spectroscopic binaries indeed for which we can determine the absolute masses. Measurements of velocities alone are not suffiicient to determine the inclination of the orbit and it comes out that all we can compute is the quotient:

$$\frac{\text{mass}}{(\text{sine of inclination})^3}$$

Furthermore if only one spectrum is measurable, we can say nothing as to the relative masses and our computations will yield only a certain function of the *sum* of the two masses. The first of these obstacles is removed in the case of an eclipsing variable, for then we know that the inclination can not be far from 90°, and both obstacles are removed if in the spectra of such variables both components are measureable. Of these latter (usually called β Lyræ variables) there are only three or four bright enough to be within reach of our present instruments. In spite of these limitations we may make certain deductions that are of considerable interest. It appears that in those cases in which both spectra have been detected at this date, the average mass of each component is several times in excess of that of our Sun. And among those in which only one spectrum has been measured, the diversity of mass is truly astonishing after making all reasonable allowance for the effects of unknown factors. Some spectroscopic binaries exceed by probably a thousand fold the masses found in others.

It is noteworthy that the δ Cepheid variables do not share in this diversity; they must be all of nearly the same mass, and the ratio of the two masses must be nearly the same for all of them.

There is another respect in which these variables present great uniformity. According to Annals of Harvard College Observatory, Volume 50, their spectral types are all between F and G, and all but one are included within the narrow limits from F5 to G.

Several astronomers have informally made the suggestion that since the δ Cepheid variables are spectroscopic binaries, it may be that others of these binaries will prove to be δ Cepheid variables whose light fluctuations have thus far escaped detection. It is however not likely that this is the case with any one of the binaries for which orbits have been computed, beyond the eleven definitely known to belong to this class of variables. We should expect one of these objects to exhibit a spectrum like that of the Sun or a little earlier in type, and to have an orbit of short period and considerable eccentricity. Polaris is the only binary that fulfills these conditions, but it is perhaps the star in all the sky of whose constancy in light we may be most certain.

7. Remarks on the Evolution of Binary Systems.

Sir George Darwin* has shown how a rapidly rotating body of small rigidity might first become elongated and then break up into two separate masses. This he supposes to have been

* Philosophical Transactions of the Royal Society, 171, 713, 1880.

the origin of the Moon.* He goes on to prove that if the two bodies thus formed are at all viscous the tides raised in each by the other would occasion, among other effects, an increase in their mean distance and their period of revolution. Dr. See† was the first to suggest a similar explanation for the origin of double stars and to point out that certain observed facts relating to visual systems agree with this idea. We are now in a position to examine spectroscopic binaries from the same point of view, and we find that all the observed geometrical characteristics of these systems are in accord with the views of Darwin and See. The components of short period binaries are as a rule very close to each other, indeed they would seem almost to touch in systems like β Lyræ and those like β Cephei. At this stage we should expect the orbit to be nearly circular, and this is also found to be the observed fact. Dr. See has shown that tidal friction not only increases the distance between the two components but makes their relative orbit more eccentric, a theorem that is very beautifully borne out by the observations, for we have traced the increase of eccentricity with period, through spectroscopic binaries and through visual systems whose periods run into centuries.

So far then as the geometrical aspects of binary evolution are concerned we may say that we have a tolerably complete account. But the case is different when we come to discuss the astrophysical questions involved. It is a well known rule, first formulated by W. Struve,‡ that if the two components of a visual binary are nearly equal in brightness a conspicuous difference in color is rarely to be found; but if one star is considerably fainter than the other it is almost always bluer, or in other words has a spectrum of earlier type. In order to account for this Huggins¶ conjectured that the faint star

* This explanation, as Nolan has pointed out and as Darwin himself is well aware, can not be regarded as complete. In its present form it involves the difficulty of accounting for the persistence of the Moon as one body just after its separation from the Earth. The same difficulty applies to spectroscopic binaries. It should also be mentioned that Moulton's recent investigations seem to indicate that if visual doubles owe their origin to the fission of a single mass, the separation must have occurred while the latter was still in the nebulous state; and that the distance apart of the two components cannot have been much smaller just after their separation than it is now.

† Inaugural Dissertation, 1892.

‡ Mensurae Micrometricae, page *lxxxii.*

¶ Presidential address, Report of British Assn. for the Advancement of Science, 1891; see also Atlas of Representative Spectra, by Sir William and Lady Huggins, Vol. I, Chapter VI.

is really the primary and has the greater mass, and that as a consequence of its smaller proportion of surface to volume it has been able to preserve much longer its original physical state. This involves the assumption that a star may increase in brightness as it ages, and to support this Huggins quotes Lane's law* according to which a body still in the gaseous state would lose less heat by radiation than it gains by gravitational contraction and would therefore grow hotter and brighter. Lewis† has recently come forward with evidence that strongly corroborates these views: he finds that of the eighteen visual systems to which the test is applicable the fainter component is the more massive in twelve cases and less massive in three, while for the remaining three the masses appear to be equal.

Overlooking for the moment certain obvious objections, the development of binary systems according to this explanation would appear to be as follows: The two stars originally formed one body that breaks up into two in the manner traced by Darwin and See. We now have a spectroscopic binary whose components are in the same physical condition and their spectra are exact duplicates of each other. In this stage the amount of light emitted is merely a matter of surface and the larger star will accordingly be the brighter. The distance between the two stars is small, the period of revolution is only a few days or even hours and the range in velocity is great. As time advances this distance is increased and the period lengthened through the agency of tidal friction. Both stars have been getting brighter, but the less massive has been radiating its energy more rapidly and runs ahead of the other in the successive stages of evolution. A time will therefore come when the less massive star will first overtake and then surpass the more massive in brightness. We now have a visual system in which the bright star is more advanced in type and is the less massive. Continuing we come to an epoch in which the less massive star has become so compressed that it no longer obeys Lane's law and it will now decrease rapidly in brightness, while the other is still on the increase, until the less massive star once more becomes the apparent as well as the real satellite. We now have a system like Sirius in which the bright star is the more massive and has a spectrum of earlier type.

* American Journal of Science, July, 1870.
† Memoirs, Royal Astronomical Society, 56, page *xxi*, 1906.

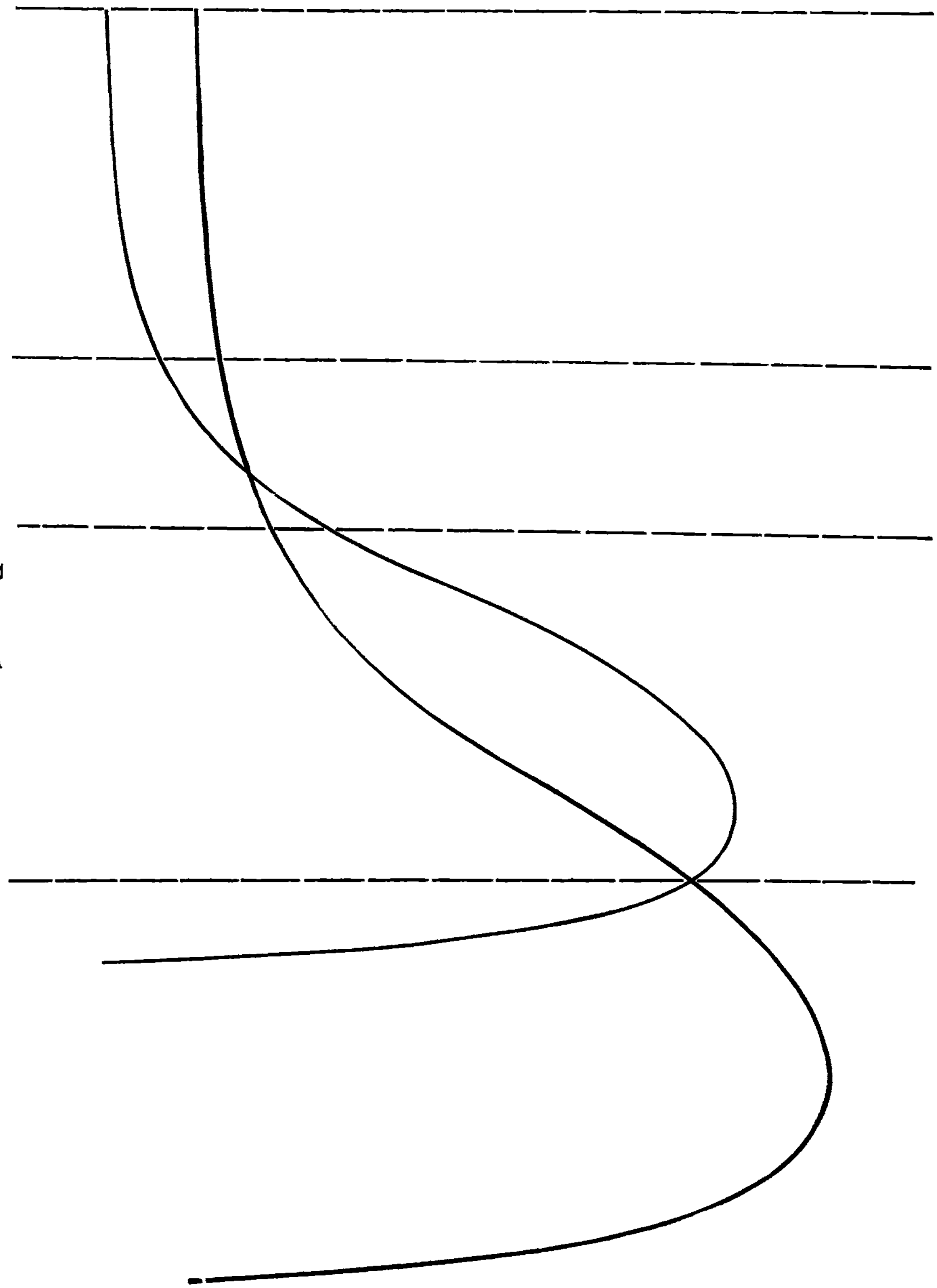

Figure 1.

In the figure we have represented the progression in brightness by two curves, the heavy one corresponding to the more massive star of a pair and the lighter curve to the less massive. These two curves are duplicates of each other except that the horizontal coördinates, representing time, have been compressed in one case by a constant factor. Let us suppose that the volumes are as two to one. The ratio of the surfaces will then be 1.6 and the two stars will differ by 0.5 magnitude. From O then to say P the system forms a spectroscopic binary, the brighter component being the more massive. The two stars have been brightening only slowly up to this time, but now between P and Q the less massive advances rapidly in this respect and from Q to R we have the usual visual binary. By the time that R is reached the less massive star has begun to decline and from this on we have the rarer visual binary in which the more massive star has resumed its place as the bright component. It is to be noted that this star attains a higher maximum than the less massive but that this maximum is reached only after the latter has begun to decline.

An objection to this explanation is urged by physicists, who have shown that the hotter a body becomes the whiter or bluer must it appear. This law was derived under certain ideal assumptions and from experiments necessarily made at comparatively low temperatures. How far they would have to be modified in order to apply them to stars we are not able to say. But it does not seem at all probable that it would fail to the extent that Huggins' hypothesis would imply. Huggins was aware of this objection but he seems to have held the opinion that solar stars are relatively richer in ultra-violet light than those of the first type.* The difficulty might be overcome by assuming that solar stars, while hotter and bluer than those of earlier type, are enveloped with atmospheres whose absorption is relatively greater in the shorter wave-lengths than in those which accompany hydrogen stars. According to this view, if we could strip two stars like Vega and Arcturus of their atmospheres, both would appear bluer than they now do; but Arcturus would gain so much more in this respect that it would appear bluer than Vega.

The question presents itself as to what stage a star will have reached by the time that Lane's law ceases to apply. This law is founded upon the assumption that the star is still a perfect gas and has not become so compressed that the usual relations between density, pressure and temperature no longer hold. According to Perry† who has behind him the great weight of Lord Kelvin's concurrence, the application of

* An Atlas of Representative Stellar Spectra, page 85.

† Nature, 75, 368, 1907.

Lane's law "ought to cease when the density of the gas at the center of the star approaches one-tenth of the density of ordinary water in the laboratory." It may be then that Lane's law concerns only bodies that antedate any with which we have here to do, and that stars of even the earliest types are now diminishing in temperature and brilliancy.

It is one of the consequences of Huggins' explanation that whenever the two spectra in a binary differ it must be the earlier type spectrum that belongs to the more massive component. Let us inquire what exceptions to this we find. Among the eight or nine* binaries in Lewis' list to which the test is applicable we find no exception certainly known to be such, although in one or two binaries the color of the companion is in doubt; thus the fainter star of η Cassiopeiæ is sometimes called purple, but other observers have described it as green, orange, rose-red, Indian red and garnet! An investigation concerning the spectral types of both stars in all those binaries for which we know the relative masses, would constitute a most valuable contribution to this subject.

If we turn to the spectroscopic binaries we find that for only two stars have we the data necessary for the present purpose, α Aurigæ and o Leonis. With regard to the former Professor Campbell says, "the spectrum of the principal star is of the solar type whereas that of the secondary is intermediate between the solar and Sirian types."† The brighter spectrum was found to belong to the more massive component, the ratio being 1.26. Mr. Plummer states that in o Leonis "the component which possesses a spectrum of the F-G type is by far the more conspicuous of the two .... The spectrum of the fainter component appears to be of the Sirian type."‡ The bright component was found to be the more massive in the ratio of 1.16 to one. It would therefore appear that both of these stars are exceptions to the rule to which the visual binaries seem to adhere, and that we have in them examples where the massive star has developed more rapidly than the other. But is it certain that the type of the fainter spectrum has been correctly assigned in these two cases? The difficulty of judging the character of a faint spectrum, seen as it were through a brighter, is considerable. There would be a tendency

* Not all the stars in the list fall under this head because in some cases the masses are said to be equal, and in others the colors are designated the same.

† *Astrophysical Journal*, 14, 261, 1901.

‡ *Lick Observatory Bulletins*, 5, 21, 1908.

to assign it to too early a type because of the apparent faintness or absence of the less conspicuous metallic lines and the diffuseness caused by the overlying continuous spectrum of the other star. It should be remembered that both these binaries were studied upon plates that do not show the critical H and K region. No better evidence of the tendency to assign the fainter spectrum to an earlier type than is really the truth could be afforded than in the case of ο Leonis itself. From an inspection of the plates the observers concluded that the type of the fainter spectrum is A, that of the other being about F5. But upon making an extended study of the measures and after imposing certain quantitative tests, Mr. Plummer decided that whenever the two components were not distinctly separated the measures had really been made upon blends and that the two spectra are much the same. Considerations like these perhaps warrant us in raising the question whether the spectra in these two binaries do not after all conform to the rule of uniformity that has been found to apply for spectroscopic binaries of earlier type, and whether for the present at least these two stars should be regarded as forming a fatal objection to Huggins' hypothesis. However even if we admit that the two spectra are the same, we should not be removing all the objections that these two stars have raised against this explanation; for the question then arises how can they have advanced so far in spectral type without showing the increase in brightness for the less massive component, that seems to be always present in equally advanced visual binaries?

An investigation with the slit spectrograph of the stars having composite spectra, to which the Harvard observers have called attention, will furnish an excellent opportunity for testing the relation between mass and type. In some of these stars the difference in spectra is considerable, and examples are present of both cases, bright star earlier type and bright star later type.

In the figure we have indicated that the light of both stars decreases very rapidly after the maxima are passed. We were led to do this by the small number of visual binaries like Sirius in which the brighter star is of the earlier type. After the small star has begun to wane it probably remains comparable with the other in brightness for only a relatively short period. This idea gains support from the small fraction of stars in general found to have spectra later than the solar type. It is a consequence of this view that there are probably many binaries in the sky of which only one component would be visible under ordinary circumstances. Sirius and Procyon really belong in this class, their faint companions having been detected only after these stars were known to be binaries from the irregularities in their proper motions. There is no reason why other stars of this description should not come to light; but it must be remembered that even if their number be great, few of them can be so favorably conditioned for detection as Sirius and Procyon, whose periods are comparatively short, whose orbits are wide and which are so bright that we have meridian observations of them reaching back for many years.

Since the spectral type of a star advances with age we should expect the visual binaries to be later in type than the spectroscopic. This is on the whole the case, for no less than sixty per cent of the spectroscopic binaries, whose orbits have been determined, are classed as either B or A, while in Aitken's catalogue of visual binaries there is no example of a helium star and only 13 per cent are of the A type. Similarly among the spectroscopic binaries we find that the four of longest periods are of the solar type.* But the spectrum of a star cannot be a very reliable guide to its absolute age. For early type may indicate that it is young or else that, being massive, it has preserved its original condition the longer. Similarly a star of late type may in truth be old, but it is equally probable that it has small mass and has run its course rapidly. In binary systems the separation of the two components is perhaps a better index to age: a close spectroscopic binary has probably been formed only recently while, according to Darwin's analysis, millions of years must elapse before the separation becomes great enough to make the system a visual one. If then we find a visual binary in which the stars are of early type we should expect to find that it is more than ordinarily massive. Unfortunately it is not possible to test this matter from the data at hand, since the determination of the masses of visual binaries demands accurate values of the parallaxes.

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* It must be remembered that some observational preference has crept in here, without which these figures might have to be modified to an important extent: the orbits of the later type spectroscopic binaries have been neglected more than the earlier; and again there may be many binaries of early type of long periods (and presumably of small range) which from the character of the lines usually found in these spectra would be difficult to discover.

Again if we find a close spectroscopic binary of the late type we should expect the masses to be small. The presence of the unknown factor $\sin^3 i$ prevents us from applying the test with certainty, but the indications strongly confirm this view.

In addition to the ideas that we have been discussing, Huggins has made an alternative suggestion to account for the colors of double stars. He asks:* "May it not be that the effect of great mass on surface density, together with the working of Lane's law . . . will favor in such stars the coming in of a solar type of spectrum at a somewhat relative earlier time?" Schuster† has also made and developed the interesting suggestion that a massive star would arrive sooner at the solar stage because its greater surface attraction would enable it to absorb its hydrogen more rapidly. With either of these explanations the observed facts in α Aurigæ and o Leonis accord better than with the hypothesis of Huggins that we have discussed more fully. On the other hand these alternative explanations would necessitate the yellow component of a visual binary being always the more massive; this is certainly not the case in some instances at least, notably Sirius.

As the results derived and collected by Lewis are of prime importance in this connection we have examined their general accuracy and have also collected other determinations of mass-ratios made by various computers. These ratios have sometimes been derived by comparing the dimensions of the relative orbit with the oscillations in the meridian places of the brighter component. It is obvious that results derived in this way are not very reliable unless the period is short and the separation wide, for systematic corrections to the early catalogues and the instrumental constants are not known wlth the requisite accuracy. In a few cases both stars have been observed with the meridian circle, and the masses thus derived are a little more trustworthy. Still better are the determinations based on micrometrical measurements of a third star in close proximity to the binary. It is a pity that more data of this kind were not provided by early observers, and it is to be hoped that observers of the present day will recognize the importance of making such measures. Our conclusion is that while most of these mass-ratios are very uncertain, it does not seem at all probable that future investigations on these systems will reverse the rule that the fainter star is usually the more mas-

* *Astrophysical Journal* 6, 326, 1897.
† *Astrophysical Journal* 17, 165, 1903.

sive of the pair unless it is more advanced in type. It is certainly true that the fainter star is much more massive than we should infer from photometric considerations alone.

Let m, D, and κ represent respectively the mass, the mean density and the mean luminosity for a unit of surface of the brighter star; m_s D_s and κ_s similar quantities for the fainter star, and Δ the difference in magnitude. These quantities may be connected by the equation

$$\frac{m_s}{m} = \left(\frac{\kappa}{\kappa_s}\right)^{\frac{3}{2}} \frac{D_s}{D} \ (0.251)\Delta.$$

Let us apply this to the system of ζ Herculis, in which the blue star is about equal in mass to the yellow star, the latter being three magnitudes brighter. We obtain

$$\left(\frac{\kappa}{\kappa_s}\right) \cdot \left(\frac{D_s}{D}\right)^{\frac{2}{3}} = 16.$$

Now since the masses are about equal it is not likely that the mean densities differ very much; indeed if there is any difference in this respect we should expect the yellow star to be the more dense. There seems then to be no escape from the conclusion that in a system like this *the luminosity for each unit of surface is greater for the yellow star than for the blue.* There are other systems, notably 85 Pegasi, to which an application of this reasoning would yield even stronger indications in the same direction. We regard such systems as forming a striking confirmation of the view that the photosphere of a solar type star may be hotter and brighter than that of an earlier type.

Huggins' explanation for the evolution of double stars, extended as we have here indicated, can certainly not be regarded as complete, but the observational material at present available would seem to raise against it no fatal objection. In our opinion it forms the most plausible working hypothesis within our reach, and the best base from which to pursue further investigations.

February 24, 1910