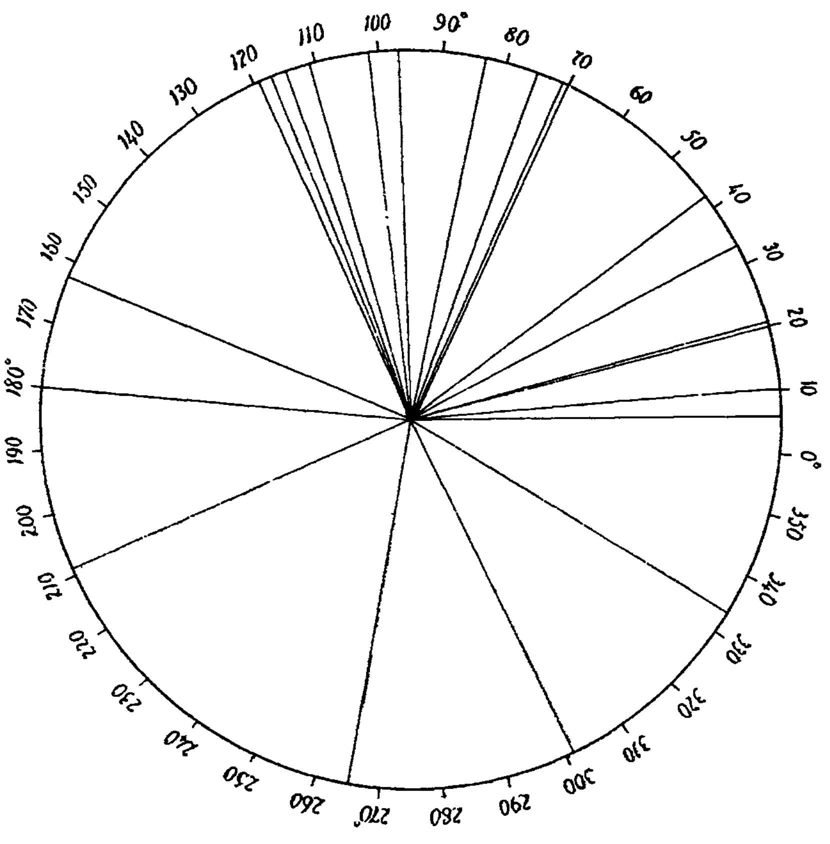
THE ORBITS AND "VELOCITY-CURVES" OF SPECTROSCOPIC BINARIES

BY J. MILLER BARR

I.

IN looking over the published papers on spectroscopic binaries, it will be remarked that the "velocity-curves"—as hitherto drawn for these objects—are often unsymmetrical. A closer examination reveals a curious *general similarity* in the form of



The radial lines represent the values of ω, (*i.e.*, the computed longitude of periastron), for twenty-three spectroscopic binaries. See accompanying table.

such curves—the *ascending* branch of the curve, with few exceptions, being of greater length than the *descending* branch. This fact, although of great theoretical interest, seems to have been hitherto overlooked by astronomers. Its significance will be apparent from an inspection of the diagram and table given below.

LIST OF SPECTROSCOPIC BINARIES.

STAR	P	e	ω	K	REFERENCE
	d			km.	
α Andromedæ	100. ?	. . .	[0° - 180°]	. . .	*Astroph. Jour.*, *20*, 146, 1904.
Polaris	3·9683	. . .	[0 – 180]	3·0	" " *14*, 52, 1901.
β Arietis	107.0	0·88	19·7	32·6	" " *25*, 320, 1907.
Capella	104·022	0·016	117·3	25·76	" " *14*, 263, 1901.
η Orionis	7·9896	0·016	42 16′	144·75	" " *17*, 68, 1903.
δ Orionis	5·7325	0·103	33 19	100·85	" " *19*, 268, 1904.
ζ Tauri	138·	0·180	9 45	14·95	" " *22*, 115, 1905.
γ Geminorum	[3·5y ?]	. . .	[0° – 180°]	. . .	" " *22*, 84, 1905.
ζ Geminorum	10·154	0·22	333·	13·2	" " *13*, 90, 1901.
α_2 Geminorum	9·2188	0·503	265·35	13·56	" " *23*, 351, 1906.
κ Cancri	6·393	0·149	162 16	67·8	" " *25*, 315, 1907.
α Carinæ	6·744	0·18	115·84	21·5	" " *26*, 268, 1907.
κ Velorum	116·65	0·19	96·23	46·5	" " *26*, 271, 1907.
η Virginis	71·9	0·254	180 0	26·8	" " *26*, 282, 1907.
Mizar	20·6	0·502	101·3	142·	" " *13*, 324, 1901.
α Draconis	51·38	0·43	19·	48·±	*Jour. R.A.S.C.*, *1*, 237, 1907.
X Sagittarii	7·0118	. . .	[0° – 180°]	. . .	*Astroph. Jour.*, *25*, 330, 1907.
Y Ophiuchi	17·1207	0·10	209·2	8·5	" " *25*, 330, 1907.
W Sagittarii	7·5946	0·320	70·0	19·5	" " *20*, 149, 1904.
μ Sagittarii	180·2	0·441	74 43	64·5	" " *26*, 157, 1907.
χ Draconis	281·8	0·423	119·0	18·15	" " *11*, 131, 1900.
β Lyræ	12·908	0·07	83·4	181·05	" " *6*, 328, 1897.
U Aquilæ	7·0240	. . .	[0° – 180°]	. . .	" " *25*, 330, 1907.
η Aquilæ	7·176	0·489	68·91	20·59	" " *9*, 59, 1899.
S Sagittæ	8·3832	. . .	[0° – 180°]	. . .	" " *20*, 231, 1904.
θ Aquilæ	17·17	0·725	20·	52·±	*Jour. R.A.S.C.*, *1*, 357, 1907.
T Vulpeculæ	4·4358	0·43	111·	17·6	*Astroph. Jour.*, *25*, 330, 1907.
δ Cephei	5·366	0·46	[0° - 180°]	20·5	" " *1*, 160, 1895.
η Pegasi	818·0	0·155	5·61	14·20	" " *14*, 20, 1901.
λ Andromedæ	20·546	0·086	301·0	7·07	" " *24*, 345, 1906.

NOTE.—The elements refer to the *brighter* component of each system, except in the case of Mizar (*ζ Ursæ Majoris*), where the value of K is that of the *relative* orbit.

The table includes all stars (so far as known to the writer) for which the "velocity-curve" appears to be certainly unsymmetrical. It contains, for each star, the computed elements P, e, ω and K, except in a few cases where such data are not available.*

* For *α Andromedæ* and *γ Geminorum* only the general form of the oscillation.

Of the thirty stars included in this table there are but *four* for which the values of ω (as calculated) lie between 180° and 360°. For these four stars we have in each case,

$$D > I,$$

where D denotes the time-interval during which the star's "radial velocity" is *decreasing*; I the interval during which it is *increasing* (algebraically). For the remaining 26 stars $D < I$, except in the case of *η Virginis* (bright component), where $D = I$, corresponding to $\omega = 180°$.*

The apparent grouping of the periastron about certain values of ω is a yet more striking feature, which is clearly shown in the annexed diagram. That such a distribution of the apses really exists is, of course, very improbable—so improbable that we are certainly justified in seeking for a different explanation of the observed facts. In other words, the elliptic elements, e and ω, as computed and published for the orbits under notice, are probably illusory; the "observed radial velocities," upon which they are based, being vitiated by some neglected source of systematic error.†

It now remains to point out the probable nature of this source of error. Two distinct hypotheses are suggested, viz.:

(1) The spectrum-lines, for the stars under notice, are

curve has been found. The elements of *U Aquilæ* and *X Sagittarii* are as yet unpublished. In the case of *δ Cephei*, Belopolsky's value for ω (272°·3), as printed in the *Astrophysical Journal* (February 1895), is erroneous. For the benefit of readers unversed in this subject it may be added that P denotes the period of revolution; e the eccentricity; ω the longitude of periastron, reckoned from the ascending node; K the "single amplitude," $2K$ the total range in the star's "radial velocity."

* Excluding those stars for which $e \lesseqgtr 0·10$, we find:

$D = I$ for one star,
$D > I$ " 2 stars,
$D < I$ " 21 "

† The period P may of course be relied upon; and it is satisfactory to note that this element has in many cases been determined with a high degree of precision. In cases such as that of Capella, for which the oscillation-curve is almost symmetrical, the computed values of K and $a \sin i$ are doubtless nearly correct.

periodically shifted from their normal positions, owing to exceptional conditions of pressure or temperature in the star's photosphere, or its surrounding atmosphere.

(2) The disks of the stars under notice are not uniformly bright. The distribution of surface-brightness in *longitude* is for each star, unequal, and for some stars, distinctly unsymmetrical. Such conditions, combined with rapid axial rotation, would result in a more or less unsymmetrical broadening of the spectral lines. The *effective* result would be a periodic shift of these lines, as measured on the spectrograms.

The possible source of error referred to in (1) has been made the subject of careful investigation.* In a few cases, such as that of *Mira Ceti*, there is evidence that a "physical shift" of certain lines in the star-spectrum does actually occur.† On any rational theory, however, it is very unlikely that physical causes would give rise to *periodic* displacements, affecting *in a similar degree* the positions of several or many lines (due to various elements) in the spectrum of a star.

The second hypothesis rests upon a much more substantial basis. It was suggested by a perusal of Dr. Albrecht's paper on "A Spectrographic Study of the Fourth-Class Variable Stars *Y Ophiuchi* and *T Vulpeculæ*."‡ In that paper, Dr. Albrecht calls attention to a most important relation which exists between the light- and velocity-curves of *δ Cephei* variables. "In every observed case," he remarks, "light-maximum and greatest velocity of approach occur within one-fifteenth of the period of each other. Likewise minimum brightness and greatest velocity of recession occur at the same time."

*Among recent papers dealing with this subject are those of Humphreys (*Astrophysical Journal*, *26*, 18, 297, 1907), Larmor (*Ibid.*, *26*, 120) and Duffield (*Ibid.*, *26*, 375). The researches of Julius on "dispersion-bands," should be considered; also the possibility that a shift of the spectral lines may result from electrical or magnetic conditions in the stellar atmosphere.

†See papers by Campbell (*Astrophysical Journal*, *9*, 31, 1899), Stebbins (*Ibid.*, *18*, 341, 1903) and Plaskett (*Jour. Roy. Ast. Soc. Can.*, *1*, 45, 1907).

‡*Astrophysical Journal*, *25*, 330, 1907.

An inspection of the accompanying table will show that it includes eight variable stars of the *δ Cephei* type.* Their orbits, according to the published elements, are more or less decidedly elliptic—the computed values for the eccentricity *e* varying from 0·10 in the case of *Y Ophiuchi* to 0·489 in that of *η Aquilæ*. If we assume that these orbits are in reality nearly circular,† it would appear that the observed facts—as graphically summarized in the light- and "velocity"-curves—may be explained on the second hypothesis outlined above. The unsymmetrical distribution of light on the discs, as postulated for these stars, is probably due to tidal action, modified by an unequal angular rotation in different latitudes, such as exists in the solar photosphere.

Accepting this theory, we must suppose that the *preceding* side of the revolving star is, on the whole, more luminous than the opposite hemisphere.‡ A similar state of things seems to exist in certain variable stars of the Algol type—notably *S Cancri*, *U Coronæ*, *δ Libræ*. For such stars, the rise from minimum to maximum brightness is less rapid than the fall from maximum to minimum. This fact would seem to admit of only one probable explanation, which is in harmony with our present theory, viz., that the *advancing front* of such a star, as it traverses its orbit, is more luminous than the *rear* side.§

Further evidence tending in the same direction is afforded by certain facts of observation, which are here summarized :

* The light-curve of one of these stars, viz., *W Sagittarii*, is apparently subject to distinct changes in form. According to Schmidt's observations (1866–76) it was formerly of the *δ Cephei* type; but the Harvard observations of 1898 give a light-curve in which the *decrease* of light is more rapid than the *increase* (*H. C. O. Annals*, *46*, Part II).

† This assumption, for theoretical reasons, seems highly probable when the shortness of the periods is considered (Cf. Darwin and See on the theory of tidal evolution).

‡ Cf. Curtiss (*Astrophysical Journal*, *20*, 186, 1904), Albrecht (*Ibid.*, *25*, 330, 1907), and Loud (*Ibid.*, *26*, 369, 1907).

§ Strictly speaking, this remark applies only to that hemisphere of the star which is turned towards the earth about the time of minimum brightness.

(*a*) Distinct irregularities occur in the "velocity-curves" of certain variable stars, and these correspond with inequalities in the *light*-curves of the stars.

(*b*) Broad, unsymmetrical lines have been noted in the spectra of several binaries—notably *δ Orionis.*

(*c*) In the interesting case of *η Virginis*,* Ichinohe has obtained *dissimilar* velocity-curves from measures of spectrograms taken, respectively, with full dispersion and with a single-prism spectrograph. This remark applies to the brighter member of the system. For the faint component, a *double* curve having the same period (71·9 days) has been found. Moreover, the deduced "radial velocity of the centre of gravity" is, for the bright component, −0·4 km., and for the fainter star +30 km.—a most significant difference.

The further discussion of this interesting subject is reserved for a future paper. In the latter I hope to deal with certain details of the theory here advocated,† and to offer some hints concerning practical methods for the separation of effects due, respectively, to axial rotation and orbital revolution of the stars under notice.

ST. CATHARINES, ONT.,
February 10, 1908.

NOTES ON MR. BARR'S PAPER

I. BY W. F. KING

IT seems remarkable that out of thirty orbits measured, only four should have their periastron nearer to us than the focus, a circumstance the odds against which, on the hypothesis of random arrangement, are of course very great.

Observed facts are, that the curve obtained by plotting the radial velocities as ordinates, with the times for abscissas, accords

* *Astrophysical Journal*, *26*, 282, 1907.

† One interesting consequence of this theory may be noted here, viz., that many—perhaps all—spectroscopic binaries are variable in brightness; though the range of this periodic light-change must, in general, be small.

well in form, at least in the great majority of orbits, with what it should be on the hypothesis of simple elliptical motion, and that this curve changes little, if at all, even after many periods. It is hardly to be supposed that such powerful tidal action as it is necessary to assume should result in a change of the apparent velocity in one part of the orbit, amounting to many kilometres per second, and still leave the general velocity curve so closely in accordance with theoretical form. Again, this powerful tidal action ought to have an effect in altering the elements of the orbit after not many complete periods; such change has not been observed.

The alternative supposition is that the displacement of the lines occurs, not in connection with the place of the body in its orbit, but from some cause not connected therewith. Under such supposition, what is the probability that in so large a proportion of the total number of cases, it should deceive us in the same way?

II. By J. S. Plaskett

Unsymmetrical Velocity Curves (First Paragraph).

Velocity curves are never true sine curves or symmetrical, except when $e = o$.

The main answer to the views expressed in the paper I think lies in the fact that the velocity curves for many of the stars under discussion have been determined from measures of plates made with high-dispersion spectrographs on spectra of the second type, which admit of very accurate measurement.

The probable error of velocity determinations is in most cases not more than a kilometre, leading to a very accurate drawing of the velocity curve. In every case the ephemeris or velocity curve drawn from the computed elements, (assuming a simple elliptic orbit without perturbations or disturbances), agrees very closely with the observations. If there were any such secondary effects as Mr. Barr speaks of it is highly improbable, to say the least, that their effect combined with the action in the orbit, presumably elliptical, would result in a velocity curve corresponding, within the errors of observation, to a simple elliptic motion of the bright star around the centre of gravity of the system.

Hypothesis (*2*) (*Page* 73).

"Rapid axial rotation." This presumably refers to more rapid rotation than the period of revolution, and the secondary curves thereby introduced would be irregular, so that the resultant curve due to elliptic motion and the axial rotation of the non-uniformly lighted star would be very irregular in outline and could not possibly result in changing the value of ω. Such has not appeared in any published curve. If on the other hand we assume a period of rotation and revolution equal, then, unless the dimensions of the body are comparable with the dimensions of the orbit not much change would occur in the resultant velocities. Further, if the *preceding* side is more luminous, would not the changes in velocity thereby produced be similar on each side of the point of zero velocity (when the star is moving parallel to the line of nodes) and result only in increasing K without changing ω appreciably?

Finally.

It must be remembered that only the inclination of the axis to the line of sight is known and it may lie in any direction whatever in the conical angle thus described. Furthermore, if the absolute directions in space are considered, it is evident that there can be no such grouping as is shown in the paper.

For example, if two stars about 12 hrs. apart have the same ω then their apses are pointed in opposite directions. Probably if all the known binaries were plotted on a globe with cones corresponding to the direction of their apses this direction would be found quite irregular and without any preponderance, other than accidental, towards any one direction.

SUPPLEMENTARY NOTE BY MR. BARR.

In dealing with a new and difficult subject, such as is brought forward in my paper on "The Orbits and 'Velocity-Curves' of Spectroscopic Binaries," it was almost inevitable that some slips and omissions should occur. Thus, in my original MS., the wording of paragraph "(2)," which defines the influence of axial rotation for a revolving star, was imperfect. This paragraph has, therefore, been carefully amended. As regards my tabular list of binaries, the case of Mizar was regarded as exceptional; the components being roughly equal in brightness and the orbital elements merely relative.* Hence it is uncertain whether the computed value of ω should be 101°·3 (as given by Vogel) or 281°·3. This fact was, unfortunately, overlooked when my table and the accompanying diagram were being prepared. The interesting case of *ι Orionis* (for which provisional elements were computed at the Dominion Observatory) also escaped my notice (*Jour. R. A. S. C.*, Nov.–Dec., 1907, p. 373).

Some results derived from a later investigation of this subject are given below (Cf. *Popular Astronomy*, April, 1908, p. 259):

Let us take the case of a rotating star which revolves in a circular orbit about the centre of gravity of the system; the axis of rotation being perpendicular to the orbit-plane, and the star rotating in the same direction as it revolves. Let us further assume that the star's surface-brightness at any point depends upon the *height of the tide* (due to a revolving satellite) at that point; *high* tide corresponding to minimum and *low* tide to maximum surface-brightness. (The axis of symmetry is here supposed to coincide with the radius vector). On these assumptions, our general expression† for the star's *apparent radial*

* Mizar must now be regarded as a quadruple system, a fourth component having recently been brought to light at the Yerkes Observatory. (*Astrophysical Journal*, March 1908, p. 166).

† In this formula i^1 is the inclination of the star's equator-plane and ν the longitude of its ascending node with reference to the "tangent-plane" from which i is reckoned. ν is measured from the ascending node of the *orbit-plane*, *i.e.*, the point for which $v + \omega = o$.

velocity (V_s), viz.,

$$V_s = V_o + V_I \left\{ e \cos \omega + \cos (v + \omega) \right\} + V_2 \varphi(e, i, i^I, \nu, v, \omega)$$

reduces to the simple form

$$V_s = V_o + V_I \cos \theta + V_2 F(\theta),$$

where V_o is the radial velocity of the centre of gravity of the system; $V_I = \frac{2a\pi}{P} \sin i$ is the *constant of orbital motion*, and $V_2 = \frac{2r_o\pi}{P^I}$ $\sin i$ is the *constant of axial motion*; r_o being the star's equatorial radius, P^I its rotation-period (which is here regarded as the same for all latitudes), and θ the orbital longitude, reckoned from the ascending node. The form of the function denoted by $F(\theta)$ is difficult to define, depending as it does upon various complex conditions—including the absorption of light by the stellar atmosphere, as modified by tidal deformation. It is easily seen, however, that $F(\theta)$ is *positive* in the first quadrant and *negative* in the second quadrant of longitude; being numerically smaller for corresponding positions in the third and fourth quadrants. Also, $F(\theta) = 0$ when θ is 90° or 270°, and

$$F(180° - \theta) = -F(\theta),$$
$$F(360° - \theta) = -F(180° + \theta),$$

where θ is taken in the first quadrant.

I have, tentatively, drawn several velocity-curves which correspond to the second formula given above. These theoretical curves are of the general type represented in my paper by the relation $D < I$. They show one peculiarity which should be noted here, viz., a more or less perceptible "hump" or secondary maximum in the longer or ascending branch of the general curve. Similar features are shown in the oscillation-curves of certain variable stars.

The influence of tidal friction is now under investigation. It will suffice to state here that a *lagging* of the stellar tidal wave (such as exists in terrestrial tides) would give rise to an oscillation-curve in fair general agreement with curves derived from spectrographic observations.

The known facts relating to the δ *Cephei* variables and to some Algol variables (as cited in my paper) make it almost certain that the surfaces of these stars are by no means of uniform brightness. For such stars, therefore, the *axial motion effect* (*i.e.*, the displacement of spectral lines due to the star's rotation) is, in all probability, quite appreciable. Thus the theory under notice may be said to "rest upon a substantial basis" (*Popular Astronomy*, *loc. cit.*)

Through the Editor's courtesy I am enabled to reply here to the critical notes of Dr. King and Mr. Plaskett, as printed above.

Dr. King seems to recognize fully the weight of the observational evidence upon which the main argument in my paper may be said to rest. But he evidently attaches greater weight to the known accordance of observation and received theory, as regards the form of the stellar oscillation-curves. I have no desire to under-estimate the force of this objection; but I think there is a natural tendency to attach undue weight to the general accordance here referred to. There are *four* independent quantities—P, $a \sin i$, e, ω—which determine the outline of the theoretical curve; and in computing an orbit by the method of least squares we practically *select* and *combine* these quantities in such a manner as to secure the *best possible agreement* between our curve and the observed "radial velocities."

Dr. King does not allude to the significant *discordances* between theory and facts which, in several important cases, (η *Virginis*, ζ *Geminorum*, *W Sagittarii*, etc.) have been found to exist. As regards the "powerful tidal action" to which he refers, it seems only necessary to point out that the consequent changes would, on the theory of simple elliptic motion, be *greater* than they would be according to the writer's hypothesis.

Turning now to Mr. Plaskett's note, it will be seen that he lays great stress upon the precision of modern spectrographic measurements. It is perhaps unnecessary to say that the writer appreciates most fully the splendid progress which has been made in this field. This progress is well exemplified by recent contributions from the Lick, Yerkes, Potsdam, and Dominion Observatories.

The argument based on the agreement of received theory with observation has already been answered in my reply to Dr. King.

In dealing with the writer's theory Mr. Plaskett says: "the resultant curve due to elliptic motion and the axial rotation of the non-uniformly lighted star would be very irregular in outline and could not possibly result in changing the value of ω." I have failed to find any basis for this statement, which is presumably due to a hasty consideration of the subject.

There is only one other point in Mr. Plaskett's criticism that seems to call for a reply. This relates to the computed positions of the periastron. My remarks on this subject had reference only to the *apparent* grouping of the periastron-points, as measured from the ascending node—a condition which Mr. Plaskett seems to have misunderstood.

It should be added that many spectroscopic binaries are now under investigation, so that further evidence bearing on this interesting subject will soon be available.

April 5, 1908.