SOME POSTULATES OF COSMOLOGY

It is clearly not true that every statement can be proved or disproved; for there are many propositions which are wholly outside of the domain of evidence. In the foundations of every system of thought there lie, and there must lie, assumptions which cannot be proved, and words that cannot be defined. These assumptions and undefined concepts constitute the very essence of the system and define its character, if we grant that the logic is faultless. They are not, in general, made at random, for otherwise the system would reduce to a mere exercise in logic and would command but a very limited interest. They arise naturally from our experience and from our efforts to coordinate the facts of experience. Many of these assumptions and undefined concepts are buried so deep in our childhood experience and early training as to entirely escape our notice, and unconsciously they shape the picture which we call our philosophy. Insofar as we can do so it is very desirable for us to recognize what is assumption in this picture and what is inference. Able thinkers will differ little in the logical processes, but they will differ widely in the assumptions and concepts with which the logical processes deal.

Certain of these assumptions have an antithetical character. Either they are true or their antitheses are true. For example, the physical universe is either infinite in extent, or it is finite. It has existed always, or it has not existed always. It always will exist, or it will not, etc. Physical evidence can never tell us which of these assumptions we should make; and since this is so we can choose among them in such a way as pleases us. The decision is an esthetic, not a scientific one.

A certain philosopher A may be pleased with the assumption that the physical universe is infinite in extent, and may be very successful in constructing a scientific system which incorporates this assumption. Philosopher B may prefer a finite universe, and may also be successful in the construction of a scientific system in harmony therewith. We may fancy A and B entering into a contest in the way of interpreting nature. At one stage of the contest A has an harmonious explanation of everything in sight and is smiling contentedly. Philosopher B is in trouble and perplexed, for certain phenomena obstinately stand out against harmony. The crowd of onlookers *believes* that A is right in his assumptions, and that B is wrong. Suddenly, however, some meddlesome experimenter or observer discovers a series of facts hitherto unknown which fits perfectly into B's scheme of thought, and brings harmony into that domain which had been discordant. Philosopher A, however, begins to feel uncomfortable. He had not thought of these things before, and there is no place in his scheme for them. He is annoyed and worried — and likewise his adherents. Philosopher B has scored a great victory, and his theory is now in accordance with all of our experience with nature. The onlookers observe this fact, shift their notions, and now *believe* that it is B who has made the correct assumptions.

And thus the contest goes on. Sometimes one is in the lead, sometimes the other; and at times one is so far in the lead that the other is even forgotten. But it must be evident to the careful thinker that it is not a question of which is right and which is wrong. It is altogether a question of success; and in this matter of success the elements of coordination of experience, simplicity and intelligibility will naturally be weighted differently by different thinkers.

Einstein has recently succeeded in constructing a system of thought which is perhaps in closer agreement with experience than our previous schemes; and this system has led him to the somewhat startling conclusion that the physical universe is finite. From a philosophical point of view his system of thought would gain much in clarity if the type of finiteness which he desires was embedded in the system as a simple postulate and some of the assumptions with which he starts were arrived at as conclusions. At least such an inversion would remove from many minds the notion that the finiteness of the universe had been proven. Einstein is the philosopher B who has forged ahead of the philosophers A of the old school, and the feat is the more surprising in that the idea of a finite universe had dropped so far behind in the contest as to be almost forgotten.

This success does not mean that we must all stampede to B's point of view. Attracted by the simplicity and beauty of the system, many will undoubtedly do so; and continued success on the part of B and continued difficulties on the part of A may eventually lead us all to adopt B's point of view as being, for the time at least, the more profitable one. But those who prefer to think of the physical universe as infinite will undoubtedly continue to work in their own system of thought, and no one can say that they are wrong. For some reason, which doubtless depends upon our own mental constitutions, the idea of an infinite universe has to many minds a charm and a beauty that will not readily be given up.

Whether we wish to choose a finite universe or an infinite one it is evident that its size must far transcend all human experience, and it is natural to ask whether the portion of it which we observe is fairly representative, or whether it is essentially peculiar. We might imagine, for example, that the galaxy is only a condensation region of stars, and that as we recede from the galaxy in any direction whatever the number of stars per unit volume decreases and approaches the limit zero at infinity. Such a universe would be infinite both physically and spatially, but the galaxy would be an essentially singular region in it. Our position near the center of the galaxy would certainly be a peculiar one, and we would have the right to wonder just how it happened that we are at the very center of things. Notwithstanding the fact that this is the way things appear to be, and, if light suffers no loss in its transmission through space, the blackness of the night sky is entirely in harmony with it, it is by no means *necessary* to suppose that this appearance in any way corresponds with the reality. There may be galaxies other than ours, and galaxies upon galaxies without limit. Moulton has shown how infinitely many galaxies can be arranged in space so that no one of them occupies an essentially peculiar position and yet the total quantity of light received from them may be as small as we please. In all such arrangements, however, the mean stellar density of a sphere (i. e., the number of stars within the sphere divided by the volume of the sphere) approaches zero as the radius of the sphere increases indefinitely. There is a certain degree of artificiality about these arrangements, but so also is there an artificiality about any arrangement whatever if perhaps we except the conception of a rough uniformity in the distribution of matter through space. There is no reason why we should ascribe any particular structure to the universe other than a more or less uniform distribution of matter provided we do not insist upon the assumption that radiant energy is transmitted through space without loss. It is true that a roughly uniform distribution of stars throughout all space would make the entire sky as bright as the disk of the sun provided light is transmitted without loss, so that both of these assumptions cannot be made at one and the same time. But if we choose to make the assumption that the stars are more or less uniformly distributed throughout all space we can lay aside the assumption that light travels without any loss, and there are no further difficulties.

Seeliger[1] has attempted to show that the assumption of a uniform distribution of stars throughout all space cannot be reconciled with a law of gravitation which varies strictly as the inverse square of the distance. The argument is as follows: The attraction of a sphere of constant density for a particle upon its surface is directly proportional to the radius of the sphere. If we imagine the sphere to become infinitely great the attraction at its surface also becomes infinitely great which is physically impossible. Therefore a uniform distribution of stars throughout infinite space is impossible. But Seeliger's conclusion is illegitimate. It is true that the attraction on the surface of such a sphere would tend towards infinity as the radius of the sphere tended towards infinity, but where in a boundless universe shall we find such a surface. Obviously, it does not exist: and Seeliger's argument is vacuous.

It is much better to focus our attention upon a definite region of space and to consider the resultant attraction of an infinite, uniformly dense, physical universe upon a particle in this region. Imagine the particle within a spherical shell of which the thickness and inner radius are so great that the shell may be considered uniform in density. Then, as Newton proved, the resultant attraction of the shell upon any particle within it is zero. Since this proposition is true however great may be the thickness of the shell, it is clear that the assump-

[1] Quoted by EINSTEIN in his recent book *Relativity*, Chapter XXX.

tion of a rough uniformity in the distribution of matter and the assumption of Newton's law of gravitation will give rise to no apparent physical difficulties. The only gravitational acceleration to which any particle is subject is due to the irregularity in the distribution of matter in the neighborhood of the particle itself. Were the distribution of matter in an infinite universe mathematically uniform throughout, gravitational acceleration would not exist. It is only the irregularities in its distribution that makes the action of gravitation manifest. The assumption of such a universe seems to be entirely legitimate.

The postulate that the distribution of matter throughout all space is roughly uniform does not imply a mere random distribution of stars and nebulae. It merely implies that the mean density of any spherical volume tends towards a finite limit that is not zero as the radius of the sphere tends towards infinity. Without doubt the mean density of different spheres of the same finite size, but widely separated in space, may differ greatly. Consider, for example, three spheres of the size of the earth, of which the first has its center at the center of the earth, the second includes an average portion of the sun, while the third is half way between the earth and the moon. The mean density of the first sphere is five and one half times the density of water; that of the second is one and four tenths times the density of water; while that of the third is very nearly if not quite zero. But if each of these spheres were expanded to a radius of 4,500 million kilometers each would include the entire solar system and their mean densities would be substantially the same. If they were still further expanded to the size of the galaxy the three spheres would be indistinguishable. The same properties will hold for any system of spheres wherever they may be located provided only that they are sufficiently expanded. The postulate merely says that the limit of the mean density is not zero.

Within that portion of the universe which man has been able to explore the organization of matter into various types of physical units is characteristic. It is not chaotic, nor is it ever perfectly uniform. Two leaves from the same tree are never exactly the same; two individuals of the same species are noticeably different; and it is probable that two molecules of water would be distinguishable upon a sufficiently fine examination. Notwithstanding this capacity for unlimited variation, organization and uniformity pervades the whole. At least science assumes that it does, and the modern doctrine of evolution is merely a postulate that this organization and uniformity persists in time, and that the duration of this persistence is unlimited.

To the mathematician it is axiomatic that size is only relative. An object is large or small only in comparison with other objects. « Absolute size » does not have any meaning. Notwithstanding this axiom of the mathematicians, scientists whose work is largely experimental or observational frequently fall into the error of thinking of their units in absolute terms. That an electron is very small and that a galaxy is very great they are willing to admit, presumably because they have been taught so or because they have become accustomed to think in terms of such units. But that an electron could be as large and as complicated as we conceive the galaxy to be would be merely an amusing idea; it would have little contact with reality.

It will be useful, perhaps, to construct a mathematical model of a universe to illustrate the relativity of size. Suppose we take 27 units of what we will call ordinary matter and arrange them in the form of a cube, the distance between the centers of adjacent units being one meter, and take the center unit as the origin of a coordinate system. Then the coordinates of these units will be (0,0,0), (0,0,1), (0,1,1,), (1,1,1), (-1,0,0) . . ., etc., 27 in all. We will call this system of 27 units of ordinary matter a *planet*. Now build around the original planet as a center a cube consisting of 27 planets altogether, with a distance of 10 meters between the centers of adjacent planets. This new unit of 27 planets we will call a *star*. About the original star as a center build a cube of 27 stars, with a distance between them of 100 meters. This new unit is a *star cluster*. A cube of 27 star clusters forms a unit of the next higher order which we call a *galaxy*. A cube of 27 galaxies is a *super-galaxy;* and so on without limit. In such a universe there are infinitely many units of ordinary matter, infinitely many planets, infinitely many stars, infinitely many units of every order: and no one of them occupies an essentially peculiar position. The coordinates of every unit of ordinary matter in this universe are integers, positive or negative; but not all integers are admissible, only those which contain no other digits than 0,1,8 and 9, and every integer expressible with these digits, except those in which the final significant digit is 8, is admissible. Thus, 9189 and 1111 are admissible, but 1980, 1108 are not admissible. Let us say that every such integer is of class A. Then it is true that at every point in space whose three coordinates are of class A there is a unit of ordinary matter, and at no other points. Every point whose three coordinates are « class A multiples » of 10 meters is the center of a planet. Every point whose coordinates are « class A multiples » of 100 meters is the center of a star, and so on.

We imagine further that each unit of ordinary matter is a cube of 27 molecules, the distance between adjacent molecules being .1 meter. Each molecule is a cube of 27 atoms .01 meter apart; each atom is a cube of 27 electrons, and so on without limit. Our imaginary universe is thus built out of an infinite sequence of physical units which differ from one another only in size. There is no largest unit, nor is there any smallest one. An inhabitant of an atom at the origin of our coordinate system using a centimeter as his measuring rod would find that all of the other atoms were at class A points, and in no way would the universe as he saw it differ from the universe as it appeared to an inhabitant of a unit of ordinary matter who used the meter as his measuring rod; and the same remark would hold for the inhabitant of an electron, or a star, or a galaxy, or any other unit located at the origin of coordinates. The universe would look exactly alike to all of them. Absolute size would have no meaning; and the inhabitant of a planet would have no difficulty in understanding that an electron would look pretty large to an inhabitant who was ten units further down the scale.

Of course the universe which nature presents to us for study does not possess the geometrical regularity of our imaginary universe. The various physical units of nature have other properties than mere size. It may be true that no two specimens of the same unit are exactly alike, nevertheless there is an idealized unit which serves as a norm for our thoughts. At the present time we have concepts, more or less definite, of such units ranging in size from an electron to a galaxy. We may imagine that there is an infinite sequence of smaller and smaller units; and that there is no smallest unit, just as there is no smallest fraction. Indeed, a smallest unit would seem to imply a unit which was not itself organized, and therefore a unit without properties. Nor need we suppose that a galaxy is the largest physical unit, nor that there is any largest unit whatever. There is no more reason for supposing that man is located in the center of the scale of physical units (as he seems to be) than there is for supposing that he is located in the center of space (as he also seems to be) unless we suppose that the center is everywhere.

Likewise, units of time are large or small only in comparison with other units. It is entirely logical to suppose that an inhabitant of an electron would live an entire lifetime, that would seem to him as long as ours does to us, in a minute fraction of a second; while another creature, to whom our galaxy would be a mere molecule, might require ten thousand times a million billion years in which to wink his eye, and even then think that he had been very quick about it. The physicist through his studies of the electrons has made us familiar with relatively short intervals of time; and it is the business of the astronomer through his studies of the larger units to make us familiar with intervals which to us seem very long. But it is only a « seeming ». Our consciousness is adapted to the rate of transformation of energy in our scale of the physical units. The units are neither long nor short in themselves, but only as we compare them with one another.

The contemplation of very extended intervals of time suggests the question as to whether the universe is being transformed progressively in one direction, or whether the transformations are of such a nature that the universe, considered from the large point of view, remains essentially unaltered. By way of illustration, we might suppose that under the law of gravitation matter is being constantly concentrated into larger and larger physical units and that the energy liberated in the process is being dissipated with equal constancy. The process would be an unending one in a universe in which there was an infinite sequence of physical units, but it would be a transformation always in one direction. This has been the assumption underlying nearly every system of cosmogony in the past, although it should be admitted that all of the cosmogonies of the past have been local cosmogonies in the sense that they have dealt only with the organization and development of particular units such as the sun, or the solar system, or even the galaxy itself.

A second illustration is contained implicitly in the second law of thermodynamics of the physicists. According to this law, which of course is based upon physical experiments, the available energy of the universe is always decreasing. According to the first law of thermodynamics, a postulate which commands almost universal assent, the total energy remains unaltered; but according to the second law the *available* energy is always decreasing, and this of course means a progressive change of the universe in one direction. According to Lord Kelvin the universe is a mechanism which is running down. Planck gives « the most general statement of the second law of thermodynamics » as follows: « Every physical or chemical process in nature takes place in such a way as to increase the sum of the entropies of all of the bodies taking any part in the process. In the limit, i. e., for reversible processes, the sum of the entropies remains unchanged ». This law is the foundation of the science of thermodynamics and it finds its justification in a vast mass of physical and chemical experience. The ability it gives us to predict the transformations that will occur under certain circumstances in physics and chemistry entitles it to the dignity of a « law ». Unfortunately the entropy of a solid body cannot in general be evaluated, but the entropy of a gas is given by the formula $M\ (A \log t + B \log v)$, where M is the mass of the gas, t is its temperature, v is its volume, and A and B are two numbers which depend upon the nature of the gas.

Notwithstanding the value of the second law of thermodynamics as a working rule in physics and chemistry, where the systems dealt with are closed (finite), it has no validity for the astronomers and cosmologists who deal with systems that are not closed. As an example of this distinction imagine a vessel of warm gas in an otherwise empty infinitude of space. If the surface enclosing the gas were an adiabatic surface (that is, one across which no energy passes) then nothing would happen according to the above formula to alter the entropy, for both the volume and the temperature would remain constant. But if the surface of the gas were not adiabatic then the temperature would be reduced owing to the radiation of heat. The volume of the gas would remain constant and therefore the entropy would decrease. There are no other bodies to be considered, for, by hypothesis, no other bodies exist. Therefore the entropy of such a universe would not increase nor remain stationary as it should do in accordance with the second law of thermodynamics.

There is another aspect of the matter which would be of interest to a cosmologist, though not to a physicist or a chemist who admits the existence only of the physical units with which he is familiar, molecules, atoms, and electrons. Let us suppose that there exists an infinite sequence of smaller and smaller units which are organized and suitably endowed with energy. It is quite conceivable that with the lapse of sufficient time a vessel of gas enclosed in an adiabatic surface might have its entropy reduced by a reduction of an appreciable fraction of its atoms to units of lower orders provided the energy of the lost atoms was added to the energy of the units of the lower orders, and therefore not manifest in the form of temperature. Such a system is again an open system, for the system of physical units is not a closed one, nor is there any limit upon the time element.

There are three stages in any scientific enquiry: *a*) What are the elements which exist in the given field? *b*) What is the general character of the relationships among these elements? and *c*) What are the quantitative expressions of these relationships? As knowledge increases the demand for mathematical formulae also increases, and we are sometimes impressed with the idea that the goal of science is a mathematical formula. Even the attainment of this goal however is but half of the struggle. The mathematical formulae themselves require interpretation. The symbols must stand for something that can be realized in thought. There must be mental images which correspond, and it is only when we have adequate mental pictures, the elements of which satisfy the mathematical formulae, that we say we *understand*. No one, for instance, would say that he understands gravitation, notwithstanding its mathematical expression is ideal in simplicity and accuracy.

The interpretation of these unambiguous mathematical relations however is not unique. Every interpretation rests upon postulates and undefined concepts, and it seems quite clear that there are as many possible interpretations as there are combinations of independent postulates, the postulates themselves being beyond physical evidence. It is in this sense that science in general, and cosmology in particular, is not unique.

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