ABERRATION OF STARS AND THE DEVIATION OF THE WAVES BY THE MOVEMENT OF THE PROPAGATION MEDIUMS

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SOMMAIRE. — ***L'auteur démontre que l'hypothèse de l'entraînement de l'éther est incompatible avec la loi de l'aberration telle qu'on l'observe. Il rappelle d'ailleurs les démonstrations classique et relativiste de la formule usuelle de l'aberration.***

SUMMARY. — ***The author shows that the carried along ether hypothesis is not compatible with the law of aberration as observed. He records the classical and the relativistic demonstrations of the usual aberration formula.***

Резюме. **— Автор показывает, что гипотеза увлечения эфира несовместима с законом аберрации' каким его наблюдают. Автор напоминает классические и релятивистские доказательства обычной формулы аберрации.**

It is often proposed, to explain the negative result of the experiment of MICHELSON – and other experiments intended to determine the movement of the earth by optic or electrical means – the assumption of the entraining of the propagation medium of the waves (ether) by the Earth around the apparatuses being used for the experiments.

This assumption encounters many difficulties, and in particular, with those which provoke the phenomenon of *aberration*. Traditional interpretation was that this phenomenon revealed the movement of the observatory compared to the ether, *by admitting that this medium was by no means entrained*.

The theory of relativity also gives an account of it, as discussed below.

Proponents of the hypothesis of "entrained ether" nevertheless maintain that it is compatible with the aberration [1]: in effect, if the light rays coming from a star pass from intersidereal ether, presumably motionless, with the "entrained" ether which surrounds the Earth, it is obvious that they undergo a deviation while passing from the one with the other. Is this deviation precisely equal to that which is observed? It is the question which we propose to solve.

THE THEORY OF ABERRATION

Above all, it is important to have a correct interpretation of aberration.

To make the theory of this phenomenon there is no need to speak about ether, as was often done before Relativity. However, if one compels oneself to speak only about relative movements, one has insurmountable difficulties, owing to the fact that the stars themselves have large movements, often badly known, and especially owing to the fact that the positions and speeds which would have to be considered are those which these stars had a few years (or often a few thousands of years) prior to this point in time when they emitted the light which we observe….

Whatever are these elements, and whatever are the movements of the star since that time, it has been the light arriving at our telecopes which is the reality recorded by these instruments, and experiments show that the direction of the light coming from a star is appreciably the same during a whole year (whatever the point of its orbit where the Earth is) compared to a frame of reference S formed by the Sun and the directions of stars known as "fixed".

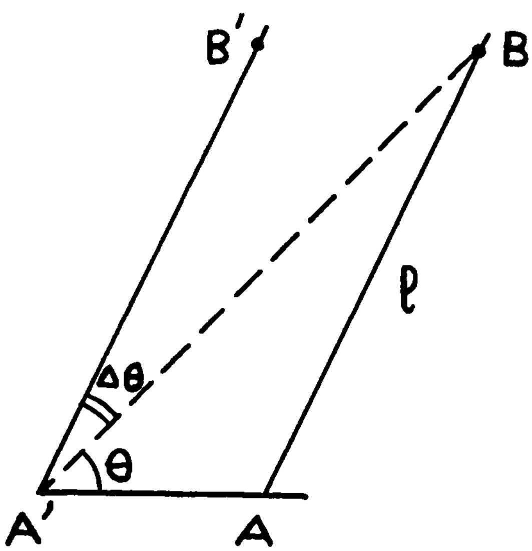

FIG. 1.

Thus let us take this system S as our reference, and AB as a telescope of length l (in which A is the eyepiece, and B is the objective) whose optical axis is directed on a star. Let us say that c is the speed of light and v is the speed of the point of the Earth where the observatory is located. The light of this star arrives initially at B, then, after a very short time, $\Delta t = l/c$, it arrives at the eyepiece which has moved to A', such that AA' = $v\Delta t = vl/c$.

The "true" direction of the light coming from star compared to S is that of right-hand side BA'. The apparent direction, i.e. that of the telescope, is AB (or A'B', after time l/c).

That is to say, θ is the angle AA'B (the angle of the direction of the light ray with the speed of the observer at the point considered). The aberration is the angle B'A'B, or A'BA = $\Delta\theta$.

In triangle ABA', one has

$$\frac{l}{\sin\theta} = \frac{\mathrm{AA}'}{\sin\Delta\theta}$$

Hence, by merging the arc $\Delta\theta$ with its sine,

$$\Delta\theta = \frac{v}{c}\sin\theta.$$

This demonstration is, with a margin of a few details, the most traditional demonstration.

In fact, one does not note a direction AB, or A'B', in the system S, but a direction of the telescope, noticed compared to the frame of reference of the Earth observatory, which is moving relative to S.

However, in two frames of reference moving one relative to another, the directions are the same ones under the old mechanics, but not according to the theory of relativity (to them, *time* is different). We say that the differences are negligible, to the first order in v/c. But this assertion is a little abstract (and perhaps debatable, owing to the fact that there are trajectories traversed with the speed of light). It seems more rigorous to proceed by analytical geometry, where we can realize in a precise way, by calculation, the elements which we are led to neglect.

Consider a light wave (or a photon) arriving at time zero at the origin of the abscissa with the angle θ compared to the system S. The equations of its movement are, in the system S

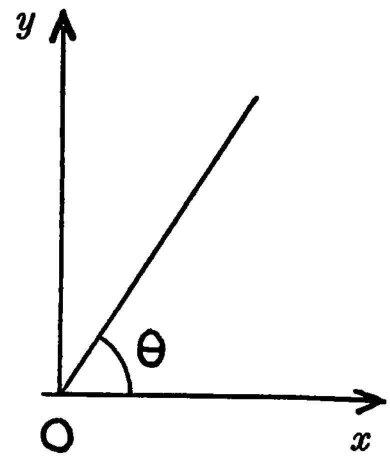

FIG. 2.

(1) $$x = -ct\cos\theta$$

(2) $$y = -ct\sin\theta .$$

Let us seek the angle of this same movement in a system S', having compared to S speed $-v$ along the x axis. One has $x' = \dfrac{x+vt}{\sqrt{1-v^2/c^2}}$, or, while neglecting v^2/c^2

(3) $$x' = x + vt$$

(4) $$y' = y .$$

Let us eliminate x, y and t from 1), 2), 3) and 4).

It becomes

$$x' = \frac{cy'\cos\theta - vy'}{c\sin\theta} \quad \text{or} \quad \frac{y'}{x'} = \frac{\sin\theta}{\cos\theta - v/c}$$

It is the equation of a line of slope $\tan\theta' = \dfrac{y'}{x'}$ or, while posing $\theta' = \theta + \Delta\theta$ and merging $\Delta\theta$ with its tangent,

$$\frac{\tan\theta + \Delta\theta}{1-\Delta\theta\tan\theta} = \frac{\sin\theta}{\cos\theta - v/c} .$$

Hence we draw

$$\Delta\theta = \frac{(v/c)\sin\theta}{1-(v/c)\cos\theta} .$$

By neglecting the terms in v^2/c^2, we find

(5) $$\Delta\theta = \frac{v}{c}\sin\theta .$$

It is the same formula as shown above. It gives very significantly the measurement of aberration as observed by astronomers.

Of course, an observation immediately does not provide the value of $\Delta\theta$ as a deviation starting from the true angle θ which is then unknown, but the whole of the observations during the year gives the variations of the angle observed around a median value, which one accepts as being the angle θ.

The aberration is maximum for $\theta = \pi/2$. It is then (v/c) what for v = 30 km/sec (speed of the Earth on its orbit) and c = 300,000 km/sec that gives 1/10,000 of a radian, that is to say approximately 20.5". This is called the *constant of the aberration*.

THE REALIGNMENT OF A WAVE BY THE MOVEMENT OF THE MEDIUM [2]

Let us consider waves related to a medium, such as mechanical or acoustic waves (and one related to EINSTEIN, the light waves). If a wave passes from a medium to another, moving compared to the first, there is a kind of refraction of the waves passing from one to the other.

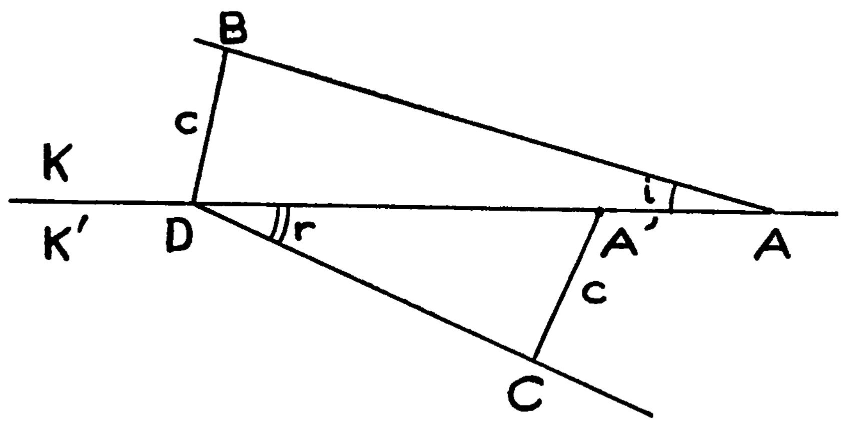

FIG. 3.

Let us consider a plane wave AB at time zero, being propagated with speed c in the medium K, and arriving under the angle of incidence i at the surface of separation with another medium K', where the propagation velocity is the same, but which is animated at speed v along this surface.

That is to say A, a point where this wave touches the surface of separation, and B a point such as BD = c.

At the end of time t = 1 the wave will pass by the point D, but the point of the medium K' which was at time zero in A will be then in A' such as AA' = v, and the line DC trace of the wave at time t = 1 is tangent in C with the circle of radius c and center A'.

Let us call angle r, ADC.

One has

$$AD = AA' + A'D$$

thus

$$\frac{c}{\sin i} = v + \frac{c}{\sin r}$$

from where

$$\sin r = \frac{c \sin i}{c - v \sin i}.$$

If v is very small compared to c, and if i does not equal $\pi/2$ the deviation is very low, and if one poses $\varepsilon = r - i$, one has, while merging $\sin\varepsilon$ with ε and $\cos\varepsilon$ with 1

$$\sin i + \varepsilon \cos i = \frac{\sin i}{1-(v/c)\sin i} = \sin i\left(1 + \frac{v}{c}\sin i + ...\right).$$

Hence, by neglecting the terms in v^2/c^2

(6) $$\varepsilon = \frac{v}{c}\sin i \tan i.$$

This formula shows that there is always a deviation, except if $i = 0$ (waves parallel at the surface of separation, i.e. perpendicular light rays).

If several layers are superimposed, the deviations are added to each surface of separation.

We can consider that the transition from one medium to another can be done via several parallel layers, with very low speeds $v_1, v_2 ... v_n$ of one layer to the next one. We then can, for calculation, regard i as appreciably constant and the deviation is on the whole (V/c) sin i tan i by calling V speed $v_1 + v_2 + \ldots + v_n$.

Generalizing, we can apply this formula to a medium of which speed passes gradually from 0 to V by ongoing changes.

COMPARISON BETWEEN ABERRATION AND THE DEVIATION BY THE MOVEMENT OF THE MEDIUMS

Formulas 5) (*Aberration*) and 6) (*Deviation*) contain v/c multiplied by trigonometric lines, and it is certain that the bringing them together is difficult.

Let us first note that if the ether was entrained by the Earth in the vicinity of the observatory, there would be no “aberration” itself with the passage through the telescope, in the sense that the optical axis of the instrument would be, exactly, and without deviation, in the same direction as the light ray which arrives there: Any deviation, would thus take place *before* this arrival, at the crossing surfaces (or the layers) of separation between motionless ether and entrained ether.

If we assume – which seems likely, at least for certain observatories and certain positions of the Earth in its orbit – that the surface (or surfaces, or layers) of separation are parallel to the plane of the ecliptic, the angles i and θ are *complementary*. The two formulas, $v/c \sin\theta$ and $v/c \sin i \tan i$, are thus incompatible.

In particular, the deviation by the movement of the mediums would be null for $i = 0$, which would correspond to $\theta = \pi/2$, i.e. with the maximum aberration.

Let us note, finally, that there is in the study of this deviation, a curious phenomenon which is akin to "total reflection", well-known in the traditional study of "refraction" itself.

Indeed, starting from a critical angle $i = \lambda$ such as $\frac{c}{\sin\lambda} = c + v$ or $\sin\lambda = \frac{c}{c+v}$ the construction of Figure 3 is no longer possible, and the waves cannot penetrate from one medium to another. This angle corresponds to the maximum deviation for speeds c and v given.

For c = 300,000 k/sec and v = 30 k/sec, this deviation would be about the size or the degree (more exactly 0°48', or $0^{Gr}90$) what is much higher than all the noted "aberrations".

In the case considered above, where the surfaces of separation are parallel to the plane of the ecliptic, there would be in the vicinity of the ecliptic, a zone of the celestial sphere where the stars would be invisible on Earth.

All of this does not correspond to anything in the astronomical observations such as they occur and are recorded in observatories.

In short, the phenomenon of the aberration is incompatible with the assumption of entrained ether.

The results of the study above about the deviation of the waves by the movement of the mediums, if they do not apply to the light, are nevertheless valid for all the waves which are related to propagation mediums. Such are the mechanical waves which occur in solid, liquid or gas bodies and in particular to sound waves and ultrasounds.

Manuscript accepted on March 5th, 1959.

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