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Newton's Second Law for Special Relativity

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Summary: The formula ***F=dP/dt*** or so-called Newton's second Law is wrong and is not applicable to the body with variable mass.

The following expression for the force acting upon the body could be found in many scientific papers:

$$F = \frac{dP}{dt} \qquad (1)$$

In relativistic physics where the mass of the body is not a constant such definition could be expanded to:

$$F = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

But the formula (1) is totally incorrect. Sometimes this relation called Newton's Second Law. This is also incorrect. Newton's Second Law states that:

$$F = ma \qquad (2)$$

The validity of expression (1) for the force could be verified very easily. Let's start with the momentum definition:

$$P = mv$$

Now we are differentiating both left and right part of this equation:

$$\frac{dP}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

The velocity derivative with respect to time is equals to acceleration also by definition and we got:

$$\frac{dP}{dt} = ma + v\frac{dm}{dt}$$

Using Newton's Second Law we obtain:

$$F = \frac{dP}{dt} - v\frac{dm}{dt} \qquad (3)$$

And we derived the right expression for the force acting upon the body of variable mass. Unfortunately many research papers are using incorrect formula for the force (1). [1], [2], [3]

Derived expression could be simplified to the expression of our interest (1) **only** in the case of constancy of the mass. It is a mistake to use expression (1) for calculation which contains variable mass such as in relativistic physics.

Also correct expression for the force (3) is giving an answer to following paradox. There are two cannons on the asteroid aimed in the opposite direction. We are firing two cannonballs absolutely synchronously. The reference frame chosen in such a way that asteroid is not moving. Nothing will change after firing. The momentums of cannonballs are opposite to each other and the momentum of asteroid will remain unchanged.

Very interesting things happens if we change our frame of reference. The asteroid is moving now with the speed of $\boldsymbol{V}$. The momentum is equals to $\boldsymbol{MV}$. The momentum of asteroid will be changed after shooting due to decreasing its mass by the mass of two cannonballs. Using incorrect formula (1) one could decide that there is a force acting upon asteroid. The direction of this force should be along to velocity vector and the sign is negative, making it braking force.

The paradox is resolved using correct formula (3).

References:

[1] Wikipedia, https://en.wikipedia.org/wiki/Four-force

[2] Ideal Rocket Equation, NASA, https://spaceflightsystems.grc.nasa.gov/education/rocket/rktpow.html

[3] Lecture Notes on Special Relativity.
http://physics.mq.edu.au/~jcresser/Phys378/LectureNotes/VectorsTensorsSR.pdf