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Gooey Ducks and Naked Physicists

Part XI
Line Dancing (or Square Dancing?) With Descartes

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Abstract: An allegory of modern science.

Part XI

The other day, I came across a movie on Hulu (or was it YouTube) called "The Stranger" (Agantuk). In the timeless, eternal charm of India, a mysterious, long-lost uncle returns to visit his family after years of traveling the world. And in the shade of a large tree, as his young nephew and friends listen entranced, the uncle begins to tell a story about truth, mathematics, and astronomy...

Uncle: *This time I'll show you a trick. I'll ask you a few questions. Let's see if you can answer them correctly. Tell me, which is large, Moon or Sun?*

The children reply: *Sun.*

Uncle: *How do you know? Just a moment. (The uncle places two coins of the same size on the ground in front of him.) Suppose this is Moon…and this is Sun. In the sky, they look the same size, don't they?*

Children: *That's because the Sun is much further away.*

Uncle: *How much? I'll tell you. The Sun is 95 million miles away. While the Moon's distance is only five hundred thousand miles.*

Children: *That's why, Sun looks bigger.*

Uncle: *Suppose the Moon had been 200,000 miles away…*

Children: *Then it would have looked bigger.*

Uncle: *(Replaces one of the coins with a bigger coin.) Like this. Isn't that so?*

Children: *Yes.*

Uncle: *And suppose the Moon had been 800,000 miles away…*

Children: *Then it would have looked smaller.*

Uncle: *(Replaces the larger coin with a smaller coin.) Somewhat like this, isn't it? But neither happens. The Moon is just so far away that makes it look the same size of Sun. (Replaces smaller coin with original size coin.) That's why, when the Moon takes its position in front of the Sun and slowly covers it…(places one coin over the other)…disc matching disc.*

Children: *Solar Eclipse!*

Uncle: *Yes, total eclipse of Sun. And when the earth's shadow falls on the Moon... then also the two discs match perfectly.*

Children: *Lunar Eclipse!*

Uncle: *Total eclipse of Moon! What d'you think made that possible?*

(Silence–children all look questioningly at each other.)

Uncle: *You don't know. Ask the wisest man on earth and even he will say, he doesn't know. Nobody knows. It's a mystery!*
I say it's one of the greatest mysteries of the universe.
Sun and Moon. King of the day! Queen of the night!
And the shadow of Earth on Moon...all exactly the same size.
Magic!

It's hard to imagine the unbelievably, precise mathematics for the Moon, the Sun, and the Earth's shadow on the Moon to appear to be exactly the same size as seen from the Earth.
What a challenge for science: God's astrophysics equation–I wonder what it would be?
An equation involving light, size of orbit, and size of sphere...a new take on gravity?
Wow! That's heavy! I look at the "old" equations of theoretical physics:

- Kepler's proportion of time squared, t^2, and distance cubed
- Newton's law of universal gravitation, $\frac{m_1 m_2}{d^2}$ or $\frac{m^2}{d^2}$
- Lorentz' time dilation, $L = L_0\sqrt{1 - v^2/c^2}$
- Einstein's, $E = mc^2$
- the equations for falling bodies, $v^2 = v_0^2 - 2gy$ and $d = \frac{1}{2}gt^2$
- and the equation for velocity squared, $v^2 = \frac{d^2}{t^2}$.

Mother Earth, Brother Sun, and Sister Moon? All I see is a common fascination with the concept of squaring (of line, time, mass, velocity, etc.)–just a wild blend of *Square-ometry*!

Uncle: What happens when the square comes between science and the light of truth?
Children: Total Eclipse of the Square? No! Total Eclipse of Science!

So I guess this brings me back to "square" one: Descartes' "proof" of squaring of line.
Does it really exist? Is the squaring of line (time, velocity, and mass) really possible?

In his book *Geometry*,

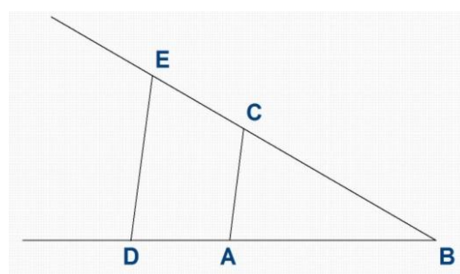
I could easily find where Descartes explains his system of mathematics for line:

> *Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction. Thus to add lines…I call one a and the other b, and write a+ b. Then a–b will indicate that b is subtracted from a; ab that a is multiplied by b; a/b that a is divided by b; aa or* a^2 *that a is multiplied by itself…*
> *Here it must be observed that by* a^2*,* b^3*, and similar expressions, I ordinarily mean only simple lines, which, however, I name squares, cubes , etc., so that I may make use of the terms employed in algebra.*

Wow! Dividing a line by another line to produce a line, and multiplying a line by another (or squaring a line) to produce a line? Yeah, Descartes sure had a radical mathematical concept! But proof? I looked and looked, and looked again; I couldn't find it anywhere!

Hmm…maybe I just missed it. I think I'll take a closer look at the diagrams.
In the first diagram, Descartes illustrates his concept of multiplying and dividing line:

> *Let AB be taken as unity, and let it be required to multiply BD by BC. I have only to join the points A and C, and draw DE parallel to CA; then BE is the product of BD and BC. If it be required to divide BE by BD, I join E and D, and draw AC parallel to DE; then BC is the result of the division.*

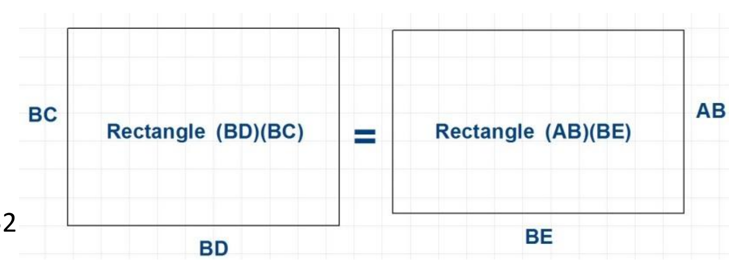


I recognize this! Descartes is using Thales' (or the intercept) theorem! No sweat!
By crossing an angle with parallel lines, Descartes constructs similar triangles whose sides are proportional, which is the same as creating equations for rectangles of equivalent areas!

$$(BD)(BC) = (AB)(BE), \quad or \quad \frac{BE}{BD} = \frac{BC}{AB}$$

Yeah, if I multiply lines BD and BC, I get a rectangle of area equal to that of rectangle AB, BE!

So that's it? What Descartes is calling multiplying, dividing, or squaring of line is just your normal, everyday area squaring? Wow!



But maybe I'm putting "Descartes" before "de horse"…(Sorry, Eva, I couldn't resist.)

In the diagram, by defining the line AB as representing unity (the number one), Descartes converts Thales' classic diagram and "similar triangle/equivalent rectangle" proportional equation of $(BD)(BC) = (AB)(BE)$, into the equation:

$$(BD)(BC) = (1)(BE), \quad or \quad \frac{BE}{BD} = \frac{BC}{1}$$

Ah, there's the mathematical sleight of hand! Alamazoo! Alacazam!
Descartes makes the line AB *appear* to disappear geometrically by calling it the number one!

Yeah, now you see it, now you don't!
By having the multiplication of lines BD and BC equal one multiplied by line BE, the result is made to appear to be simply line BE.

But in reality, as the line AB always represents one, the result of multiplying BD by BC is always going to be AB multiplied by BE–the area of a rectangle, not a line!

So Descartes really hasn't cut the lady in half! Whew!
He hasn't "squared" line, and line AB hasn't disappeared. Quite the opposite!

By making line AB the unit one, it becomes the identity element that defines the number values of all the other lines and ends up–like the ghost in the machine, or the invisible man–part of every calculation within the diagram! Line AB: it's the number one!

Every line, then, becomes a number and every number a line–a multiple of line AB.

Now I get it! Descartes is seeing his numbers as lines! Amazing!

So with all the lines representing numbers, all the classic diagrams become number equations (and, in essence, mini-calculators), and when Descartes "squares" line, he's never squaring line, he's squaring number and area!

Ay, Caramba! Multiplying and dividing numbers by using *geometry*?
That's truly inspired!

So Descartes hasn't broken from classic Euclidian geometry at all!
He just likes his geometry with a twist–like I like my martinis, "shaken, not stirred"!

Twenty-three

I had a teacher who told me it was possible to represent math in any way with any symbol. But an algebra of lines–where a length of line represents the basic numerical unit of one, all lines represent numbers, and lines can also represent unknowns? Impressive!
Hats off to Descartes: the original cipher-meister!
What a story! A New York Times bestseller…"The Descartes Code," an intellectual thriller:

> *A mathematical mystery protected for centuries by a secret society…which could shake the very foundations of science.*
>
> *While in Paris on business, renowned Cambridge mathematician Emil Schaffhausen receives an urgent late-night phone call: The curator of the City of Science and Industry has disappeared… and so begins the quest to crack the Descartes Code!*

Yeah, codebreaking 101! Descartes' "secret code"? His mathematical invisible ink? No sweat! By using a line as his base symbol to represent numeric values, you don't see the numbers you just see the lines!
Reminds me of the computer's binary code–where the two base symbols 0 (zero) and 1 (one) don't represent their actual numerical values, but are used to express any numerical value. But Descartes ingeniously uses only one symbol–a line–to represent any number!
Astounding! One symbol! I wonder if there's a name for that?
I looked on the internet…there is! Ooh! The unary system!

> *A base-1 numeral system…to represent a number N, an arbitrarily chosen symbol representing 1 is repeated N times. For example, the numbers 1,2,3,4,5,…would be represented as 1,11,111,1111,11111,…*

Ah, counting fingers and toes–the first math! Everything based on the first digit–numero uno! But Descartes breaks the mold of the ordinary unary system. He tallies by placing his symbol for one sidewise in a line, and declares his tally mark (the line) to be infinitely adjustable! Ooh, you can't help but appreciate the simple beauty of it! Any length of line can represent one, and one line can represent any number to infinity.
Wow! Descartes has devised the ultimate mathematical symbol to signify number: the line!

(Drum roll) Move over Stretch Armstrong and Mr. Fantastic!
Introducing Descartes–the world's greatest elastic superhero! He can stretch his tally mark (the line representing one) from here to eternity! Now that's a stretch!