Universal Reference Frame

Alejandro A. Torassa

(2013) Buenos Aires, Argentina
atorassa@gmail.com

Abstract

In classical mechanics, this paper presents the universal reference frame.

Universal Reference Frame

The universal reference frame is a reference frame fixed to the center of mass of the universe.

The position $\mathring{\mathbf{r}}_a$, the velocity $\mathring{\mathbf{v}}_a$, and the acceleration $\mathring{\mathbf{a}}_a$ of a particle A of mass m_a relative to the universal reference frame $\mathring{\mathrm{S}}$, are given by:

$$\mathring{\mathbf{r}}_a = \int\int (\mathbf{F}_a/m_a)\; dt\; dt$$

$$\mathring{\mathbf{v}}_a = \int (\mathbf{F}_a/m_a)\; dt$$

$$\mathring{\mathbf{a}}_a = (\mathbf{F}_a/m_a)$$

where $\mathbf{F}_a$ is the net force acting on particle A.

From the above equations the following equations are obtained:

$$\begin{array}{ccc}
\boxed{m_a\mathring{\mathbf{r}}_a - \int\int \mathbf{F}_a\; dt\; dt = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_a\mathring{\mathbf{r}}_a^2 - {}^1\!/_2\, m_a(\int\int (\mathbf{F}_a/m_a)\; dt\; dt)^2 = 0} \\
\downarrow & & \downarrow \\
\boxed{m_a\mathring{\mathbf{v}}_a - \int \mathbf{F}_a\; dt = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_a\mathring{\mathbf{v}}_a^2 - \int \mathbf{F}_a\; d\mathring{\mathbf{r}}_a = 0} \\
\downarrow & \nearrow & \downarrow \\
\boxed{m_a\mathring{\mathbf{a}}_a - \mathbf{F}_a = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_a\mathring{\mathbf{a}}_a^2 - {}^1\!/_2\, m_a(\mathbf{F}_a/m_a)^2 = 0}
\end{array}$$

where ${}^1\!/_2\, \mathring{\mathbf{v}}_a^2 = \int \mathring{\mathbf{a}}_a\; d\mathring{\mathbf{r}}_a \;\rightarrow\; {}^1\!/_2\, m_a\mathring{\mathbf{v}}_a^2 = \int m_a\, \mathring{\mathbf{a}}_a\; d\mathring{\mathbf{r}}_a \;\rightarrow\; {}^1\!/_2\, m_a\mathring{\mathbf{v}}_a^2 = \int \mathbf{F}_a\; d\mathring{\mathbf{r}}_a$

Reference Frame

The position $\mathring{\mathbf{r}}_a$, the velocity $\mathring{\mathbf{v}}_a$, and the acceleration $\mathring{\mathbf{a}}_a$ of a particle A of mass m_a relative to a reference frame S, are given by:

$$\mathring{\mathbf{r}}_a = \mathbf{r}_a + \mathring{\mathbf{r}}_S$$

$$\mathring{\mathbf{v}}_a = \mathbf{v}_a + \mathring{\omega}_S \times \mathbf{r}_a + \mathring{\mathbf{v}}_S$$

$$\mathring{\mathbf{a}}_a = \mathbf{a}_a + 2\,\mathring{\omega}_S \times \mathbf{v}_a + \mathring{\omega}_S \times (\mathring{\omega}_S \times \mathbf{r}_a) + \mathring{\alpha}_S \times \mathbf{r}_a + \mathring{\mathbf{a}}_S$$

where $\mathbf{r}_a$, $\mathbf{v}_a$, and $\mathbf{a}_a$ are the position, the velocity, and the acceleration of particle A relative to the reference frame S; $\mathring{\mathbf{r}}_S$, $\mathring{\mathbf{v}}_S$, $\mathring{\mathbf{a}}_S$, $\mathring{\omega}_S$, and $\mathring{\alpha}_S$ are the position, the velocity, the acceleration, the angular velocity, and the angular acceleration of the reference frame S relative to the universal reference frame $\mathring{\text{S}}$.

The position $\mathring{\mathbf{r}}_S$, the velocity $\mathring{\mathbf{v}}_S$, the acceleration $\mathring{\mathbf{a}}_S$, the angular velocity $\mathring{\omega}_S$, and the angular acceleration $\mathring{\alpha}_S$ of a reference frame S fixed to a particle S relative to the universal reference frame $\mathring{\text{S}}$, are given by:

$$\mathring{\mathbf{r}}_S = \int\int (\mathbf{F}_0/m_s)\; dt\; dt$$

$$\mathring{\mathbf{v}}_S = \int (\mathbf{F}_0/m_s)\; dt$$

$$\mathring{\mathbf{a}}_S = (\mathbf{F}_0/m_s)$$

$$\mathring{\omega}_S = \left|(\mathbf{F}_1/m_s - \mathbf{F}_0/m_s)/(\mathbf{r}_1 - \mathbf{r}_0)\right|^{1/2}$$

$$\mathring{\alpha}_S = d(\mathring{\omega}_S)/dt$$

where $\mathbf{F}_0$ is the net force acting on the reference frame S in a point 0, $\mathbf{F}_1$ is the net force acting on the reference frame S in a point 1, $\mathbf{r}_0$ is the position of the point 0 relative to the reference frame S (the point 0 is the center of mass of particle S and the origin of the reference frame S) $\mathbf{r}_1$ is the position of the point 1 relative to the reference frame S (the point 1 does not belong to the axis of rotation) and m_s is the mass of particle S (the vector $\mathring{\omega}_S$ is along the axis of rotation)

On the other hand, the position $\mathring{\mathbf{r}}_S$, the velocity $\mathring{\mathbf{v}}_S$, and the acceleration $\mathring{\mathbf{a}}_S$ of a reference frame S relative to the universal reference frame $\mathring{\text{S}}$ are related to the position $\mathbf{r}_{cm}$, the velocity $\mathbf{v}_{cm}$, and the acceleration $\mathbf{a}_{cm}$ of the center of mass of the universe relative to the reference frame S.

Kinetic Force

The kinetic force $\mathbf{K}_{ab}$ exerted on a particle A of mass m_a by another particle B of mass m_b, caused by the interaction between particle A and particle B, is given by:

$$\mathbf{K}_{ab} = \frac{m_a m_b}{m_{cm}} (\mathring{\mathbf{a}}_a - \mathring{\mathbf{a}}_b)$$

where m_{cm} is the mass of the center of mass of the universe, $\mathring{\mathbf{a}}_a$ and $\mathring{\mathbf{a}}_b$ are the accelerations of particles A and B relative to the universal reference frame $\mathring{\mathrm{S}}$.

From the above equation it follows that the net kinetic force $\mathbf{K}_a$ acting on a particle A of mass m_a, is given by:

$$\mathbf{K}_a = m_a \mathring{\mathbf{a}}_a$$

where $\mathring{\mathbf{a}}_a$ is the acceleration of particle A relative to the universal reference frame $\mathring{\mathrm{S}}$.

From page [1], we have:

$$m_a \mathring{\mathbf{a}}_a - \mathbf{F}_a = 0$$

That is:

$$\mathbf{K}_a - \mathbf{F}_a = 0$$

Therefore, the total force $(\mathbf{K}_a - \mathbf{F}_a)$ acting on a particle A is always in equilibrium.

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