Now,Gauss himself suggested that cyclotomic numbers might be useful for the kind of research he was involved with, but he never really pursued this direction seriously.

In cyclotomic numbers Jiang discovered automorphic functions.

Automorphic functions (Complex hyperbolic function) and proofs of Fermat last theorem are the greatest mathematical discovery that was ever made

In 1991 Fermat Last Theorem Has Been Proved(II)

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Abstract

In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. Fermat proved FLT for exponent 4.Therefore Fermat proved his last theorem.

In this paper using automorphic functions we prove FLT for exponents $4P$ and P, where P is an odd prime. We rediscover the Fermat proof. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{4m-1} t_i J^i\right) = \sum_{i=1}^{4m} S_i J^{i-1}, \qquad (1)$$

where J denotes a $4m$ th root of unity, $J^{4m} = 1$, m=1,2,3,…, t_i are the real numbers.

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S_i is called the automorphic functions(complex hyperbolic functions) of order $4m$ with $4m-1$ variables [2,5,7].

$$S_i = \frac{1}{4m}\left[e^{A_1} + 2e^H \cos\left(\beta + \frac{(i-1)\pi}{2}\right) + 2\sum_{j=1}^{m-1} e^{B_j} \cos\left(\theta_j + \frac{(i-1)j\pi}{2m}\right)\right]$$

$$+\frac{(-1)^{(i-1)}}{4m}\left[e^{A_2} + 2\sum_{j=1}^{m-1} e^{D_j} \cos\left(\phi_j - \frac{(i-1)j\pi}{2m}\right)\right] \qquad (2)$$

where $i = 1, \ldots, 4m$;

$$A_1=\sum_{\alpha=1}^{4m-1} t_\alpha,\quad A_2=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha,\qquad H=\sum_{\alpha=1}^{2m-1} t_{2\alpha}(-1)^\alpha,\quad \beta=\sum_{\alpha=1}^{2m} t_{2\alpha-1}(-1)^\alpha,$$

$$B_j=\sum_{\alpha=1}^{4m-1} t_\alpha\cos\frac{\alpha j\pi}{2m},\quad \theta_j=-\sum_{\alpha=1}^{4m-1} t_\alpha\sin\frac{\alpha j\pi}{2m},$$

$$D_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha\cos\frac{\alpha j\pi}{2m},\quad \phi_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha\sin\frac{\alpha j\pi}{2m},$$

$$A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)=0. \qquad (3)$$

From (2) we have its inverse transformation[5,7]

$$e^{A_1}=\sum_{i=1}^{4m} S_i,\quad e^{A_2}=\sum_{i=1}^{4m} S_i(-1)^{1+i}$$

$$e^H\cos\beta=\sum_{i=1}^{2m} S_{2i-1}(-1)^{1+i},\quad e^H\sin\beta=\sum_{i=1}^{2m} S_{2i}(-1)^i,$$

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}\cos\frac{ij\pi}{2m},\quad e^{B_j}\sin\theta_j=-\sum_{i=1}^{4m-1} S_{1+i}\sin\frac{ij\pi}{2m},$$

$$e^{D_j}\cos\phi_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}(-1)^i\cos\frac{ij\pi}{2m},\quad e^{D_j}\sin\phi_j=\sum_{i=1}^{4m-1} S_{1+i}(-1)^i\sin\frac{ij\pi}{2m}. \qquad (4)$$

(3) and (4) have the same form.

From (3) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=1 \qquad (5)$$

From (4) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=\begin{vmatrix} S_1 & S_{4m} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix}$$

$$=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{4m-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{4m-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & (S_{4m})_1 & \cdots & (S_{4m})_{4m-1} \end{vmatrix} \qquad (6)$$

where

$$(S_i)_j=\frac{\partial S_i}{\partial t_j}\ [7]$$

From (5) and (6) we have circulant determinant

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=\begin{vmatrix} S_1 & S_{4m} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix}=1 \qquad (7)$$

Assume $S_1 \neq 0, S_2 \neq 0, S_i = 0$, where $i = 3,...,4m.$ $S_i = 0$ are $(4m-2)$ indeterminate equations with $(4m-1)$ variables. From (4) we have

$$e^{A_1}=S_1+S_2, \quad e^{A_2}=S_1-S_2, \quad e^{2H}=S_1^2+S_2^2$$

$$e^{2B_j}=S_1^2+S_2^2+2S_1S_2\cos\frac{j\pi}{2m}, \quad e^{2D_j}=S_1^2+S_2^2-2S_1S_2\cos\frac{j\pi}{2m} \qquad (8)$$

Example [2]. Let $4m=12$. From (3) we have

$A_1=(t_1+t_{11})+(t_2+t_{10})+(t_3+t_9)+(t_4+t_8)+(t_5+t_7)+t_6$,

$A_2=-(t_1+t_{11})+(t_2+t_{10})-(t_3+t_9)+(t_4+t_8)-(t_5+t_7)+t_6$,

$H=-(t_2+t_{10})+(t_4+t_8)-t_6$,

$B_1=(t_1+t_{11})\cos\frac{\pi}{6}+(t_2+t_{10})\cos\frac{2\pi}{6}+(t_3+t_9)\cos\frac{3\pi}{6}+(t_4+t_8)\cos\frac{4\pi}{6}+(t_5+t_7)\cos\frac{5\pi}{6}-t_6$,

$B_2=(t_1+t_{11})\cos\frac{2\pi}{6}+(t_2+t_{10})\cos\frac{4\pi}{6}+(t_3+t_9)\cos\frac{6\pi}{6}+(t_4+t_8)\cos\frac{8\pi}{6}+(t_5+t_7)\cos\frac{10\pi}{6}+t_6$,

$D_1=-(t_1+t_{11})\cos\frac{\pi}{6}+(t_2+t_{10})\cos\frac{2\pi}{6}-(t_3+t_9)\cos\frac{3\pi}{6}+(t_4+t_8)\cos\frac{4\pi}{6}-(t_5+t_7)\cos\frac{5\pi}{6}-t_6$,

$D_2=-(t_1+t_{11})\cos\frac{2\pi}{6}+(t_2+t_{10})\cos\frac{4\pi}{6}-(t_3+t_9)\cos\frac{6\pi}{6}+(t_4+t_8)\cos\frac{8\pi}{6}-(t_5+t_7)\cos\frac{10\pi}{6}+t_6$,

$$A_1+A_2+2(H+B_1+B_2+D_1+D_2)=0, \quad A_2+2B_2=3(-t_3+t_6-t_9). \qquad (9)$$

From (8) and (9) we have

$$\exp[A_1+A_2+2(H+B_1+B_2+D_1+D_2)]=S_1^{12}-S_2^{12}=(S_1^3)^4-(S_2^3)^4=1. \qquad (10)$$

From (9) we have

$$\exp(A_2+2B_2)=[\exp(-t_3+t_6-t_9)]^3. \qquad (11)$$

From (8) we have

$$\exp(A_2+2B_2)=(S_1-S_2)(S_1^2+S_2^2+S_1S_2)=S_1^3-S_2^3. \qquad (12)$$

From (11) and (12) we have Fermat's equation

$$\exp(A_2+2B_2)=S_1^3-S_2^3=[\exp(-t_3+t_6-t_9)]^3. \qquad (13)$$

Fermat proved that (10) has no rational solutions for exponent 4 [8].

Therefore we prove we prove that (13) has no rational solutions for exponent 3. [2]

Theorem . Let $4m = 4P$, where P is an odd prime, $(P-1)/2$ is an even number.

From (3) and (8) we have

$$\exp[A_1 + A_2 + 2H + 2\sum_{j=1}^{P-1} (B_j + D_j)] = S_1^{4P} - S_2^{4P} = (S_1^P)^4 - (S_2^P)^4 = 1 . \quad (14)$$

From (3) we have

$$\exp[A_2 + 2\sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = [\exp(-t_P + t_{2P} - t_{3P})]^P . \quad (15)$$

From (8) we have

$$\exp[A_2 + 2\sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = S_1^P - S_2^P . \quad (16)$$

From (15) and (16) we have Fermat's equation

$$\exp[A_2 + 2\sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = S_1^P - S_2^P = [\exp(-t_P + t_{2P} - t_{3P})]^P . \quad (17)$$

Fermat proved that (14) has no rational solutions for exponent 4 [8]. Therefor we prove that (17) has no rational solutions for prime exponent P .

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