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DISPROOF OF THE GENERALIZED CONTINUUM HYPOTHESIS IN THE SYSTEM OF ZF OR ZFC AND CANTOR'S SET THEORY

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ABSTRACT; Since Cantor made mathematical proof, it has been "believed" that sometimes cardinality of set and its subset become equal. But I discovered a good way to disprove this idea. This disproof is depending only one axiom of ZF or ZFC and Cantor's set theory. Some of old achievements will be forced to reconsider.

1.Introduction.....plan of this disproof

Most reader of this paper will wonder if it is possible to dsprove GCH(Generalized Continuum Hypothesis[1]) in the system of ZFC set theory though professor P.J.Cohen has proved that GCH is independent from ZFC set theory[1].

Following is the plan of this disproof.

Step1;Prove cardinality of set becomes always larger than cardinality of its subset even if it were infinite set.

Step2;Create a new set which has intermediate cardinality between power sets.⇔Disproof of GCH

Step1 is the new idea in the history of set theory. Cantor's definition of comparison of cardinality was not perfect. And some of old achievements about set theory has been done under this insufficient understanding, so they may lose value as works.

And also, only one axiom of ZF or ZFC[2] and Cantor's set theory[3] is used is the feature of this disproof(regarding the concept, "a map between two sets becomes one to one ⇔ cardinality of those two sets are equal" as the axiom of Cantor's set theory).

2.Proofs

2.1 Proof of "step1"

Cardinality of set is always truly larger than cardinality of its subset even if it were infinite set in the system of ZF or ZFC and Cantor's set theory.

Axioms;

(a) "A map *f* maps set A into set B by one to one" ⇔" Cardinality of set A and set B are equal"(Cantor's set theory)

(b) "Axiom of extensionality"(ZF or ZFC set theory)

Proof;

Suppose set A, B, C and they satisfy following condition.

(A⊃B)∧(A⊃C)∧(B∩C=[empty set]), in other words, (A=B∪C)∧(B∩C=[empty set]).

And also, define A', B', C' as duplication of A, B, C.

If a map f_1 maps B' into A one to one and a map f_2 maps C' into C one to one, composite map $f_3=f_1\circ f_2$ becomes many to one. But, f_3 is a map between A and A', so it must be one to one for the axioms of (a) and (b). This is inconsistent.

Therefore, f_1 does not exists in real and cardinality of A is always truly larger than cardinality of B(**Fig**).

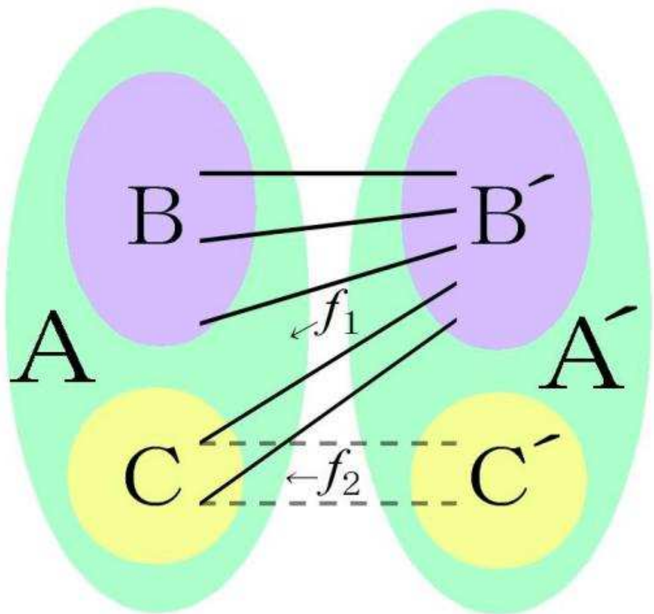

Fig. Inconsistent mapping

THEOREM1;

Irrespective of finite set or infinite set, cardinality of set is always truly larger than cardinality of its subset.

2.2 Proof of "step2"

Theme;

Disproove GCH in the system of ZF or ZFC and Cantor's set theory.

Axioms;

(a) "A map *f* maps set A into set B by one to one" $\Leftrightarrow$" Cardinality of set A and set B are equal"(Cantor's set theory)

(b) "Axiom of extensionality"(ZF or ZFC set theory)

Proof;

Define "P_n" as n-th power set. Such are, P_0=N, $P_1=2^N$, $P_2=2^{P_1}$, ..., $P_{n+1}=2^{P_n}$, ... And define $p_n = P_{n+1} \setminus \{1\}$.

For **THEOREM1**,

$P_n \subset p_n \subset P_{n+1} \Leftrightarrow \aleph_{n+1} \neq 2^{\aleph_n}$

GCH has been disproven.

3.Conclusion

"Cardinality of set and its subset sometimes become equal" is the false and insufficient idea of old era. The most important thing of this paper is that it was disproovable and disprooved from axiom.

References

[1]Wikipedia, "Continuum Hypothesis", https://en.wikipedia.org/wiki/Continuum_hypothesis

[2]Wikipedia, "Zermelo-Fraenkel set theory", https://en.wikipedia.org/wiki/Zermelo%E2%80%93Fraenkel_set_theory

[3]Wikipedia, "Georg Cantor", https://en.wikipedia.org/wiki/Georg_Cantor