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Gooey Ducks and Naked Physicists

Part L
Rectangular Reasoning

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Abstract: An allegory of modern science.

Part L

So…Bye, bye Miss Mathematical Pi?

What a cryin' shame! All that time and energy lost trying to resolve pi.
And now the startling truth (just the facts, ma'am?):

DON'T TRY TO RESOLVE PI AND EVERYTHING RESOLVES!

So much for "The Case of the Pernicious, Pestiferous Pi." Mystery solved! Case closed!
But can life go on without pi? Sure 'nuff! I already got me a new equation:

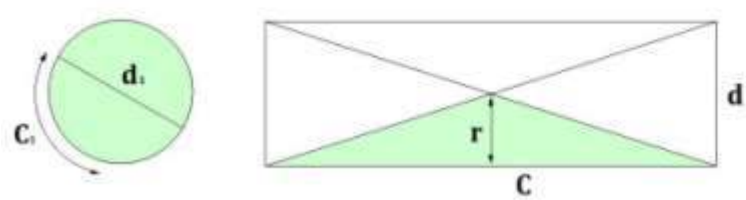
Take away pi and everything becomes a piece of cake!

Ahh…so close your eyes.
Take a bite, and let the velvety taste and texture take you to a place you never expected.
A place of wildly permissible indulgence. Of pure math-driven desire. That place without pi!

So, ready for somethin' soft and sultry, or maybe a little "hair of the dog" that bit you?
The old "pi" equations show the area of a circle, $A = \pi r^2$, to be one-fourth the surface area of a sphere, $A = 4\pi r^2$.

But as those equations don't resolve, if I "*take away pi*" by replacing π with $\frac{C}{d}$,
I can go from $A = 4\pi r^2$ (to $A = 4\frac{C}{d}r^2$, to $A = 4\frac{C}{2r}r^2$, to $A = 2\ C\ r$) to arrive at my new "no pi" equation for the surface area of a sphere, $A = C\ d$, that does resolve:

- Then, by drawing a rectangle with the sides as the lengths of the circumference and the diameter (to give me the surface area of the sphere),



- And drawing diagonals from the corners to create four triangles,
- As each triangle would be ¼ of the surface area of the sphere, each of the four triangles would be equal to the area of a circle!

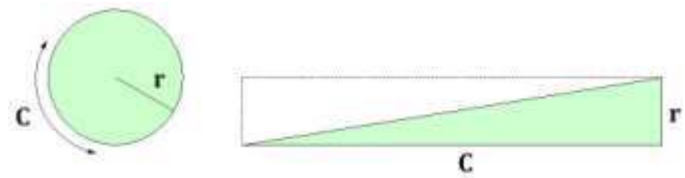
Whoop-te-doop, in one fell swoop!
A twofer!

With one simple diagram, I can "rectangle" the surface area of a sphere and "triangle" the area of a circle at the same time!

Eureka!

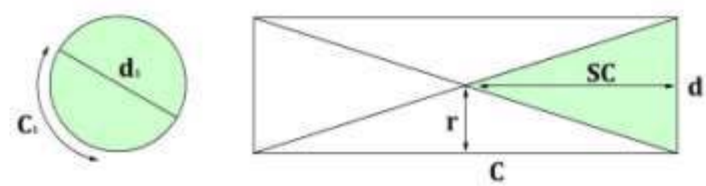
Wait just a doggone minute! I remember what our good friend Archimedes had to say:

The area of any circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius, and the other to the circumference, of the circle.



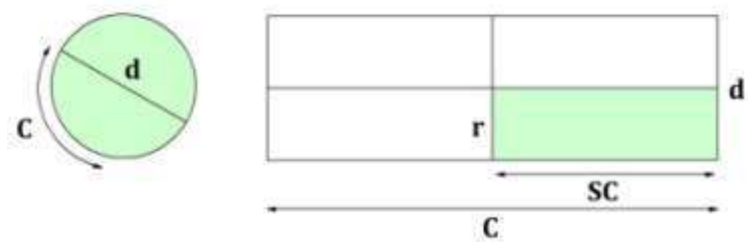
But comparing the two diagrams, you'll have to admit, Archie, your way isn't the only way! The area of a circle doesn't have to equal *only* a right-angled triangle:

The area of a circle is equal to *any triangle* with a base of the circumference and a height of the radius, and also to *any triangle* with a base of the diameter and a height of the semicircle!



Brain flash! What if–instead of triangles–I decided to cut my rectangle into rectangular quarters? As each ¼ of the surface area of a sphere equals the area of a circle:

The area of a circle is also equal to *any rectangle* with sides the lengths of the semicircle and the radius!



Man, I hear ya, Archie. There's certainly more than one way to skin a cat, huh?
But now the fun part: a multiple choice question! (My favorite kind!)

<u>Question: The area of a circle is equal to which of the following</u>:

A. A triangle with a base of the circumference and a height of the radius, $A = \left(\frac{1}{2}\right) C\, r$,
B. A triangle with a base of the diameter and a height of the semicircle, $A = \left(\frac{1}{2}\right) d\, SC$,
C. A rectangle with sides the lengths of the semicircle and the radius, $A = (SC)\, r$,
D. All of the above.

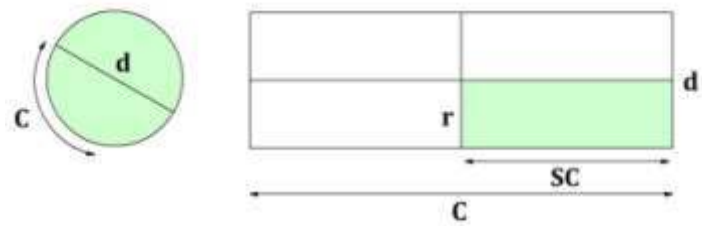
Times up! Easy peasy lemon squeezy! That one's just too easy! I'm gonna have to go with D! Yeah, they're all different ways of saying the same thing!

The area of any circle is equal to the semicircle times the radius:

$$\boldsymbol{A = SC\, r.}$$

So, what's next, Gertrude?

If I look at the diagram of the rectangle cut into rectangles, looks like each quarter (the area of a circle) is similar in proportion to the large rectangle (the surface area of a sphere)!

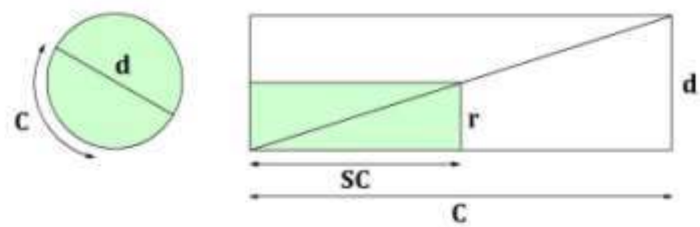


Sure, being based on the same proportions (the radius, diameter, semicircle, and the circumference) it makes sense they'd be proportional.

But can I prove that with a diagram? Lemme see:

Using one of the small rectangles and drawing a diagonal…Look at that, Thales!

I just converted the surface area of a sphere and the area of a circle into similar rectangles and triangles! Hooray! Hooray! What a day!



There's something to write home about! I think I just earned my math merit badge!

And the ratios that prove the similarity would be:

- The radius is to the semicircle as the diameter is to the circumference, $\left(\frac{r}{SC}\right) = \left(\frac{d}{C}\right)$,
- The radius is to the diameter as the semicircle is to the circumference, $\left(\frac{r}{d}\right) = \left(\frac{SC}{C}\right)$,
- The circumference is to the diameter as the semicircle is to the radius, $\left(\frac{C}{d}\right) = \left(\frac{SC}{r}\right)$, and
- The circumference is to the semicircle as the diameter is to the radius, $\left(\frac{C}{SC}\right) = \left(\frac{d}{r}\right)$.

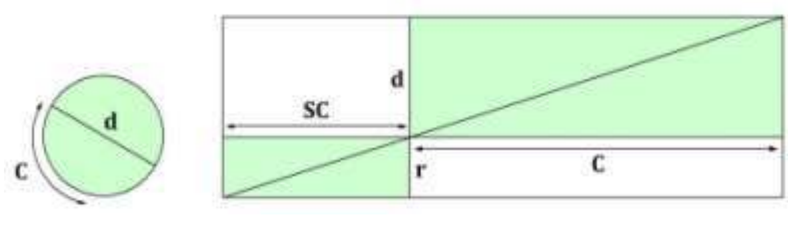
So any way you look at it, all the ratios end up as the same equation of proportionality:

$$(C)(r) = (SC)(d).$$

But by golly, that's an equation for a rectangle with sides of the circumference and the radius, $(C)(r)$, equal in area to a rectangle with sides of the semicircle and the diameter, $(SC)(d)$.

Huh, so how do I show these two new rectangles in a diagram? I got it!

If I slide my small rectangle down the diagonal…Bam! I get complementary "innie" and "outie" rectangles!



Wow! Picture-perfect proportionality! (Try to say that three times fast!)

I just realized…My diagram is the "In and Out Complementary Principle" from the ancient Chinese book, *Nine Chapters on the Mathematical Art.* How about them fortune cookies? Don't speak Chinese? No sweat, man! No need to try. Confucius say: "Man who run before bus get tired; man who run behind bus get exhausted." (Couldn't resist the Chinese chuckle!) So observe and learn, young grasshopper. The math diagram "speaks" for itself:

- The surface area of a sphere is equal to a rectangle with sides the lengths of the circumference and the diameter, $A = C\ d$.
- The area of a circle is equal to ¼ the surface area of a sphere, or a rectangle with sides the lengths of the semicircle and the radius, $A = SC\ r$.
- And the rectangle with sides the lengths of the semicircle and the diameter, $(SC)(d)$, and the rectangle with sides the lengths of the circumference and the radius, $(C)(r)$, are each equal to half the area of rectangle, $C\ d$:

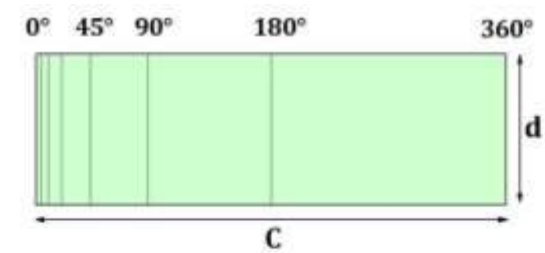
  $$(SC)\ (d) = \left(\frac{1}{2}\right) Cd, \text{ and } (C)(r) = \left(\frac{1}{2}\right) Cd$$

  Which means:

  $$(C)(r) + (SC)(d) = the\ surface\ area\ of\ a\ sphere = (C)(d).$$

Holy Huey, Dewey, and Liu Hui! I think I just made math history!
I've just applied the concept of complementary rectangles to the surface area of a sphere and the area of a circle! And the bonus? My "innie and outie diagram" gives me an easy way to determine the surface area of the whole, the half (the hemisphere), and the quarter-sphere. But what if I wanted to calculate an eighth, a sixteenth, etc.?

I got it! Now that I know the surface area of a sphere is equal to a rectangle (with sides the lengths of the circumference and the diameter, $C\ d$), I can use the rectangle, itself, to calculate any angular portion of the sphere! Sure, by dividing the rectangle into the same 360 degrees as the sphere: 180° would be ½ the rectangle so it'd be ½ the surface area of a sphere, 90° would be ¼ , 45° an ⅛ , etc.



Wow! I'm gellin' like Magellan! A new way to circumnavigate the globe and calculate the surface area of a sphere! 1° or 360°, and anything in between! And if you'll allow me a little, ahem, "latitude": *Will this change the world*? You bet your sweet bippy! Slick and quick! So are you ready, World? Yeah, let's combobulate, ambulate, let loose and…*Rectangulate*!