Quantized time and frequency

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Let $\{|\psi(t_0 + j\Delta t)\rangle : j \in Z\}$ be a set that remains to be made orthogonal. So it remains to show that $\langle\psi(t_0 + n\Delta t)|\psi(t_0 + m\Delta t)\rangle = \delta_{n.m}$ for all integers n,m. Therefore, we shall make it orthogonal as follows. We know that each function can be separated into time and space dependent functions.

$$|\psi(t_0 + n\Delta t)\rangle = e^{\frac{-iEn\Delta t}{\hbar}} |\psi(x, y, z)\rangle \tag{1}$$

Therefore, to force the orthogonality condition we find that,

$$\langle\psi(t_0 + n\Delta t)|\psi(t_0 + m\Delta t)\rangle = e^{\frac{iE(n-m)\Delta t}{\hbar}} \langle\psi(x, y, z)|\psi(x, y, z)\rangle \tag{2}$$

Where $e^{\frac{iE(n-m)\Delta t}{\hbar}} \langle\psi(x, y, z)|\psi(x, y, z)\rangle = e^{\frac{iE(n-m)\Delta t}{\hbar}}$ because we assume the time-independent solutions are already orthogonalized. Now, with this fact in hand we may show that

$$e^{\frac{iE(n-m)\Delta t}{\hbar}} = \delta_{n.m} \tag{3}$$

When $n = m$, the equation holds. We must use the Euler identity to force the equation for $n \neq m$, and set it equal to zero.

$$e^{\frac{iE(n-m)\Delta t}{\hbar}} = \cos\frac{E(n-m)\Delta t}{\hbar} + i\sin\frac{E(n-m)\Delta t}{\hbar} = 0 \tag{4}$$

There are two possibilities with the same outcome. First take the real part and we notice that $\frac{E(n-m)\Delta t}{\hbar} = \frac{2k-1}{2}\pi$. Let $\hbar = \frac{h}{2\pi}$ and say the energy (and in consequence the wavefunction) is that of a photon, so $E = jh\nu$. Plugging this into the real condition, we get:

$$4j\nu(n-m)\Delta t \in Z \tag{5}$$

Therefore $\nu\Delta t \in Z$. Now try for the imaginary condition: $\frac{E(n-m)\Delta t}{\hbar} = k\pi$. Plugging in the values for the photon, we have again $\nu\Delta t \in Z$. So we conclude that the timestep chosen Δt would be $\Delta t = \frac{n}{\nu} = nT$ where T is the period of the photon wave.

Consequently, when you write the wavefunction in terms of all its future, present, and past states, which are orthogonal, you retain your original function because $A_0 = \langle\psi(t_0 + n\Delta t)|\psi(t_0 + M\Delta t)\rangle = \delta_{n,M}$.

$$|\psi(t_0 + M\Delta t)\rangle = \sum_{j=-\infty}^{\infty} A_0 |\psi(t_0 + j\Delta t)\rangle \quad (6)$$

M, j an integer.