Time Dependence

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Abstract

A fundamental relationship involving two pairs of objects that all have some unique proper time relative to one inertial reference frame is studied. It is found that the proper times of any two objects added together are linearly dependent with any other two objects' proper times added together (where the objects' velocities are measured relative to the same reference frame).

The method of the proof is as follows: First, a theorem involving the proof of two separate conditional statements is shown to be true for both conditional statements. Then a truth table analysis of the fundamental statements involved in the theorem is conducted. A conclusion is reached by considering from all possible combinations of truths and falsehoods only the possibilities that agreed with the proved theorem and what was logically possible, i.e. a possibility was not considered if it declared the same entity to be linearly dependent and independent at the same time.

Theorem of Temporal Linear Dependence. 1. *Let there be four objects moving relative to some inertial frame with respective proper times* $\Delta t_1' = \Gamma_1 \Delta t_1$, $\Delta t_2' = \Gamma_2 \Delta t_2$, $\Delta t_3' = \Gamma_3 \Delta t_1$, $\Delta t_4' = \Gamma_4 \Delta t_2$, *where* $\Delta t_1 = \Delta t_2$ *is the time measured in the reference frame, and* $\Gamma_n = \sqrt{1 - \frac{v_n^2}{c^2}}$ *. If* $\Delta T_\eta = \Delta t_1' + \Delta t_2'$ *and* $\Delta T_\nu = \Delta t_3' + \Delta t_4'$ *are linearly independent, then* $v_1 = v_2$ *and* $v_3 = v_4$*. Also, if* $v_1 = v_2$ *and* $v_3 = v_4$*, then* $\Delta t_1' + \Delta t_2'$ *and* $\Delta t_3' + \Delta t_4'$ *are linearly dependent.*

First Conditional Statement. For the first conditional statment, consider the following system of equations:

$$\begin{aligned} \Gamma_1 \Delta t_1 + \Gamma_2 \Delta t_2 &= \Delta T_\eta \\ \Gamma_3 \Delta t_1 + \Gamma_4 \Delta t_2 &= \Delta T_\nu \end{aligned}$$

Solving for Δt_1 and Δt_2 and equating the two one arrives at:

$$\Delta T_\eta (\Gamma_4 + \Gamma_3) = \Delta T_\nu (\Gamma_1 + \Gamma_2). \quad (1)$$

Since for the first conditional statement ΔT_η and ΔT_ν are linearly independent, it follows that $\Gamma_1 = -\Gamma_2$ and $\Gamma_3 = -\Gamma_4$. Therefore $v_1 = v_2$ and $v_3 = v_4$. $\square$

Second Conditional Statement. For the second conditional statement, consider Equation **??** and that $v_1 = v_2$ and $v_3 = v_4$. It follows that ΔT_η and ΔT_ν are linearly dependent. □

Truth Tables. Now a truth table analysis is performed to ascertain the final result. Let P be the statement, ΔT_η and ΔT_ν are linearly dependent, let Q be the statement, ΔT_η and ΔT_ν are linearly independent, and let S be the statement, $[v_1 = v_2$ and $v_3 = v_4]$. There are 3 statements that can either be true or false. This means there will be $2^3 = 8$ possibilities or scenarios.

	P	Q	S	$Q \rightarrow S$	$S \rightarrow P$	$(Q \rightarrow S) \wedge (S \rightarrow P)$
1.	T	T	T	T	T	T
2.	T	T	F	F	T	F
3.	T	F	T	T	T	T
4.	T	F	F	T	T	T
5.	F	T	T	T	F	F
6.	F	T	F	F	T	F
7.	F	F	T	T	F	F
8.	F	F	F	T	T	T

First direct your attention to the last column in this table. We exclude all the scenarios that end up with the last column as false because we proved that both the conditional statments in the theorem are true previously. This excludes scenario 2, 5, 6, and 7. Next, scenario number 1 and 8 are excluded because they both involve either P and Q both being false or both being true. This would be a paradox, so they are excluded. The only remaining scenarios are 3 and 4. Doing a thorough study of the logical truth table involved in the three statements (scenario 3 and 4); Q: ΔT_η and ΔT_ν are linearly independent, P: ΔT_η and ΔT_ν are linearly dependent, and S: $[v_1 = v_2$ and $v_3 = v_4]$, it is found that ΔT_η and ΔT_ν are linearly dependent for any four values of v_n. This conclusion is reached by way of taking scenario 3 and 4 to be the only logically possible realities at the same time. □